The Government of the Republic of the Union of Myanmar

Ministry of Education

Year 1 Semester 2

EDU1206

Curriculum and Pedagogy Studies: Mathematics

Student Teacher Textbook

PREFACE

The Myanmar Ministry of Education developed the four-year Education College Curriculum, in line with the pre-service teacher education reform as specified in the 'National Education Strategic Plan' (NESP) 2016-2021.

The Myanmar Education College Curriculum consists of several components: the curriculum framework, syllabi, Student Teacher Textbooks, and Teacher Educator Guides. This curriculum for the four-year Education College was designed and structured to align with the Basic Education Curriculum and to equip student teachers with the competencies needed to teach effectively in Myanmar's primary and middle school classrooms. It is based on a Teacher Competency Standards Framework (TCSF) which articulates the expectations for what a teacher should know and be able to do in the classroom.

The curriculum follows a spiral curriculum approach, which means that throughout the four years, student teachers return to familiar concepts, each time deepening their knowledge and understanding. To achieve this, the four-year Education College programme is divided into two cycles. The first cycle (Years 1 and 2) is repeated at a deeper level in the second cycle (Years 3 and 4) to enable student teachers to return to ideas, experiment with them, and share with their peers a wider range of practices in the classroom, with the option to follow up on specific aspects of their teaching at a deeper level.

The curriculum structure provides an integrated approach, where teaching of subject knowledge and understanding educational theories are learnt through a supportive learning process of relevant preparation and practical application and experience. The focus is, therefore, not just on subject content, but also on the skills and attitudes needed to effectively apply their knowledge, skills, and attitudes in teaching and learning situations, with specific age groups. As the focus is on all components of a 'competency' – knowledge, skills, attitudes and their effective application – it is referred to as a competency-based curriculum.

Accordingly, a competency-based curriculum is learner-centred and adaptive to the changing needs of students, teachers, and society. Where new concepts are learnt, they are then applied and reflected on:

1. Learn (plan what and how to teach);
2. Apply (practise teaching and learning behaviours); and
3. Reflect (evaluate teaching practice).

Beyond the Education College coursework, it is intended that student teacher graduates will be able to take and apply this cycle of 'learn, apply, and reflect' to their own teaching to effectively facilitate the learning and development of Myanmar's next generation.

The Myanmar Education College Curriculum was developed by a curriculum core team, which is a Ministry of Education-appointed team of Myanmar Education College teacher educators supported by the Ministry of Education, resource persons from the Universities of Education, University for the Development of National Races of the Union and a team of national and international experts. Overall guidance of the work was provided by the Department of Higher Education, Ministry of Education.

The curriculum development was also supported by the Strengthening Pre-Service Teacher Education in Myanmar project, with technical assistance from the United Nations Educational, Scientific and Cultural Organization (UNESCO) and financial contributions from Australia, Finland, and UK Governments. Substantial input to the drafting process was also provided by Japan International Cooperation Agency and the primary education curriculum development team through the Project for Curriculum Reform at Primary Level of Basic Education (CREATE) team.

TABLE OF CONTENTS

HOW TO USE THIS TEXTBOOK

Who will use this Mathematics Student Teacher Textbook?

This textbook has been designed to guide you, as a student teacher, through Year 1 of the Mathematics subject. In this textbook, you will find foundational information about Mathematics. The textbook also includes learning activities and additional resources to help you develop the knowledge, skills and attitudes you need to be an effective teacher in Myanmar. You will use the textbook as a key resource in class; you can also use the textbook for independent self-study.

While the content in the textbook is addressed to you, as the student teacher, it is also a resource for your teacher educators, who will serve as your facilitators and mentors as you develop key competencies in Mathematics. Throughout this course, you and your teacher educator will work together, using this textbook as a tool for learning.

When and where does Mathematics take place?

The learning area of Mathematics has been allotted 72 periods of teaching for each year of your four-year Education College programme. Classes will be held on your Education College campus.

What is included in the Year 1 Mathematics textbook?

The textbook organisation and content align with the syllabus of the four-year Education College Curriculum on Mathematics.

This textbook contains the following topics for Year 1 Mathematics:

- Introduction to Mathematics
- Problem-Solving and Misconceptions
- Understanding Mathematics
- How We Learn and How We Teach Mathematics
- Mathematical Modelling and Representation

For each unit, you will be working through learning activities, both individually and with your peers and teacher educator, to deepen your knowledge, skills, and attitudes on the topic. The Content Map, in the table below, highlights the expected learning outcomes and time allocations for each unit in this textbook.

Table A. Year 1, Semester 2, Mathematics content map

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
4. How We Learn and How We Teach Mathematics	4.1 The Mathematics Curriculum (Primary)	4.1.1. Understand-ing the structure and contents of learning strands	• Describe the primary Mathematics curriculum structure and explain how the main learning areas are interconnected and link to the overarching subject objectives	A4.1	1
		4.1.2. The cur-riculum of whole numbers	• Explain the scope and sequences of 'Whole Numbers' in the curriculum and how they are described in the primary textbooks	A4.1	1
		4.1.3. Fractions and decimals	• Explain the scope and sequencing of 'Fractions' and 'Decimals' in the curriculum and how they are described in the primary textbooks	A4.1	1
		4.1.4. Geometry	• Explain the scope and sequences of 'Geometry' in the curriculum and how they are described in the primary textbooks	A4.1	1
		4.1.5. Measure-ment and Mathe-matical Relations	• Explain the scope and sequences of 'Measurement' and 'Mathematical relations' in the curriculum and how they are described in the primary textbooks	A4.1	1
		4.1.6 Review of whole numbers, fractions and decimals	• Explain reasons for the scope and sequencing of whole numbers, fractions and decimals in the curriculum	A4.1	1
		4.1.7. Review of Geometry and Measurement	• Explain the reasons for the scope and sequencing of 'Geometry and Measurement' in the curriculum	A4.1	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
	4.2. Planning Learning	4.2.1. The ingredients of a Mathematics lesson	• Use action verbs for writing learning outcomes	A4.1 B1.1 B1.3	1
		4.2.2. Planning a sequence of lessons	• Construct a sequence of lessons using appropriate teaching and learning strategies to achieve learning outcomes	A4.1 B1.1 B1.3	1
		4.2.3. Planning an introductory lesson	• Use action verbs for writing learning outcomes • Use appropriate teaching and learning strategies to achieve learning outcomes • Construct a lesson	A4.1 B1.1 B1.3	1
	4.3. Motivation	4.3.1. Understanding motivation: theory and practice	• Explain theory of psychology on motivation for primary school teaching • Identify some of the prerequisites for facilitating motivation in Mathematics classes	A1.1 A4.1 B1.1 B1.3	1
		4.3.2. Understanding motivation: understanding the implications of theory	• Explain theory of psychology on motivation for primary school teaching • Identify some of the points to consider and practice in the Mathematics classroom	A1.1 A4.1 B1.1 B1.3	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		4.3.3. Understanding motivation: cognitive development of children and teaching of Mathematics	• Describe key ideas about the development of thinking of children and of how they learn • Explain how particularly important Mathematical learning and thinking is for the cognitive development of primary school children • Discuss strategies to motivate primary students for effective teaching and learning • Demonstrate strategies to motivate primary students for active learning in your micro-teaching and peer-teaching	A1.1 A4.1 B1.1 B1.3	2
	4.4. Learning and Teaching Resources	4.4.1. Posters, charts, handouts and equipment	• Classify teaching and learning resources (in other words, handouts, worksheets, books, charts, model, representations, real objects and so on) • Demonstrate how to access teaching and learning resources from outside sources or develop them	A2.1 A4.1 A5.1	1
		4.4.2. What's on the Internet?	• Demonstrate the kinds of useful teaching and learning resources that are available for primary Mathematics on the Internet • Access useful teaching and learning resources for primary Mathematics from the internet	A2.1 A4.1 A5.1	1
		4.4.3. Collaboration: teachers' communities of practice	• Identify what communities of practice are and how they enable teachers to collaborate and share resources in a variety of ways including through social media	A2.1 A4.1 A5.1 D2.1	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
	4.5. Assessment for and of Learning	4.5.1. Assessing for and of learning: formative and summative	• Discuss what is assessment for and assessment of learning in primary Mathematics • Develop an assessment strategy for mathematics	A1.1 A4.1 B2.1	1
		4.5.2. Developing assessment instruments for Grade 2	• Develop formative and summative assessment instruments for Grade 2 primary Mathematics	A1.1 A4.1 A5.1 B2.1	1
		4.5.3. Developing assessment instrument for Grade 5	• Develop formative and summative assessment instruments for Grade 5 primary Mathematics	A1.1 A4.1 A5.1 B2.1	1
5: Mathematical Modelling and Rep-resentation	5.1. Algebra	5.1.1. What is algebra?	• Describe algebraic thinking • Explain that algebraic thinking can be developed from Grade 1, long before letters/ variables are introduced	A1.1 A2.1 A5.1 B1.1 B1.2	2
		5.1.2. Teaching algebraic thinking in the early grades: the notion of equality	• Describe that a sound conceptual understanding of the equal sign and equality is an important building block of algebra • Describe that some arithmetic activities develop a conceptual understanding of equality • Facilitate activities that develop a relational rather than a computational understanding of the equal sign	A1.1 A2.1 A5.1 B1.1 B1.2	2

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		5.1.3. Teaching algebraic thinking in the early grades: generalising arithmetic	• Generalise arithmetic from particular examples • Explain that generalised arithmetic is one aspect of algebraic thinking • Identify that arithmetic can be generalised without using variables (letters) • Explain how primary school students can generalise arithmetic	A1.1 A2.1 A5.1 B1.1 B1.2	1
		5.1.4 Algebra in the early years: generalising towards the idea of a function	• Generalise patterns with shapes • Identify that patterns can be represented in different ways • Demonstrate that different representations can make different aspects of a pattern visible • Represent patterns in pictures or diagrams, in words, in function tables and function diagrams and in equations and graphs • Teach children how to generalise from patterns with shapes	A1.1 A2.1 A5.1 B1.1 B1.2	1
	5.2. Geometry	5.2.1. Lines	• Teach primary school students about right angles, perpendicular lines and parallel lines • Teach primary students to fold right angles, perpendicular lines and parallel lines • Teach primary students to draw parallel and perpendicular lines	A1.1 A2.1 A5.1 B1.1 B1.2	1
		5.2.2. Circles	• Explain how a circle is defined, and name the parts of a circle • Draw circles with or without pairs of compasses and make patterns by drawing circles	A1.1 A2.1 A5.1 B1.1 B1.2	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		5.2.3. Triangles	• Classify and name triangles according to the sizes of their vertices • Classify and name triangles according to the relative lengths of their sides • Draw specified triangles on grid paper or with a pair of compasses or a circle-maker • Draw lines that connect dots or point where grid lines intersect, and understand that slanting lines are longer than vertical or horizontal lines between the same number of dots or points of intersection	A1.1 A2.1 A5.1 B1.1 B1.2	1
		5.2.4. Quadrilat-erals	• Classify and name quadrilaterals • Explain the relationship between different quadrilaterals, in other words, that some of subsets of each other • Facilitate students about quadrilaterals, trapezia, parallelograms, rectangles, rhombuses and squares	A1.1 A2.1 A5.1 B1.1 B1.2	1
		5.2.5. Pentagons and hexagons	• Identify both regular and irregular pentagons and hexagons • Use paper folding to make regular pentagons and hexagons • Construct hexagons and pentagons using a pair of compasses or a circle-maker card strip • Teach Grade 5 students about pentagons and hexagons	A1.1 A2.1 A5.1 B1.1 B1.2	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		5.2.6. Sum of interior angles of polygons	• Find the sum of the interior angles of a triangle • Teach primary students to find the sum of the interior angles of a triangle • Explain the importance in Mathematics of using known facts to find unknown facts	A1.1 A2.1 A5.1 B1.1 B1.2	1
	5.3. Time and Space	5.3.1. Using physical Mathematics (1)	• Understand how to use physical Mathematics to teach various aspects of the primary curriculum • Develop some hands-on exercises/ lessons to help students practise counting and making various polygons	A1.1 A2.1 A5.1 B1.1 B1.2	1
		5.3.2. Using physical mathematics (2)	• Have experience of learning in a co-constructive way • Develop some strategies for teaching the concept of time to young children	A1.1 A2.1 A5.1 B1.1 B1.2	1
	5.4. Data Handling	5.4.1. Using bar graphs and pie charts across subjects	• Understand the number and operations skills that are required to develop graphic pictures used in other subjects • Develop skills to read and analyse graphical representations in informative articles and reports • Develop some strategies for teaching the concept of analyse graphical representations to young children	A1.1 A2.1 A5.1 B1.1 B1.2	1
		5.4.2. Teaching tables and graphs in Grade 2	• Explain what primary students have to learn in the areas of ‘data collection and arrangement’ and ‘data analysis’ • Develop some strategies for teaching the concept to young children	A1.1 A2.1 A5.1 B1.1 B1.2	1
Total Number of Periods					36

The overall objective of Mathematics is to ensure that you are prepared to teach the Mathematics curriculum as defined for basic education in Myanmar. You will learn the academic standard equivalent to primary and middle school level in order to ensure a strong subject proficiency foundation for being effective teachers for primary school students (Education College Curriculum Framework, 2018).

Learning objectives for primary school students for Mathematics subject (primary Mathematics syllabus):

- To have basic mathematical knowledge and skills regarding numbers, quantities, geometrical figures and data representation (knowledge and understanding, skills);
- To reason and explain logically in problem-solving (thinking);
- To apply mathematical knowledge and skills to the problems in our daily life as well as in their learning; (knowledge and understanding, skills); and
- To appreciate the usefulness of mathematical ideas and approaches (attitude).

This course will prepare student teachers to equip primary students with Mathematics as a life skill for understanding the world we live in. The 21st century citizen needs the skills of Mathematics to solve problems in work and other life situations. Everyone has the potential to be a mathematician and teacher is the key in helping students achieve this. The vision for teacher education in Myanmar, is to produce graduates who over the course of their four-year degree have developed deep understanding and skills in collaborative problem-solving. Both academic (what we teach) and methodology (how we teach) content is important and is presented in an integrated approach in this course. In order to become teachers who can adapt their teaching to the needs of different students, student teachers will learn how to develop learning outcomes of subject topics so as to be able to create activities that will assist primary students' deeper learning. They will know how to create their own teaching aids making them appropriate for the age of students they will be teaching. With reference to the Educational College Curriculum Framework, in Year 1 and 2, student teachers are expected to develop fundamental knowledge of Mathematics and develop basic pedagogical content knowledge for teaching Mathematics. In Year 3 and 4, they will further develop deeper understanding of Mathematics subject knowledge and to gain a more systematic grasp of primary Mathematics curriculum, instruction and assessment.

The content of this textbook is based on the Myanmar Teacher Competency Standards Framework (TCSF), which articulates the expectations for what you should know and be able to do in the classroom. The teacher competencies in focus for the Mathematics course include:

Table B. Mathematics teacher competencies in focus

Competency standard	Minimum requirement	Indicators
A1: Know how students learn	A1.1 Demonstrate understanding of different theories of how students learn relevant to their age and developmental stage	A1.1.1 Give examples of how the students' cognitive, physical, social, emotional and moral development may affect their learning A1.1.2 Prepare learning activities according to students' level of cognitive, physical, social and emotional development
	A1.2 Demonstrate understanding of how different teaching methods can meet students' individual learning needs	A1.2.2 Identify focused and sequenced learning activities to assist students to link new concepts with their prior knowledge and experiences
A2: Know appropriate use of education technologies	A2.1 Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources	A2.1.1 Plan learning experiences that provide opportunities for student interaction, inquiry, problem-solving and creativity A2.1.2 Use teaching methods, strategies and materials as specified in the textbooks and additional low cost support materials to support student learning
	A2.2 Demonstrate understanding of appropriate use of Information Communication	A2.2.1 Describe the function and purpose of online and offline educational tools and materials to support the teaching and learning process
A4: Know the curriculum	A4.1 Demonstrate understanding of the structure, content and expected learning outcomes of the basic education curriculum	A4.1.2 Prepare lesson plans reflecting the requirements of the curriculum and include relevant teaching and learning activities and materials A4.1.3 Describe the assessment principles underpinning the primary curriculum

Competency standard	Minimum requirement	Indicators
A5: Know the subject content	A5.1 Demonstrate understanding of the subject matter to teach the subject/s for the specified grade level/s	A5.1.1 Describe the key concepts, skills, techniques and applications for the subjects covered in the grade levels taught
	A5.2 Demonstrate understanding of how to vary delivery of subject content to meet students' learning needs and the learning context	A5.2.1 Describe ways to contextualise learning activities for the age, language, ability and culture of students to develop understanding of subject related principles, ideas and concepts A5.2.2 Explain how lessons are contextualised to include localised information and examples related to the subject content, concepts and themes
B1: Teach curriculum content using various teaching strategies	B1.1 Demonstrate capacity to teach subject-related concepts and content clearly and engagingly	B1.1.1 Clearly explains the curriculum content and intended learning outcomes
	B1.2 Demonstrate capacity to apply educational technologies and different strategies for teaching and learning	B1.2.1 Use teaching methods and learning strategies appropriate for the class-culture size and type B1.2.2 Use knowledge of literacy and numeracy instructional strategies to support students' learning in different subject areas B1.2.3 Create opportunities for students to investigate subject-related content and concepts through practical activities
	B1.3. Demonstrate good lesson planning and preparation in line with students' learning ability and experience	B1.3.1 Plan and structure lesson to ensure all of the lesson time is used effectively B1.3.2 Provide lesson introductions to link new learning to prior learning, to engage students' interest and to motivate them in learning B1.3.3 Prepare focused and sequential learning experiences that integrate learning areas and are responsive to students' interests and experience B1.3.4 Use questioning techniques and examples to introduce and illustrate concepts to be learnt

Competency standard	Minimum requirement	Indicators
B2: Assess, monitor and report on students' learning	B2.1 Demonstrate capacity to monitor and assess student learning	B2.1.2 Use assessment information to plan lessons
D2: Engage with colleagues in improving teaching practice	D2.1 Improve own teaching practice through learning from other teachers and professional development opportunities	D2.1.1 Discuss teaching practices with supervisors, colleagues and willingly seek constructive feedback D2.1.3 Establish goals for own professional development as a teacher D2.1.4 Participate in professional activities conducted by recognised professional associations

Source: Myanmar Teacher Competency Standards Framework (TCSF), Beginning Teachers, Draft Version 3.2. (May 2019) (pp. 30-36)

How do I use this textbook?

You can use this textbook both for your own self-study and as an in-class resource for learning activities facilitated by your teacher educator. Each unit in the textbook includes:

Expected Learning Outcomes: These are listed at the beginning of each unit and at the beginning of each lesson. The expected learning outcomes indicate what you should know and be able to do by the end of the lesson or unit.

Learning Content: The learning content for each unit is broken down into units and lessons that cover subject content knowledge that is important for teachers to know.

Learning Activities: The learning activities included in the textbook are individual activities that you can do to help reinforce and deepen your knowledge and understanding of a topic. Your teacher educator will also facilitate learning activities during class. These may be individual, partner, small group, or whole class activities designed to help you achieve the learning outcomes for each lesson.

Review questions: You can use the unit review questions to test your own understanding of the unit content, or to help you study for an exam.

Unit Summary: At the end of the unit, there is a brief summary of the main points of the unit to help you review and remember the most important information.

Unit reflection: Taking the time to deliberately think about, or reflect, on what you have learnt will help you remember and apply that learning, and make connections with other subject areas and real-life. Each unit ends with some suggestions on how you can reflect and follow-up on what you have learnt in the unit.

Further reading: Each unit lists suggestions of additional resources on the topic. You can look these up in the library, on the internet, or in your Education College's e-library to learn more about the topic.

At the end of this textbook, you will find a **Glossary** with the definitions of words found throughout the textbook that might be new to you. These words are listed in alphabetical order. You will also find a list of all the **Bibliography,** which are the original sources of information used throughout the textbook.

Remember, your teacher educator is there to help facilitate your learning in this course. If there is material you do not understand in the textbook, be sure to ask your teacher educator, or your classmates, for help. As a student teacher, you are part of a community of collaborative learning within your Education College as you work – together with your peers and guided by your teacher educators – to earn your teaching qualification.

Unit 4

How We Learn and How We Teach Mathematics

The lessons in Unit 4 set out to provide some of the answers to the question posed by the title of the unit. The first sub-unit with seven lessons should provide you with a very good understanding of, in a good level of detail, the Mathematics curriculum at the primary level. In the sub-unit on Planning Learning, we progress from considering what goes into planning Mathematics lessons to planning a sequence of lessons to planning an inspiring introductory lesson. A series of four lessons on **Motivation** provides some theory and analysis and then focuses on how teachers could use music to work on motivating young students. In the sub-unit on Learning and Teaching Resources, we cover sourcing resources and making **posters**, **handouts** and equipment to use in the Mathematics classes. In the third lesson in this sub-unit, we explore what's on the Internet. Finally, we review various forms of assessment and understand practical application using problems and our knowledge to develop both formative and **summative assessments** for Grade 2 and Grade 5.

Expected learning outcomes

By the end of this unit, you will be able to:

- Describe the primary Mathematics curriculum structure and explain how the main learning areas are interconnected and link to the overarching subject objectives;
- Explain the scope and sequences of 'Whole Numbers' in the curriculum and how they are described in the primary textbooks;
- Explain the scope and sequencing of 'Fractions' and 'Decimals' in the curriculum and how they are described in the primary textbooks;
- Explain the scope and sequences of 'Geometry' in the curriculum and how they are described in the primary textbooks;

- Explain the scope and sequences of ‘Measurement’ and ‘Mathematical relations’ in the curriculum and how they are described in the primary textbooks;
- Explain reasons for the scope and sequencing of whole numbers, fractions and decimals in the curriculum;
- Explain the reasons for the scope and sequencing of ‘Geometry’ and ‘Measurement in the curriculum;
- Use action verbs for writing learning outcomes;
- Construct a sequence of lessons using appropriate teaching and learning strategies to achieve learning outcomes;
- Use appropriate teaching and learning strategies to achieve learning outcomes;
- Construct a lesson;
- Explain theory of psychology on motivation for primary school teaching;
- Identify some of the prerequisites for facilitating motivation in Mathematics classes;
- Identify some of the points to consider and practice in the Mathematics classroom;
- Describe key ideas about the development of thinking of children and of how they learn;
- Explain how particularly important Mathematical learning and thinking is for the cognitive development of primary school children;
- Discuss strategies to motivate primary students for effective teaching and learning;
- Demonstrate strategies to motivate primary students for active learning in your micro-teaching and peer-teaching;
- Classify teaching and learning resources (in other words, handouts, worksheets, books, charts, model, representations, real objects and so on);
- Demonstrate how to access teaching and learning resources from outside sources or develop them;
- Demonstrate the kinds of useful teaching and learning resources that are available for primary Mathematics on the internet;
- Access useful teaching and learning resources for primary Mathematics from the internet;
- Identify what communities of practice are and how they enable teachers to collaborate and share resources in a variety of ways including through social media;

- Discuss what is assessment for and assessment of learning in primary Mathematics;
- Develop an assessment strategy for Mathematics;
- Develop formative and summative assessment instruments for Grade 2 primary Mathematics; and
- Develop formative and summative assessment instruments for Grade 5 primary Mathematics.

4.1. The Mathematics Curriculum (Primary)

The set of seven lessons in this unit are designed to ensure that student teachers have a very good grasp of what is in the Mathematics curriculum in primary school. After examining the **learning strands,** we go through each strand and the descriptions of the topics in detail in order to cover the strand. We look at how these are dealt with in the textbooks and look at how the teaching of these topics is staged over the grades of primary education. Then, in two review lessons, we probe our understanding of contents and logic of the curriculum.

4.1.1. Understanding the structure and contents of learning strands

This lesson looks in detail at the strands.

Expected learning outcome

By the end of the lesson, you will be able to:

- Describe the primary Mathematics curriculum structure and explain how the main learning areas are interconnected and link to the overarching subject objectives.

Introduction

You should recall that there are four strands in new primary Mathematics: Number, Geometry, Measurement and Mathematical Relationships.

Learning activity 1: Group Work

In your groups, study and discuss the grade-wise objectives of the new primary Mathematics curriculum and analyse the structure of each strand and the learning contents included in each strand. Each student should complete the worksheet that follows. One representative from each group will make a presentation to the class after this discussion.

Number strand

- There are three sub-strands: whole numbers, fractions and decimals;
- For each type of number concept - addition, subtraction, multiplication and division will be learnt; and
- For the whole number, four operations will be learnt.

'Four operations' here means that primary students will solve mathematical problems which require combinations of addition, subtraction, multiplication and division. This combination of the four operations will start from Grade 3 after the students have learnt the basics of addition, subtraction, multiplication and division.

Geometry strand

- There are two sub-strands: plane geometry and solid geometry;
- Plane geometry will cover lines and angles, polygons, circles, and symmetry;
- There are mainly three types of polygon: triangles, quadrilaterals, and other polygons; (to include pentagon and hexagon);
- Right-angled triangles, isosceles triangles, and equilateral triangles will be learnt;
- Rectangles, squares, parallelograms, rhombus and trapeziums will be studied in the quadrilaterals units;
- Drawing circles will be learnt; and
- In solid geometry, solid figures (three-dimensional shapes) will be covered.

Measurement strand

- There are four sub-strands: Basic SI units, SI derived units, British units and Myanmar units;
- The basic SI units of length, weight and time will be learnt - mm, cm, m, km for length; g and kg for weight; and hours, minutes and seconds for time;
- SI derived units will cover area (cm^2, m^2, km^2), volume (litres, millilitres, cm^3, m^3), and angle (degrees);
- In British units, length will include inches, feet and yards and weight will cover pounds and ounces; and
- The Myanmar units of *taung* and *htwa* will be learnt in length. For weight, *peithar* and *kyathar* will be learnt.

Mathematical relations strand

- There are two sub-strands: data collection arrangement, and data analysis; and
- Data collection arrangement will cover tables, pictograms, bar graphs and line graphs. Data analysis will cover bar graphs, line graphs, and average.

Learning activity 2

Your teacher educator will distribute an answer sheet after the presentations. Compare the answers you filled in the worksheet and compare them with the answer sheet.

4.1.2.
The curriculum of whole numbers

Through examining the **grade-wise curriculum** and the Mathematics textbooks in this lesson, we should develop a good understanding of what should be covered in whole numbers and how to stage this over the grades.

Expected learning outcome

By the end of the lesson, you will be able to:

- Explain the scope and sequences of 'Whole Numbers' in the curriculum and how they are described in the primary textbooks.

Learning activity 1: Introducing the Lesson

You should recall the following points:

- There are three sub-strands: whole numbers, fractions and decimals; and
- For whole numbers, there are five sub-portions to learn; number concepts, addition and subtraction, multiplication and division, and calculations involving all four operations.

Grade-wise objectives

Grade-wise objectives show what children should be able to do by the end of each grade while the topic diagram details the steps of learning. The worksheet you will complete is designed to help you understand the overall scope and sequence of primary Mathematics curriculum based on the grade-wise objectives. To support your understanding of the sequence, some main learning steps which are not described in the grade-wise objectives (but described in topic diagram) are given. For those points, answers are shown on the worksheet.

Examining the textbooks

It can be difficult to understand the contents of the curriculum by only looking at the objectives. A thorough examination of the new textbooks that are currently available is useful.

Learning activity 2: Group Work and Presentation

1. Examine the scope and sequencing of whole numbers in the curriculum and in the textbooks.
2. Complete the worksheet while studying the grade-wise diagram and the textbooks. You are expected to discuss your findings while filling in the worksheet.
3. Be prepared to make a presentation of the findings.

The following are important points:

- The concept of whole numbers will be taught in all grades.
- In Grade 1, students will learn about the concept of number starting from 0 to 100. The number will increase gradually in upper grades, up to 1,000,000,000
- In Grade 5, students will learn about the least common multiple and greatest common factors.
- In Grade 1, after learning about the concept of numbers, students will learn addition and subtraction with those numbers.
- In Grade 2, students will learn about the concept of numbers up to 1000. Subsequently, they will learn addition and subtraction with those numbers
- In Grade 3, students will learn about numbers up to 10,000 and addition and subtraction up to that number
- In Grade 2, students will start to learn about multiplication. In Grade 3, multiplication is learnt up to 100,000
- In Grade 2, after learning the multiplication table of 10x10, students will learn about division within this table.
- After learning numbers up to 100,000, students will learn addition, subtraction, and multiplications.
- In Grade 3, students will learn multiplication with remainders within the multiplication table of 10x10 and outside said table within 100.
- In Grade 4, students will learn up to 4 digits / 3 digits within remainders.
- After learning addition subtraction, multiplication and division, students will learn how to solve problems involving all of four operations.

4.1.3. Fractions and decimals

We follow the same process in this lesson using the grade-wise curriculum and the textbooks as our references to understand what needs to be covered concerning fractions and decimals and the sequencing of topics across the grades.

Expected learning outcome

By the end of the lesson, you will be able to:

- Explain the scope and sequencing of 'Fractions' and 'Decimals' in the curriculum and how they are described in the primary textbooks.

Learning activity 1: Introduction

Recall the following points:

- Fractions and decimals are included in the numbers strand; and
- Fractions and decimals both contain the same four sub-portions: concepts, addition, subtraction, multiplication, and division.

Learning activity 2: Group work

In the lesson, you will study and analyse the scope and sequences of fractions and decimals in primary Mathematics and identify the features of this area. The grade-wise objectives show what children should be able to do by the end of each grade while the topic diagram shows the detailed steps of learning. To understand the overall scope and sequencing of primary Mathematics based on the grade-wise objectives you will need to study the topic diagram, the grade-wise diagram and the textbooks.

In your groups:

1. Study the grade-wise objectives of fractions and decimals and identify the scope and sequences by completing the worksheet; and
2. Your group will make a presentation of the findings later. Discuss your findings while you are filling in the worksheet.

The scope and sequencing of fractions and decimals

Fractions

- In Grade 3, students will start to learn about fractions.
- In Grades 3 and 4, students will learn about proper, improper, and mixed fractions.
- In Grade 3, after learning about the concept of proper fractions, students will learn about adding and subtracting fractions with the same denominator within the scope of the fractions.
- In Grade 4, after learning about the concept of improper and mixed fractions, students will learn about adding and subtracting fractions with denominators where one denominator is a multiple of the other.
- In Grade 3, students will learn about adding and subtracting fractions with the same denominator. In Grade 4, students will learn how to do the same operations with fractions where one denominator is a multiple of the other. In Grade 5, students will learn how to do the same operations with fractions with different denominators.
- After comparing fractions with different denominators, students learn addition and subtraction of improper and mixed fractions with different denominators.
- In Grade 5, students will learn about multiplying and dividing fractions.
- In Grade 5, students will learn about fractions as a division of two whole numbers and fractions having different denominators.

Decimals

- In Grade 4, students will start to learn about decimals
- In Grade 4, students will learn the concept of decimals up to the 100ths and their addition and subtraction.
- In Grade 5, students will learn about decimals up to the 1000ths.
- Subsequently, students will learn about rounding off decimals, and division of two whole numbers by rounding off the answer up to 100th place.
- In Grade 5, students will learn about multiplication and division of decimals.

4.1.4. Geometry

In this lesson, we will get to grips with the topics in geometry in primary education and how we approach these over the grades.

Expected learning outcome

By the end of the lesson, you will be able to:

- Explain the scope and sequences of 'Geometry' in the curriculum and how they are described in the primary textbooks.

Learning activity 1: Introducing the lesson

Recall the following points:

- There are two sub-strands: Plane geometry and solid geometry.
- Plane geometry contains four sub-portions: Lines and angles, polygons, circles, and symmetry.
- Solid geometry contains the sub-portion solid figures.

Learning activity 2: Group work and presentation

Your teacher educator will provide you a worksheet to complete this task. Note th following instructions:

1. In the worksheet, topics of each grade and their linkages are described.
2. Fill in the blanks on the worksheet, by referring to the given answers.
3. Look at the linkages carefully to understand the sequences.

Grade-wise objectives show what students should be able to do by the end of each grade while the topic diagram, which you studied in previous lessons, shows the detailed steps of learning. Some main learning steps which are not described in grade-wise objectives (but described in topic diagram) are given in the worksheet. For those points, answers are already shown in the worksheet. Look at the primary textbooks while analysing the scope and sequences.

The primary school textbooks

Use the textbooks to make sure that you are familiar with the contents of each objective. It can be difficult to understand the contents only by looking at the objectives. At this point, only Grade 1 and Grade 2 textbooks are available, so focus on the topics at these grades.

Scope and sequencing of geometry presentation

Plane geometry

- In Grade 1, students will start to learn about identifying triangular, rectangular and circular shapes in the surroundings.
- In Grade 2, students will start to learn about lines and angles from constructing right angles. This will lead to students drawing parallel lines and perpendicular lines, and describing the properties of parallel lines in Grade 4.
- From Grade 2 to Grade 5, after learning the properties of polygons, students will draw polygons.

- In Grade 2, students will learn about right-angled triangles, squares and rectangles after learning how to describe triangles and quadrilaterals based on the number of vertices and sides. (This point is not described in the grade-wise objectives, but it is described in the topic diagram).
- In Grade 2, after learning about right angles, students will learn about polygons with right angles such as right-angled triangles, squares and rectangles.
- In Grade 3, students will learn how to draw quilateral and isoceles triangles with a compass after learning how to draw circles with a compass.
- In Grade 3, students will learn how to draw circles
- In Grade 4, after learning about parallel lines, students will learn about polygons with parallel lines such as parallelograms, rhombuses and trapezia.
- In Grade 4, students will learn how to distinguish between a pair of figures which are line symmetric.
- In Grade 5, students will learn how to draw polygons after learning how to describe polygons including pentagons and hexagons. Students will learn how to explain the rule on the sum of interior angles of polygons.

Solid geometry

- In Grade 1, students will learn about solid figures by starting to look at polygons in the student's surroundings. Students will also learn to identify shapes like boxes, cans, and balls.
- In Grade 3, students will learn about cubes and cuboids and their nets (newly introduced topics).

4.1.5. Measurement and mathematical relations

Lastly, we look at the measurement and mathematical relations strands by reviewing the grade-wise curriculum and how the topics are presented in the textbooks.

Expected learning outcome

By the end of the lesson, you will be able to:

- Explain the scope and sequences of 'Measurement' and 'Mathematical relations' in the curriculum and how they are described in the primary textbooks.

Learning activity 1: Introducing the lesson

You should recall the following points:

- There are four sub-strands in the measurement strand: Basic SI unit, SI derived unit, British unit and Myanmar unit.
- In the sub-strand of basic SI unit, there are three sub-portions, which are length, weight and time.
- In the sub-strand of SI derived unit, there are three sub-portions, which are area, volume and angle.
- In the sub-strands of British unit and Myanmar unit, there are two sub-portions, which are length and weight.
- There are two sub-strands in the mathematical relations strand, which are data collection and arrangement and data analysis.

Grade-wise objectives

Grade-wise objectives show what students should be able to do by the end of each grade while the topic diagram details the steps of learning. The worksheet you will complete is designed to help you understand the overall scope and sequence of primary Mathematics based on the grade-wise objectives. To support your understanding of the sequence, some main learning steps which are not described in grade-wise objectives (but described in topic diagram) are given. For those points, answers are already shown in the worksheet.

Examining the textbooks

It can be difficult to understand the contents of the curriculum by only looking at the objectives. A thorough examination of the new textbooks that are currently available is useful.

Learning activity 2: Group Work and Presentation

Examine the scope and sequencing of measurement and mathematical relations in the curriculum and in the textbooks. Complete the worksheet while studying the grade-wise diagram and the textbooks. Be prepared to make a presentation on your worksheet and findings.

The following are important points:

Basic SI units

- From Grade 1 to Grade 4, students will learn about basic SI units.
- In Grade 1, students will start by learning how to compare the lengths of objects, and measurement with arbitrary units (see topic diagram). In Grade 2, students will learn about millimetres, centimetres, and metres. In Grade 3, students will learn about kilometres.
- In Grade 4, students will learn how to calculate the perimeters of polygons
- In Grade 2, students will learn about weight by comparing the weight of objects. In Grade 3, standard units of grams and kilograms will be learnt.
- In Grade 1, students will learn about time by starting to read “o’clock’ and ‘half past’.
- In Grade 2, students will about time duration with hours and minutes, and the relationship between day and hour. In Grade 3, students will learn about time duration with seconds (see topic diagram).

SI derived units

- Learning of SI derived unit should occur between Grade 2 and Grade 5.
- Learning of area starts from rectangular area using cm^2, m^2 and km^2 in Grade 4. Triangular areas and quadrilateral areas will be learnt in Grade 5.
- Learning of volume starts from litres in Grade 2, millilitres in Grade 3, and cm^3 and m^3 will be learnt in Grade 5.
- Students learn comparing volumes first, then learn litre as a standard unit of volume in Grade 2. Learning of measurement starts by comparing things.
- Learning of angles starts by comparing the sizes of angles in Grade 3, while and Grade 4 students measure and draw angles up to 360^o by using a protractor.

British units

- Learning starts with inches and feet in Grade 2, and students will learn how to use inch, feet, and yard in Grade 4.
- Pounds and ounce will be learnt as British unit of weight in Grade 4.

Myanmar units

- *Taung* and *htwa* will be learnt as Myanmar units of length, *Peithar* and *kyathar* will be learnt as Myanmar units of weight, all in Grade 3.

Data collection and arrangement

- Learning of data collection and arrangement takes place from Grade 2 to Grade 4.
- Students will learn tables and pictograms in Grade 2, bar graphs in Grade 3, and line graphs in Grade 4.

Data analysis

- Data analysis is taught from Grade 3 to Grade 5.
- It starts with simple analysis using bar graphs in Grade 3, and becomes more complicated in upper grades. Grade 4 students will learn to interpret line graphs in terms of the changes with time, and Grade 5 students will learn to analyse given data with use of the idea of average.

4.1.6. Review of whole numbers, fractions and decimals

You have now studied the strands and sub-strands of the Mathematics curriculum in detail for primary school. In the following two lessons, you will discuss some questions to ensure that you have a good overview and understanding of this curriculum, and are able to analyse why topics are sequenced in the way they are over the years of primary education.

Expected learning outcome

By the end of the lesson, you will be able to:

- Explain reasons for the scope and sequencing of whole numbers, fractions and decimals in the curriculum.

The following two lessons are designed for you to review the new curriculum focus of the previous lessons. In this lesson, you will review whole numbers, fractions and decimals.

Learning activity 1: Discussion questions

During the lesson, discuss these questions in your groups and in the class. After the lesson as a homework assignment, complete an individual assignment recording your answers to the questions.

Number concepts

1. What do you understand about number concept?
2. Why do you think that number concept is studied throughout the grades?
3. What are students expected to know and be able to do in the development of number concept?

Addition, subtraction, multiplication and division

1. What are students expected to do and to know while learning addition and subtraction?
2. Why do you think that there is no addition and subtraction in Grade 4 and 5?
3. What are the calculation strategies suggested by the syllabus for each grade? It will be useful to use a textbook to answer the question.

Fractions and decimals

1. Generally, in the primary Mathematics syllabus, decimals are only learnt in Grade 3. Why do you think this is?
2. What content do students need to know in fractions in order to work with decimals?
3. Fractions and decimals are often taught separately. Examine a textbook to show how the relationship/connection between the two is suggested.
4. The understanding of place value is the key to working effectively with decimals. Where in the syllabus is the teaching of place value done? Examine a textbook to show the place value of decimals can be taught.

4.1.7. Review of geometry and measurement

In this last lesson on this unit, we will review the geometry and measurement covered in the primary curriculum and test your understanding of the scope and sequencing.

Expected learning outcome

By the end of the lesson, you will be able to:

- Explain the reasons for the scope and sequencing of 'Geometry and Measurement' in the curriculum.

Learning activity 1: Group work

In this lesson, you will reflect on the curriculum for geometry and measurement. Reflect on the questions that follow in the text. During the lesson, discuss these questions in your groups and as a class. After the lesson for homework, complete an individual assignment recording your answers to the questions.

Lines and angles

- What does a student have to know and be able to do by the end of Grade 4 with regards to lines and angles?

Polygons

- Circle all the verbs in the curriculum description for geometry. What do you think these verbs mean in the teaching of geometry? Can you suggest other verbs with similar meanings?
- What do students have to know, in terms of properties and prior knowledge, in order to:
  - Describe triangles and quadrilaterals based on the number of vertices and sides.
  - Classify triangles based on angles and sides.
  - Describe the properties of parallelograms and rhombuses.
  - Describe polygons.
  - Explain the rule about the sum of interior angles of polygons.
- In each grade, how are students expected to draw these?

Circles

- Why do you think the syllabus suggests the introduction of circles be taught in Grade 4?
- What are the given conditions for students to learn how to draw circles? Check the grade-wise section by strands.

Symmetry

- Why do you think that the teaching of symmetry only begins in Grade 3 and not earlier?
- What does the verb 'distinguish' mean in the teaching of symmetry? What are students expected to do? Use a Grade 3 textbook to help you answer this question.

Cubes and cuboids

- What do students have to know in order to identify cubes and cuboids?
- The building of nets allows a focus on the properties of cubes and cuboids. What are these properties?

Length, weight and time

- What do you think students learn by comparing the length of objects?
- What do you think students learn from comparing the weight of objects?
- What everyday contexts do the textbooks suggest we use in the teaching length, weight and time?

Area

- The study of area is introduced in Grade 4. Why do you think this is?
- What prior knowledge do students need before they are able to solve problems on rectangular area using such units as km2, cm^2 and m^2?

Polygons

- Examine Grade 3 and 4 textbooks to see how students are expected to use units of volume?
- What everyday contexts do the textbooks suggest we use in the teaching of volume?

Angles

- The teaching of angles is done in measurement. Students also need an understanding of angles in geometry. How is this knowledge and understanding used in geometry?
- Students are required to measure and draw angles up to 360 degrees. List all the angles that they should know and be able to draw.

Review questions

1. List the strands of the Mathematics curriculum in Myanmar.
2. Explain how the curriculum is organised for the measurement strand.
3. When teaching a topic, what should a teacher pay attention to in the curriculum?

4.2. Planning Learning

Now that we are familiar with the curriculum for primary school Mathematics and the sequencing of topics throughout the years, we cant tackle the topic of planning learning. In the first of these three lessons in the sub-unit, we will construct a framework of questions and matters to consider when planning. In the fourth lesson, we will plan a series of lessons around a topic. In the final lesson, we will plan in detail the introductory lesson to this series.

4.2.1. The ingredients of a Mathematics lesson

In this lesson, we will explore the key questions to be posed when planning lessons and the key ingredients of lesson plans. We will explore the range of action words that are used in composing learning objectives for Mathematics lessons.

Expected learning outcome

By the end of the lesson, you will be able to:

- Use action verbs for writing learning outcomes.

Learning activity 1

You will review the grade-wise objectives of the primary Mathematics curriculum and the section of the Grade 2 textbook covering the topic you have been assigned.
Once you have been assigned a topic from the Grade 2 curriculum:

1. Review the grade-wise objectivities of the primary Mathematics curriculum and the section of the Grade 2 textbook covering the topic you have been assigned.
2. List as many action words in the document and text as you can find.
3. Create two flipcharts, one with general learning outcomes from the curriculum and another with mathematical action words and explain these actions with examples of learning activities that could be used for each of these action words.

Learning activity 2: Group work

Now review the curriculum and the textbooks, and explain the structure and the learning contents of your allocated topic. Consider the following points and answer the questions:

Learning outcomes

- Planning learning starts with planning learning outcomes. To do so we must use a range of verbs to describe, at the outset, what kinds of learning activities are best suited to meeting the expected outcomes?
- There are a range of general verbs that we might use across all subjects. You will remember these from your exploration of Bloom's taxonomy. They are verbs that depict a hierarchy of thinking skills, words like identify, understand, explain, explore, describe, analyse.
- For Mathematics, there is a special set of verbs, such as calculate, simplify, solve, compute, differentiate, prove. Mathematics teachers need to be familiar with both sets of verbs.

The curriculum

- When planning lessons for a year or a term, a month or a week, teachers need to consult the curriculum. You should use the Mathematics textbook to get an idea of what is covered and to get guidance on your approach.
- Write a set of learning outcomes for a topic using appropriate verbs. If you were planning to teach the topic over five lessons, what would the title be for each lesson, and what would the specific learning outcomes be for these lessons?

Assessing learning

- What do students need to know before embarking on this topic?
- What do students need to be able to do as the result of learning?
- How are teachers able to know whether students have the prior skills and knowledge?
- How do teachers know whether students have grasped what you set out to teach?

Methodologies

- What variety of approaches and methods are good ways to teach the various aspects of the topic? What will you need to prepare (equipment, handouts, and so on)?
- Do you have the right balance of activities to keep students interested and also to give them sufficient time to grasp the ideas and to practise?

Changing plans

- Is it ever good and/or necessary to change your plan?
- What factors would make changes necessary?
- What would you do if you run out of time in a lesson?
- What would you do if some students are finished while others are still working?

4.2.2. Planning a sequence of lessons

In this lesson, we will explore the key questions to be posed when planning lessons and the key ingredients of lesson plans. We will explore the range of action words that are used in composing learning objectives for Mathematics lessons.

Expected learning outcome

By the end of the lesson, you will be able to:

- Construct a sequence of lessons using appropriate teaching and learning strategies to achieve learning outcomes.

Learning activity 1: Introducing the lesson

Considering your learning in Unit 4 so far, in your opinion, waht would make a good Mathematics lesson?

Learning activity 2: Group work and presentation

Using the learning outcomes, you developed in the previous lesson, you will continue to work in groups to plan the series of lessons in more detail. You should consider the content and approach for each lesson to ensure students are interested and engaged. You should also consider all the questions raised in the previous lesson.

4.2.3.
Planning an introductory lesson

In this lesson, you will design, describe and set out in detail a 40-minute lesson that should be the introductory lesson to the series of lessons you planned for in the previous lesson.

Expected learning outcomes

By the end of the lesson, you will be able to:
- Use action verbs for writing learning outcomes;
- Use appropriate teaching and learning strategies to achieve learning outcomes; and
- Construct a lesson.

Learning activity 1: Introducing the lesson

- What do you remember about lesson planning from the last lesson?
- What elements of the planning process do you find challenging and what do you feel confident doing?

Learning activity 2: Individual work and presentation

In this learning activity, you will develop a plan an introductory class. Consider the following as you develop your plan:

- This is the first time you have taught the topic. In considering your approaches, you can start by looking at the textbooks available to you.
- Consult with other student teachers and review each other's plan. Make suggestions.
- Conduct research on the internet to get ideas.
- Use your group's plan to cover the topic.

Review questions

1. When writing learning outcomes, teachers should use verbs. For writing learning outcomes for Mathematics, what verbs should be considered?
2. Writing a lesson plan and considering different teaching and learning approaches is a very important part to ensure effective teaching and learning. Where could you get inspiration and ideas?

4.3. Motivation

Mathematics teachers are challenged to motivate their students to be the best mathematicians they can be, and to be interested in, and intrigued by Mathematics. This involves, as has been covered extensively in the first semester, a range of teaching strategies and approaches that engages students and develops their mathematical thinking. However, teaching methods alone are not sufficient. The four lessons that follow in this sub-unit explore some theories of motivation and how teachers can teach while being conscious of motivation, the implications for Mathematics teachers of the level of cognitive development of students in psychology and some creative ways to motivate and teach using music.

4.3.1. Understanding motivation: theory and practice

In this introductory lesson, you will read, review and discuss a paper that seeks to summarise theory and practice in the area of motivation. The lesson is designed to cover the key prerequisites for creating a motivating classroom and sets the tone for the lessons that follow.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Explain theory of psychology on motivation for primary school teaching; and
- Identify some of the prerequisites for facilitating motivation in Mathematics classes.

Introduction

In this introduction lesson, we consider a perspective on what motivation is and what its roots are in human psychology. In the three lessons that follow, we will consider how motivation reaches its greatest positive objective only if it is appreciated by all those involved in the process of teaching and learning.

Learning activity 1

Think back to your Mathematics lessons at school. What motivated you to work hard in those lessons in primary school? Perhaps you were not motivated to work hard – if so, why were you not motivated?

Increasing motivation

Once we have a clear perspective on the psychology of motivation, we will look at using methods like **'Physical Mathematics'** to contribute to the motivation of young students. Young students usually hold authority figures in high esteem, and, as such, are more likely to use them as models. For this reason, it is important that teachers are also motivated to participate in the lessons and learning. The lessons must therefore work in the **zone of proximal development** (Vygotsky, 1978) of all participants. If teaching and learning is to reach growth objectives, lessons that motivate must:

1. Engage with what both students and teachers already know;
2. Recruit students and teachers to learn from the 'other'. Although teachers will guide the lessons, students and teachers build on knowledge of one another. Sometimes the teacher fills the role of a 'student' and vice versa; and
3. The outcome for every student and every teacher, as well as the class culture, should be positive and help growth.

What is motivation?

Leadership-central (2018) defines motivation as 'a general desire, need or want that generates the energy required for someone to behave in a particular way. This means that there's actually a motive behind the action, which is really an impulse that will trigger the motivational cognitive process which will lead an individual to act on that impulse.'

Motivation is different from 'agency'. Agency comes from the conscious perception of the elements in a situation. When we experience our agency, then we use our will to make a choice and direct behaviour. (Wilbraham, 2004).

Motivation involves elements of which one is conscious, as well as those which operate beneath consciousness, such as impulses, that which is taken for granted, but not questioned or considered consciously. The definition of motivation shows that all living beings are motivated to behave in certain ways, but does not suggest *why we are motivated at all*. It is proposed that motivation is core to existence and that because human beings are social, this motivation relates to physical and psychological acknowledgement.

Survival: The root of motivation

To understand and work with the motivation of primary school students, we must understand the interaction between opposites. In learning and teaching, students often experience 'stuck-ness' (Oakley, 2014) before they are able to learn and grow. Motivation is intimately related to survival and the need for recognition. Children, like all humans including teachers, fear alienation and want to be recognised. This is why the process of motivated learning and teaching is so important. This is not only significant in relation to the mental and physical health of the child, but also to the teacher, the class dynamic, the community and society.
The concepts of recognition and alienation are seldom consciously thought or even spoken about, except in moments of unusual achievement and/or isolation in moments of crisis. Neither of these concepts can be reduced solely to an internalised, individual struggle, nor can they be understood as operating separately from each other. Sembou uses Hegel's conception of the struggle for recognition by defining it as: 'By the term "struggle for recognition" is meant the struggle of individuals for the recognition of their person by others.' (Sembou, 2003). This concept requires the 'other' to see and value that which is

unique and real. A similar concept about the struggle for recognition is that of Ubuntu, an African philosophical concept. Ubuntu has been defined by the critical psychologist Nhlanhla Mkhize as: '[the] concrete or practical realisation of the knowledge that the possession of the qualities of personhood is reflected in people's relationship with others. *Ubuntu* is characterised by caring, just and respectful relationships.' (Mkhize, 2004).

Mamphele Ramphele (2002) says of *Ubuntu* that 'one's personhood makes sense only in relation to others'. The meaning Ramphele ascribes to *Ubuntu* emphasises the significance of integrating individual uniqueness *with* common good, rather than one at the expense of the other. Ramphele's meaning concurs with Nick Crossley's conception that consciousness of self can only occur during the process of also becoming conscious of the other, that which is not self: 'A consciousness of self, in other words, always already entails a sense of the other' (Crossley 1996). The ability to relate to self and other is both a cause and a consequence of each other – consciousness of self and other is a mutually constitutive process. Thus, the success of *ubuntu* (and recognition) as a matter of human health, growth, learning and ultimately well-being demands that the needs of the individual and collective (at all levels) must be integrated rather than one being realised at the expense of the other.

Alienation is the experience emerging during withholding of recognition, or following negation. It is an experience of having little to no personal agency over one's life. Alienation is reflected in the Marxist concept, cited in Hook as a 'loss of reality, to the situation where human beings are estranged from their own bodies, from the natural world and from their potentially universal essences.' (Hook, 2004). The dynamic relationship between this struggle for recognition, and its opposite, the fear of alienation, is inevitable within human relationships (Crossley, 1996; Hoffman, 2017). This is particularly clear in the learning and teaching situation. Motivating lessons seek to bring out that which is real and unique in students and teachers in order to remove a culture of alienation between students, teachers and learning content so that it is possible for recognition to occur.

Different kinds of motivation

Motivation can have destructive or constructive results. For example, a criminal requires motivation and this mostly has destructive results. Destructive motivation is strongly influenced by feeling a hidden shame (Gilligan, 2003; Hoffman, 2010). Hidden motivations powerfully affect the relationships of individuals and societies which culminate in violations against self and others. In other words, Gilligan (1996, 2003) claims that substantial aspects of violations are often deeply embedded in complex, covert and disguised (from self and other) emotions of shame and thus are, by definition, often lacking in agency.
On the other hand, wise leadership also requires motivation, but has constructive results. In general, destructive motivation relies on disrespect or alienation of the other, while constructive alienation relies on respect or acknowledgement of the other as an integral part of personal growth (Hoffman, 2017).

Growth of constructive motivation

A healthy body and a healthy mind create conditions necessary for to grow from constructive motivation. This section suggests a model of identity as a basis to provide constructive motivation for growth and learning within the zone of proximal development of both teachers and students. Our identities capture the way in which we know ourselves, the way in which others know us, and the way we would like (and not like) to be known. An identity captures our perceptions of our pasts, our present and our futures. Another's knowledge of our identity captures recognition and builds confidence. Their engagement with it in a respectful manner is most likely to recruit us to use that confidence to learn and grow in confidence.

For practical purposes, a model of identity includes five elements: spiritual self (the way in which we are unique, the feeling of purpose); physical self (what we eat, exercise); emotional self (the feelings which play into being motivated to do or not do something); creational self (constructive, creative ways in which we leave unique footprints in the world); and intellectual self (how we think, the intellectual logic we apply to our decisions) (Copeland & Harris, 2000). All these aspects are parts of who we are, and play an active role in whether we are motivated to learn or stay stuck.

All of these elements have ‘hungers’. To grow them, we need to satisfy their needs. By feeding all the elements on a balanced level, we are more likely to experience overall health and well-being in all areas, resulting in growing confidence. Confidence increases the likelihood that students will want to be recruited into learning.

Engage with what students and teachers already know

The core of what each person knows, both as a member of the social structure (what they have in common), as well as the way in which they are unique (the unique purpose everyone has which can contribute to the building and growing of families, schools, communities and societies) lies within their identities. Identities consist of what we are born with and what we learn from relationships as we grow. There are three tiers of what we know about ourselves, and therefore show to others:

- Tier one is explicit knowledge that can be recalled and produced when asked. This part is relatively easy to acknowledge, to show, and to work with.
- Tier two is unconscious. It is part of what we take for granted and seldom question. This is more difficult to work with because it is known only at a deeper level and more difficult to access. It takes personal reflection, mindfulness and the existence of safe relationships to discover.
- Tier three - the most difficult part of what we know constitutes that which we deny and do not want to know or claim as part of ourselves. This takes courage to come to know. Even though we practise denial, it does not disappear. It arises and is met at a personal level with anxiety and depression, and on a social level with defences such as withdrawal and explosive behaviour. When this level is triggered, there is motivation to prevent this knowledge from becoming accessible both to ourselves and to others.

Motivation maintains and grows a more constructive path when unconscious elements are brought to the surface, and more of its elements start to operate on a conscious level. Fear of failure and punishment encourages withdrawal and ‘stuck-ness’. Constructive motivation is enhanced when potentially shameful incidents are worked with directly and respectfully (Hoffman, 2010). To enable the depth of engagement and recognition of another necessary to motivate for learning and growth, trust is essential. Trust is created in relationships mainly by ensuring containment, and removal of judgment and condemnation.

Recruit students and teachers to consciously learn from the other

Teachers contain and guide the lessons. Nonetheless, students and teachers can be recruited to learn from the 'other' and to build on the knowledge of the other.

Since students and teachers want recognition and fight against alienation, the quality of their interaction and learning occurs in a mirror-like fashion. In other words, students and teachers *do* learn from each other. When teachers are bored, it is more likely that students will become bored and vice versa. When students are passionate, it is more likely that teachers will be more passionate about teaching them, and vice versa.

Sometimes, an aspect of teaching intended to motivate has an opposite result. For example, routine can be intended to enhance safety and containment. In practice, it can lead to 'stuck-ness' when the value of routine is not balanced with the value of the possibilities of newness, creativity and growth (Oakley, 2014).

When teachers are unaware of how much their level of motivation affects the class, it is more likely that they will blame the students for lack of learning, thereby halting growth. When teachers become explicitly aware of how powerfully they affect motivation in children, and that their power to motivate depends crucially on how well they know their students, teachers are more likely to listen to the voices, needs and stories of the children they teach. When students experience the recognition that comes from their teacher actively engages with them, they are more likely to model the behaviour of their teacher, and are more likely to actively listen to and respect others (teachers, classmates and then, society).

Learning from the other will only happen if it is safe to do so. If it is unsafe, students and teachers will defend against possible threats. Defence involves 'closed-ness' while learning requires 'openness' to new knowledge (Crossley, 1996).

For teachers to be motivated and feel passionate about their work, they need to know that they will benefit from more than just a salary. When salary alone is the benefit, teaching can become routine and teachers can enter the 'zombie' zone. The culture of 'stuck-ness' is easily transferred to students. To enhance the openness necessary to motivate learning, trust is essential. Trust emerges when students and teachers feel safe both physically and psychologically. Threats such as fear of failure, shame, punishment and humiliation must be minimised.

Learning activity 2: What is motivation?

When you have read the text above, discuss in groups the main ideas presented under the following topics and capture these ideas on a flipchart.

a. What is motivation?
b. The roots of motivation.
c. Different kinds of motivation
d. Growth of constructive motivation.

Self-study

Reflecting on the texts you read, write a few ideas in your notebook on how you could motivate yourself as a student teacher to maintain your interest and to become a passionate and motivated teacher.

4.3.2. Understanding motivation: understanding the implications of theory

Expected learning outcomes

By the end of the lesson, you will be able to:

- Explain theory of psychology on motivation for primary school teaching; and
- Identify some of the points to consider and practice in the Mathematics classroom.

Learning activity 1: Introducing the lesson

During the lessons on motivation, you will be required to do some personal reflection on your own motivation. You should keep a personal journal in which you write your ideas and reflections. This journal is for your own reflection and is private.

Key learning points

The introductory paper on positive motivation was presented, read and discussed in class in the first lesson.

- In your groups from your notes and recollections, discuss the question: Why is the issue of motivation specifically important in the Mathematics class?
- Capture your ideas for presentation.
- Appoint a member of your group to present to the class.
- During the presentations, think about different ideas to carry forward into the classroom to help the children become positively motivated in Mathematics.
- When you do group work, feel free to write in your diary if you differ in any way, or have something to add. It is equally important to know that sometimes you are the same as others, and sometimes you are different.

Practical applications

Consider how we can use the ideas on positive motivation and translate them into practice. During this seminar, you will be asked to be clear and to work with your emotions, so that when you work on these elements with the students you will be able to understand them better. You will also know when to recognize when students are struggling to understand. It will draw out how you can contribute to your work in a unique way, using your mind, your creativity, your emotions, your body, in a way which hopefully will make you the best teacher (and healthiest person) that you can be. Once you understand the power of your own motivation, it will be much easier, and more fun to teach the children how to realise theirs.

Feelings

We are born with the basic elements to survive: our bodies which help us fight or flee or freeze, core emotions such as anger, and expressions (such as smiling or frowning).
Children remember things in terms of emotions. Feelings can be intense, and since the brain is not completely developed until our late teen years, children tend to be aggressive, or want to hide, or pretend nothing is wrong. They still have to learn to use their intellect to help them work more strategically with their emotions.

Learning activity 2

With a partner, discuss the following questions:

- On a scale of 0 to 10 (where 0 is equal to none at all and 10 means always), how much are you positively motivated?
- Do you know what the main difference is between 'good' feelings and 'uncomfortable' ones?
- Which feelings allow you to learn more?
- Which feelings discourage you and lead to you being less productive? For example, trouble at home may cause you to feel intense sadness which impacts your intellectual capabilities.
- Which feeling do you feel most?
- How easy will it be to do a maths sum, or listen to another person, under these opposing circumstances ("good" and "uncomfortable" feelings)?

Physical activity

One of the methods you can use to help cope with intense emotions is physical activity. For example, you may read the introductory paper and it makes no sense to you. Your immediate reaction after reading it first may be: ‘It is no use, I am tired’, or ‘I feel defeated.

Try going for a run or doing some form of exercise for 10 minutes. Then come back and break down the task of understanding the paper. Read it through one paragraph at a time, making a note of what is most important to you, in each paragraph. When you have finished, go for a walk around the block, come back and read your notes. Did your initial feelings of tiredness and feeling defeated change?

Your body is smart; it talks to you in its own language. For example, your body makes you feel ill if you have eaten too much. If there is something that is too big for you to manage, your body will try to make it easier for you to tackle the problems. Therefore, when you exercise, your body releases endorphins –nature’s anti-depressant – in order to make you feel better about the process. You may not like exercise, but you will love the feeling that comes after you have done it. To get your body to work well with you, you have to respect your body. You have to feed it properly and exercise it.

The other part of our identity that helps us work with intense feelings so that we do not immediately react destructively is to work with our emotions through doing something creative. This will give time for the intensity to come down a bit.

Intellect

When you go through an emotionally difficult time, you may find that you cannot concentrate on anything which is intellectually stimulating. This is because your mind is overwhelmed by your intense emotions. These emotions take up much of your energy. If the intense emotion is an uncomfortable one, you are likely to feel tired and not want to go out and you may even feel as if you want to just go to sleep for a long time. But even when you wake up, you are not likely to feel properly rested. It may be that you feel ‘less’ or fear that you are not worth much.

If the intense emotion is a 'good', comfortable one, you may feel a burst of extra energy. You may want to go and celebrate (for example, if it is because you did well in your exams, if you received a certificate, if you have been acknowledged as special in some way). It is probably still unlikely that you will want to concentrate too much on your academics.

Working with your body will not directly resolve all issues related to emotionally difficult times, but it will give you the motivation you need to more effectively use your mind, with your emotions, creatively to resolve obstacles so that you can reach your dreams.

4.3.3. Understanding motivation: cognitive development of children and teaching of Mathematics

In this lesson, you will explore the thinking of Piaget and Vygotsky and the implications for Mathematic teachers of their ideas about what young students are able to understand and how they learn.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Describe key ideas about the development of thinking of children and of how they learn;
- Explain how particularly important Mathematical learning and thinking is for the cognitive development of primary school children;
- Discuss strategies to motivate primary students for effective teaching and learning; and
- Demonstrate strategies to motivate primary students for active learning in your micro-teaching and peer-teaching.

Introduction

Children do not think and learn in the same way as adults do. Surprisingly, this idea is fairly new to society's thinking and practice in education. The theories that are considered in this unit were only developed last century. For most of history, society believed that children were little adults and that they should, therefore, learn in the same way as adults do. Because children do not think like adults, teachers need to understand how children think and learn to find the best ways possible to help them learn and to encourage the development of their thinking.

Piaget's stages of cognitive development[1]

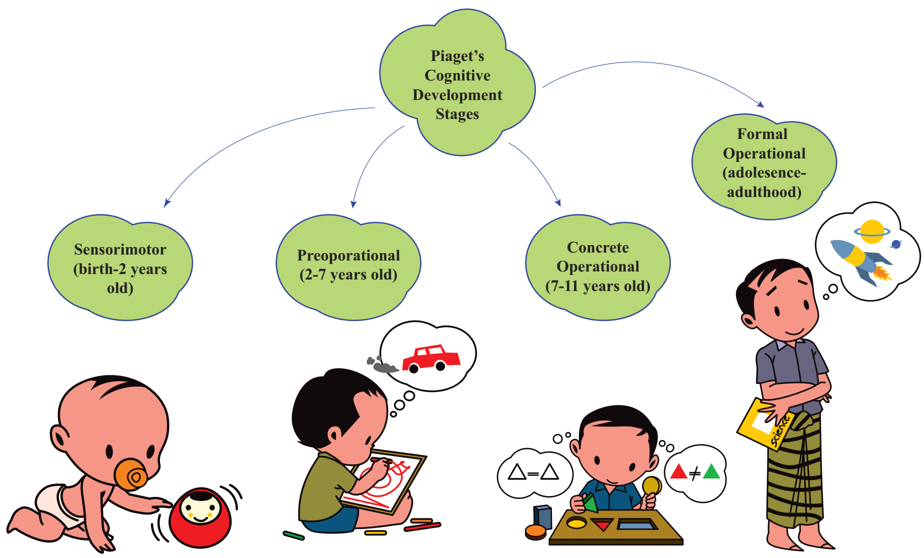

Figure 4.1. Four stages of cognitive development

Jean Piaget was a Swiss psychologist (1896-1980). His theory of the cognitive development of children is that they develop mentally through four separate stages:

The stage between birth and two years old is called the **sensorimotor stage**. The infant experiences her/his world through movement and sensation. They learn through basic actions such as sucking, grasping, looking and listening. They will learn things that they cannot see any longer still exist. They learn that they are separate from other people and things around them. They will learn that they can cause things to happen around them through their actions.

[1] Additional information can be found through video demonstration of Piaget's Theory of Cognitive Development - www.youtube.com/watch?v=IhcgYgx7aAA

The stage between two and seven years old is called the **preoperational stage**. Children can start to use pictures and words to represent objects. They can think symbolically. They tend to focus on themselves and find it difficult to see the world from the perspective of others. Their language and thinking skills are developing. They see and think at this stage in concrete terms.

The stage between seven and eleven years old is called the **concrete operational stage**. Children start to think logically about concrete things. They can start to grasp the concept of 'conservation'. As an example, this means that they understand that the same amount of liquid in a wide short glass is equal to that in a tall, thin glass. Since their thinking becomes more logical and organised they start to be able to be inductive – they have the ability develop a general rule from specific observations.

The stage starting from twelve years old and older is termed the **formal operational stage**. The young person begins to think in abstract terms and considers hypothetical matters ('what if…?'). The young person starts to find interest in philosophical, moral, social, economic and political issues. These require abstract reasoning and theoretical ideas. At this stage, the young person is able to make deductions using logical thought to reason from a general principle to specific information.

These stages explain why, for example, a teacher will have great difficulty trying to teach algebra to a child of eight. The child is still in the concrete operational stage and cannot work with abstractions. This only normally becomes possible after about 12 or so when the formal operational (or hypothetical-deductive) stage is being reached. The brain is only completely developed around 18 to 21 years so there is constant inner development over the entire period of becoming an adult.

Piaget draws an important distinction between the psychological concepts of development and learning. For him, development is an organic process, happening from nature. This would include things like the development of the brain and nervous system. It would also include the influence of the physical environment and, finally, culture and custom (what Piaget calls social transmission).

Learning, on the other hand, is intentionally set by the facilitator and is carried out in a way which creates, enhances and adds skills. It often brings cognitive changes and adds to development in the child.

This means for Piaget:

- Learning and development are related but are not the same.
- Learning cannot take place way beyond the level of development of a person.
- Learning will also not happen if the person is completely familiar with the task.
- Learning will only happen if the task can be partly understood by the student who, with a little help from the facilitator, will be able to complete the task. In so doing, transformation happens within the student.
- The teacher has also to learn from the student to find out from where to start, what the student already knows.
- This also means that the teacher and student can and should constantly swap roles.

Vygotsky's concepts of the zone of proximal development and scaffolding

Lev Vygotsky (1896-1934) was a Soviet psychologist. He argued that children follow the example of adults and develop through the ability to do things without assistance. He developed the concept of the zone of proximal development (ZPD). This is the difference (space or gap) between what a student can do without help and what this student can do with help.

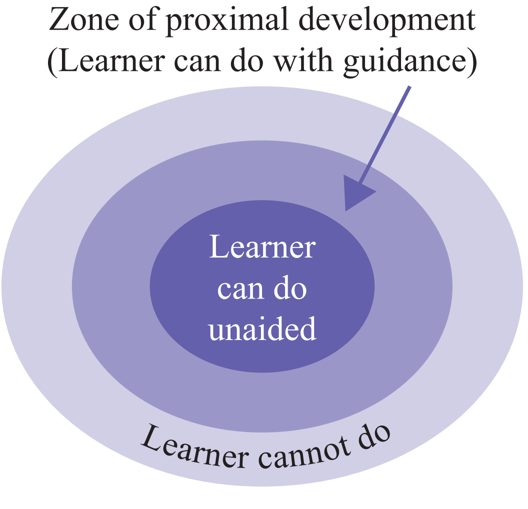

Figure 4.2. Zone of proximal development

Vygotsky believed that the role of education (and educators) was to provide children with experiences that were within their ZPD. This is to encourage and advance learning.

Other theories in applying Vygotsky's thinking around ZPDs developed the concept of **scaffolding**. Just as scaffolding on a building is slowly withdrawn as the structure goes up, so a skilled teacher gives scaffolding support to the student and then slowly withdraws this when the support is no longer needed.

Self-study

Now that you have read and considered the text, work in your groups of four. Develop a summary of the key concepts of either Piaget's concepts of the stages of development and of his understanding of learning and development, or Vygotsky's ZPD and scaffolding. You should present your summary in words and diagrams. As well as presenting your findings, compose two questions on the subject to pose to another group.

Psychological interiorisation

There is a particular expression on the face of a student who has finally grasped a concept, idea or pattern in Mathematics. Often this is referred to as an 'aha!' moment. The experience is a pleasure for the student and the expression is a reward for the teacher.

Both Piaget and Vygotsky suggest frameworks within which teachers, as the facilitators of learning, can think about what they should be doing to best enhance the chances of real learning for children.

Piaget argues that during the process of gaining knowledge, an internal change happens at the start and at the end,

- The facilitator must firstly 'recruit' the student, by finding out what they are already familiar with. This baseline forms the 'starting point.
- Since the teacher starts with something the student is familiar and confident with, the student is 'recruited'.
- The facilitator then acknowledges the student's knowledge (provides credit and pride) and suggests something which the student can do with what is already known, but is not yet completely known. The facilitator provides a challenge. The challenge must involve elements of 'newness', 'unfamiliarity' and 'tension'.

- The outcome offered by the facilitator must be of interest to the student. One element to motivate is the inner need to find equilibrium. This Piaget calls the 'equilibration process';
- If the student's interest has been captured, s/he must then for a moment give up their own knowledge and follow the steps of the facilitator and 'place their own knowledge in abeyance';
- The student must trust to place her/his own knowledge in suspension. The facilitator must understand the value of incentive and recognition rather than punishment and
- Each step in the process of learning must be given space for the student to try to 'operate', try their own hand. The facilitator should always 'recognise and respect' to keep the value of achievement as an emotional end goal.

In this lesson, we will explore some of the ideas highlighted in this process as well as consider Vygotsky's ZPD and scaffolding. Eventually, we will model ways in which these ideas can be used in the primary school classes where you teach.

Ordering and counting

Reflect on Piaget's stages of cognitive development from the previous lesson, particularly the illustration that sums up how children understand their world at different stages.

In the following extract, Piaget is talking about children's learning from experience:
Piaget, J. (1964). Cognitive Development in Children: Development and Learning; Journal of Research in Science Teaching

'But there is a second type of experience which I shall call logical mathematical experience where the knowledge is not drawn from objects, but is drawn by actions effected on the objects…"

He goes on to tell the story of a mathematician friend who told him about a childhood experience when he was five.

“He was seated on the ground and he was counting pebbles. He put them in a row and counted them one, two, three up to ten. Then he finished counting and started to count them in the other direction. He began by the end and once again found ten. He found that it was marvellous that there were ten in one direction and ten in the other. So he put them in a circle and counted them that way and found ten once again. Then he counted them in the other direction and found ten once more. So he put them in some other arrangement and kept counting them and kept finding ten. There was the discovery he made.

He did not discover a property of pebbles. He discovered a property of the action of ordering. The pebbles had no order. It was his action which introduced a linear order or a cyclical order or any kind of order. The pebbles had no sum; they were simply a pile. To make a sum action was necessary, the action of putting together and counting. He found that the sum was independent of the order.

So it is not the physical property of pebbles which the experience uncovered, it is the properties of the actions carried out on the pebbles and this is quite another experience. It is the point of departure of mathematical deduction. The subsequent deduction will consist of interiorising these actions and then combining them without needing any pebbles! ” (Piaget, 1964)

Learning activity 1

Now in your groups, consider this story. Try to draw a diagram that illustrates the point that Piaget is making about the learning.

Learning activity 2

Now imagine a lesson that you could devise for teaching in the local context that uses the lessons here and that encourages the discovery of ordering and counting. Capture this on a flipchart.

Mathematics, fractions and music

In the previous example the mathematician as a child was around five years old. Now consider a child of eight or nine. The topic is fractions. Watch and listen to the playing of the Dance of the Sugar Plum Fairy by the Russian composer Tchaikovsky played by a duo on a 'glass harp'. https://www.youtube.com/watch?v=QdoTdG_VNV4

Learning activities 3 and 4

In groups, consider the Mathematics involved in music and the local context that you are familiar with. Devise lessons for teaching fractions that would encourage your students to discover the concept of fractions.

Review questions

1. What are the four stages of cognitive development of children that Jean Piaget identified?
2. What is Vygotsky's concept of the zone of proximal development (ZPD)?
3. What is Vygotsky's concept of 'scaffolding'?

4.4. Learning and Teaching Resources

If teachers are well equipped with the skills to develop and knowledge to access learning and teaching resources, they will have the tools to conduct challenging and productive lessons with their students. Effective teaching and learning resources contribute to motivating students.

In this sequence of three lessons, we will examine the development of teaching instruments and posters (or the sourcing of these). We will also develop an overview of what is available on the internet. Then we will look at how working with fellow teachers in your school and in schools in your area can enhance your teaching and learning and make preparation more efficient.

4.4.1. Posters, charts, handouts and equipment

Good quality posters and charts in the classroom are useful for information and sometimes for teaching. Handouts are generally used to present enrichment material that is not covered in the textbook or that presents information in a different way. Similarly, worksheets are usually developed by teachers to help students learn how to do some Mathematics through various steps. Good equipment (for numbers and operations, for geometry and measurement) is very valuable for primary Mathematics teachers. In this lesson we examine these equipments.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Classify teaching and learning resources; (in other words, handouts, worksheets, books, charts, model, representations, real objects and so on); and
- Demonstrate how to access teaching and learning resources from outside sources or develop them.

Learning activity 1: Introducing the lesson

Reflect on your experiences during the Education College Mathematics subject. What teaching and learning resources has your teacher educator used to help you in your learning?

Charts and posters

Charts are mounted on the wall and are usually large enough for students to see from anywhere in the classroom. They are generally useful for teaching purposes. Charts for Mathematics may summarise information on a variety of topics: Numbers, fractions, decimals, measurement, geometry, algebra, and data handling. They can be used for teaching or revision of key points of a topic. A good set of charts could be kept by a teacher and changed and stored based on the topics being covered at the time. For example, you may have a chart that summarises information on polygons. It could be a conversion chart for measurement using different units. It could be a chart that represents information about time. Charts can be made by the teacher or they could be sourced through the education department or non-government agencies. Posters are generally used to represent information of interest and can form part of a display. There could be a poster display on interesting Mathematicians.

Worksheets and handouts

A worksheet is an assignment of sorts. It contains description and explanation and then a series of exercises for the student to complete. For Mathematics, a worksheet would usually cover aspects of a topic and can be used as an assessment. Handouts are printed additions to the textbook on a topic not covered in the text or on a topic for enrichment.

Equipment

You may require specific equipment to teach topics that make learning easier. Think of teaching quadrilaterals, measuring weight, telling time, and measuring volume. Do you remember the equipment suggested in lessons in the first semester?

Learning activity 2: Production of posters, charts, equipment, handouts

Your teacher educator will assign a topic to your group. Your group must design and develop at least two teaching and learning resources suitable for the topic.

4.4.2. What's on the Internet?

There are many resources available to be used for teaching and learning Mathematics on the internet. Some are useful and of good quality. Others are not. As a teacher, you need to know how to access these resources and to use them. You also need to be able to differentiate between those that are good quality and those that are not. In this lesson, we will look at some of the resources available and explore ways to use them.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Demonstrate the kinds of useful teaching and learning resources that are available for primary Mathematics on the Internet; and
- Demonstrate how to access teaching and learning resources from outside sources or develop them.

Introduction

Some would argue that the internet is the answer to the educational challenges of the 21st century. This would suggest that learning could happen without teachers. This argument is, however, debatable. The Internet may not be able to replace teachers but it can be a very useful resource that teachers can utilise to enhance their students' learning.

Learning activity 1: Introducing the lesson

What do you think could be some of the main drawbacks in using the Internet as a source of education as a teacher?

What's on the Internet?

The internet can be overwhelming because there is so much information, almost too much for us to digest. Busy teachers need information to be organised for them in the same way as a good librarian organises resources in a library so that you can tell what is useful for your purposes.

Here are some useful examples from the Internet that primary Mathematics teachers can use:

- Mathematics teaching sites such as Khan Academy;
- Academic articles on various aspects of Mathematics and teaching in primary schools (from various university units, government education departments, schools);
- Various websites and blogs that provide useful teaching tips and ideas on covering various topics in the Mathematics; and
- YouTube videos on all aspects of Mathematics.

Learning activity 2: Exploring and searching the internet

Explore the following sites to see what resources they offer and to think about how you could use them in the classroom:

- https://www.khanacademy.org
- https://www.ixl.com/math
- https://vitalmaths.com

Provide a summary, on flipchart for the class, of what you discovered was available on these sites. Make suggestions of how you could use them to support teaching and learning.

You may be looking for something specific on the internet; 'Ideas for teaching a particular topic' or 'teaching resources for primary Mathematics teachers'. All the search engines have the powerful ability to find a range of sites that might provide answers to your questions. You then have to shift through these to find the appropriate information.

Each member of your group should compose a question for teaching a specific topic in Mathematics, do an internet search and summarise what you found.

4.4.3. Collaboration: teachers' communities of practice[2]

In this lesson, we explore how teachers are collaborating in different ways to share resources and knowledge. We will seek to experience a **community of practice** by working in groups and as a class to research, share and organise information for teach and learning Mathematics at primary school.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Identify what communities of practice are and how they enable teachers to collaborate and share resources in a variety of ways including through social media.
- Demonstrate how to access teaching and learning resources from outside sources or develop them.

Learning activity 1: Introducing the lesson

Your teacher educator will introduce you to the history and development of communities of practice.

Introduction

The most useful resources for teaching and learning can often be suggested by fellow teachers. Effective learning between teachers is an invaluable resource. There are significant benefits to be had for primary Mathematics teachers if energy and effort are put into building and sustaining communities of practice.

A community of practice comprises a group of people who do the same job. The group generally shares experience, resources and knowledge. The group serves to motivate and

[2] https://en.wikipedia.org/wiki/Community_of_practice

encourage its members. Communities of practice have existed in some shape or form since the term and theory for the concept was first proposed by Jean Lave and Etienne Wenger in 1991 and has since been expanded.[3]

Communities of practice sometimes form organically because a group of people have a common interest and want to explore it. The community is sometimes deliberately formed so that its members can gain knowledge in their shared area of occupation or interest. Learning happens through sharing knowledge, information and experience.

Gatherings of communities of practice can take place at work – at a school, a factory, or a site – or they can be conducted online (Dubé, Bourhis & Jacob 2005).[4] If members use their mobile phones to communicate, they are described as mobile communities (Kietzmann et al, 2013).[5]

In our technology-driven society, Information and Communication Technology (ICT) provides practitioners of all kinds with a range of options to use to share information and knowledge.

Learning activity 2: Investigating and planning communities of practice

Watch the video on the theory and reality of communities of practice and make notes: https://www.youtube.com/watch?v=jJQDY-qdatU

Compare and share your notes within your group and nominate a representative to present these collected notes to the class.

We noted in the last lesson that a fair amount of research is required to be able to use the resources available on the internet effectively, to know what is there, to know what is good quality and useful.

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3 Lave, J. & Wenger, E. (1991). Situated Learning: Legitimate Peripheral Participation. Cambridge: Cambridge University Press.

4 Dube, L. & Bourhis, A. & Jacob, R. (2005). The impact of structuring characteristics on the launching of virutal communities of practice. Journal of Organisational Change Management. Vol 18. 145-166. doi: 10.1108/09534810510589570.

5 Kietzmann, J., Plangger, K., Eaton, B., Heilgenberg, K., Ptt, L. & Berthon, P. (2013). Mobility at work: A typology of mobile communities of practice and contextual ambidexterity. The Journal of Strategic Information Systems. Vol 22. 282-297. doi: 10.1016/j.jsis.2013.03.003

For this project, you will form a class community of practice. The goal of this community practice is for all members of the class to have a useful resources file to take with them at the end of Year 1 to help them teach primary Mathematics. As we get into examining teaching and learning aspects of the curriculum in the next unit, it will be useful for class members to take part in a joint project to research and organise information around useful resources for primary Mathematics teachers.

- Plan the project so that all members are collecting, assessing and curating resources from the internet. It may be useful to assign topics or grades for each group;
- Members should only recommend resources if they are relevant and of high quality. So they must review them and give a brief description of their contents and how they could be used;
- Members may wish to organise translations of materials so that they can be used with greater ease in Myanmar;
- Members may also wish to experiment with animation software and make local resources for Myanmar teachers and students;
- Plan the project so that each group has a coordinator who meets with the coordinators of other groups;
- Your community of practice (or coordinators) should schedule to meet periodically; and
- The objective of the community of practice is to access and assemble high quality teaching and learning resources and to organise and store these in a way that makes them easy to access and use by fellow students.

Review questions

1. Why is it important to consider using learning and teaching resources?
2. What is important to consider when searching the internet for learning and teaching resource?

4.5. Assessment for and of Learning

Although assessment is a topic that is covered extensively in your Educational Studies learning area, developing the skills for assessing Mathematics learning requires a specific set of skills. Teachers should be well-equipped with a range of assessment tools firstly to assess students as they are learning new concepts and consolidating their skills and then to test how much they have learnt at the end of a cycle.

In this series of three lessons, we will firstly, through reading and studying some articles, examine various strategies for assessment. Then in the subsequent lessons, we will practise developing some assessment instruments for both formative and summative assessment using primary Mathematics problems pitched at Grade 3 and Grade 6.

4.5.1. Assessing for and of learning: formative and summative

During this lesson, we will read and discuss four articles on assessment and use this information to develop knowledge of different strategies for assessment.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Discuss what is assessment for and assessment of learning in primary Mathematics; and
- Develop an assessment strategy for Mathematics.

Although assessment is a topic that is covered extensively in your Educational Studies learning area, developing the skills for assessing Mathematics learning requires a specific set of skills. Teachers should be well-equipped with a range of assessment tools firstly to assess students as they are learning new concepts and consolidating their skills and then to test how much they have learnt at the end of a cycle.

In this first of a series of three lessons, we will firstly, through reading and studying some articles, examine various strategies for assessment. Then in the subsequent lessons, we will practise developing some assessment instruments for both formative and summative assessment using primary Mathematics problems pitched at Grade 3 and Grade 6.

Learning activity 1

Express in your own words why it is important for teachers to know how to assess students.

Learning activity 2: Group Discussion

Reading 1

Your teacher educator will give you a copy of 'Formative and Summative Assessments in the Classroom'[6] and time to read it before this lesson. Prepare to discuss the following questions:

1. How do the writers describe summative assessments? In groups, create a flipchart summary of the points made in the article;
2. How do the authors describe **formative assessments**?
3. Describe the various strategies that good teachers can use for formative assessment that the authors raise?
4. What do the authors say about balancing assessments?

[6] Garrison, C. & Ehringhuas, M. (2007). Alberta Education, Alberta, Canada.

Reading 2

Your teacher educator will give you a copy of *Alberta Assessment*.[7] You will have time to read the article before this lesson. In your groups, discuss the following questions:

1. How do the writers describe 'continuous assessment' and 'collaborative assessment'?
2. What are the key points that the writers make about planning assessment?
3. What do they understand by 'performance assessment'?
4. What do they understand by 'portfolio-based assessment'?
5. What do they say about tests?

Reading 3

Your teacher educator will give you a copy of *Self and Peer Assessment* (2011), research conducted by the Primas project at the University of Nottingham's Centre for Research in Mathematics Education. You will have time to read the article before this lesson. In your groups, discuss the following questions:

1. What is the purpose of this article?
2. What should you, the reader, learn in the process?
3. What are the five activities that the writers explore to achieve the learning objectives?
4. Focus on Activity A in the document: Exploring how students may be made aware of IBL (Inquiry-Based Learning) processes.

[7] Assessment, Knowledge and Employability Studio Teacher Workstation. (2007). Alberta Assessment. Alberta Education, Alberta, Canada. Retrieved from www.learnalberta.ca

4.5.2. Developing assessment instruments for Grade 2

In this lesson, we will try to develop different assessment instruments specifically for a formative/diagnostic assessment and a summative assessment using examples of Grade 2 type problems and examples and exercises from the textbook.

Expected learning outcome

By the end of the lesson, you will be able to:

- Develop formative and summative assessment instruments for Grade 2 primary Mathematics.

Learning activity 1

Developing a formative assessment tool

Your group will be assigned a topic from the Grade 2 curriculum different from the topic that you worked on during sub-unit 4.2. These topics include:

- Tables and graphs
- Time and duration
- Addition 1
- Addition 2
- Subtraction 1
- Subtraction 2
- Measurement of length 1
- Measurement of length 2
- Numbers up to 1,000
- Solving problems using diagrams
- Triangles and quadrilaterals

- Multiplication
- Division
- Volume
- Weight

Consider what would be included in a series of lessons to cover your topic at Grade 2. Using the resources in the exemplar test papers and the examples and exercises in the Grade 2 textbook, compose a formative assessment instrument. This could be a tool that a teacher would use throughout a series of lessons to check that students have understood examples, are able to solve problems independently. Or it could be a diagnostic tool that could be used to determine where students still have problems.

Learning activity 2

Developing a summative assessment tool

Your group should now develop a summative assessment test for when the series of lessons have been completed. This must be a balanced assessment with questions covering all aspects of the topic for Grade 2. In addition, there should be a balance in the degree of difficulty of the questions. There should be some that are straightforward, some that are moderately difficult and some that provide a test for the more advanced students in the class.

4.5.3. Developing assessment tools for Grade 5

In this lesson, we will try to develop different assessment instruments specifically for a formative/diagnostic assessment and a summative assessment using exemplars of Grade 5 type problems and examples and exercises from the textbook.

Expected learning outcome

By the end of the lesson, you will be able to:

- Develop formative and summative assessment instruments for Grade 5 primary Mathematics.

Learning activity 1

Developing a formative assessment tool

Your group will be assigned a topic from the Grade 5 curriculum. These topics include:

- Number concepts
- Addition and subtraction of fractions
- Multiplication and division of fractions
- Multiplication and division of decimals
- Polygons
- Areas of triangles and quadrilaterals
- Volumes of cubes
- Data analysis

Consider what would be included in a series of lessons to cover your topic at Grade 5. Using the resources in the exemplar test papers and the examples and exercises in the Grade 5 textbook (if available), compose a formative assessment instrument. This could be a tool a teacher would use throughout a series of lessons to check that students have understood examples and are able to solve problems independently. Or it could be a diagnostic tool that could be used to determine where students still have problems.

Learning activity 2

Developing a summative assessment tool

Your group should now develop a summative assessment test for when the series of lessons have been completed. This must be a balanced assessment with questions covering all aspects of the topic for Grade 5. In addition, there should be a balance in the degree of difficulty of the questions. There should be some that are straightforward, some that are moderately difficult and some that provide a test for the more advanced students in the class.

Review questions

1. What is summative assessment?
2. What is formative assessment?

Unit Summary

Key message

- The lessons in Unit 4 set out to provide some of the answers to the question posed by the title of the unit;
- The first sub-unit with seven lessons should have provided student teachers with a very good understanding in a level of detail of the Mathematics Curriculum at the primary level and opportunity to practice using the curriculum statement;
- In the unit on Planning Learning, we considered what goes into planning Mathematics lessons and then proceed to planning a sequence of lessons and then planning an inspiring introductory lesson;
- A series of four lessons on Motivation provided some theory and analysis. It then focuses on how teachers could use music to work on motivating young students;
- The sub-unit on Learning and Teaching Resources covered sourcing resources and making posters, handouts and equipment to use in the Mathematics classes. In the third lesson in this sub-unit we explored what's on the internet; and
- In the lessons in the final sub-unit, we reviewed various forms of assessment and then practiced application by using problems and knowledge to develop both formative and summative assessments for Grade 2 and Grade5.

Unit reflection

Reflect on the lessons in the sub-unit exploring the Mathematics Curriculum. What were the key lessons you learnt in this sub-unit? Detailed knowledge of the curriculum in each of the topic areas is vital for teachers. It is also important to understand sequencing.

Reflect on the lessons in the sub-unit on Planning Learning. What were the key lessons you learnt about Planning Learning? Planning initially involves drawing on knowledge of the curriculum and thinking though the best strategies to teach aspects. Detailed planning helps to pace the learning over a cycle of lessons.

Reflect on the lessons in the sub-unit exploring Motivation. What are your first thoughts about this sub-unit? What were the lessons you learnt in this sub-unit? Both teachers and students need to be motivated for really effective learning to take place.

What were the memorable lessons learnt in the sub-unit that focused on Learning and Teaching Resources? Do you think 'communities of practice' could work for teachers? There is a wealth of materials and ideas on the internet to help teachers. Teachers are skilled and talented and can develop their own resources.

What skills do you think you have developed during the lessons on assessment? Look back at how you have developed formative and summative assessment tools, and check your understanding of these terms by referring to your Educational Studies learning area.

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Unit 5 Mathematical Modelling and Representation

In this unit, we explore topics of early algebra, geometry, statistics, probability and data handling. The lessons in this unit are designed to provide you with ideas of how to embark on teaching these topics to young students. The lessons also explore the complexities of the topics so that teachers understand better in relevant with the challenges young students are having when faced with these topics.

Expected learning outcomes

By the end of this unit, you will be able to:

- Describe algebraic thinking;
- Explain that algebraic thinking can be developed from Grade 1, long before letters/variables are introduced;
- Describe that a sound conceptual understanding of the equal sign and equality is an important building block of algebra;
- Describe that some arithmetic activities develop a conceptual understanding of equality;
- Facilitate activities that develop a relational rather than a computational understanding of the equal sign;
- Generalise arithmetic from particular examples;
- Explain that generalised arithmetic is one aspect of algebraic thinking;
- Identify that arithmetic can be generalised without using variables (letters);
- Explain how primary school students can generalise arithmetic;
- Generalise patterns with shapes;
- Identify that patterns can be represented in different ways;
- Demonstrate that different representations can make different aspects of a pattern visible;

- Represent patterns in pictures or diagrams, in words, in function tables and function diagrams and in equations and graphs;
- Teach children how to generalise from patterns with shapes;
- Teach primary school students about right angles, perpendicular lines and parallel lines;
- Teach primary students to fold right angles, perpendicular lines and parallel lines;
- Teach primary students to draw parallel and perpendicular lines;
- Explain how a circle is defined, and name the parts of a circle;
- Draw circles with or without pairs of compasses and make patterns by drawing circles;
- Classify and name triangles according to the sizes of their vertices;
- Classify and name triangles according to the relative lengths of their sides;
- Draw specified triangles on grid paper or with a pair of compasses or a circle-maker;
- Draw lines that connect dots or point where grid lines intersect, and understand that slanting lines are longer than vertical or horizontal lines between the same number of dots or points of intersection;
- Classify and name quadrilaterals;
- Explain the relationship between different quadrilaterals, in other words, some of subsets of each other;
- Facilitate students about quadrilaterals, trapezia, parallelograms, rectangles, rhombuses and squares;
- Identify both regular and irregular pentagons and hexagons;
- Use paper folding to make regular pentagons and hexagons;
- Construct hexagons and pentagons using a variety of methods (for example, paper folding, a pair of compasses or a circle-maker card strip);
- Teach Grade 5 students about pentagons and hexagons;
- Find the sum of the interior angles of a triangle;
- Teach primary students to find the sum of the interior angles of a triangle;
- Explain the importance in Mathematics of using known facts to find unknown facts;
- Understand how to use physical Mathematics to teach various aspects of the primary curriculum;
- Develop some hands-on physical exercises/ lessons to help students practise counting and making various polygons;

- Have experience of learning in a co-constructive way;
- Develop some strategies for teaching the concept of time to young children;
- Understand the number and operations skills that are required to develop graphic pictures used in other subjects;
- Develop skills to read and analyse graphical representations in informative articles and reports;
- Develop some strategies for teaching the concept of analyse graphical representations to young children;
- Explain what primary students have to learn in the areas of 'data collection and arrangement' and 'data analysis'; and
- Develop some strategies for teaching the concept to young children.

5.1. Algebra

5.1.1. What is algebra?

In this lesson, we explore what algebra is and how teachers can develop students' algebraic thinking.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Describe algebraic thinking; and
- Explain that algebraic thinking can be developed from Grade 1, long before letters/variables are introduced.

Learning activity 1: Diagnostic assessment

1. Think for a moment: What is algebra?
2. Talk to the student teacher next to you. Discuss the question what is algebra?
3. Write your ideas on a piece of paper.
4. Paste the notes from the whole class on the wall or board.

Learning activity 2: Pair Work

1. Think on your own about high school algebra;
    a. What do you remember doing?
    b. How did you experience algebra at school? Be honest!
    c. How did most people at your school experience algebra? Be honest!
2. Now talk to the student teacher next to you. Discuss, the questions above.
3. Write your ideas on a piece of paper.
4. Paste the notes from the whole class on the wall or board.

Learning activity 3

Your lecturer will do a presentation about what Mathematics educators say about algebra and helping students learn algebra. Listen and take some notes.

Reading

What is algebra? What is early algebra? Making the transition to algebra easier. Making the learning of algebra more meaningful.

How do students experience algebra?

When you ask adults about school algebra, many can remember seeing x's and y's, some can remember that they worked with equations. Many people do not enjoy learning algebra at school. Some people who found algebra easy at school are not sure what the purpose of algebra is.

'For most people learning algebra is a bit like being taught how to kick, dribble and head a ball without even knowing about the game of soccer, or practising musical scales without ever playing a tune. Isolated skills are learnt to no apparent purpose. Perhaps it is not surprising that for most people the algebra experience is boring, difficult, pointless and confusing.' Mason et al (1985)[8]

How can we change teaching and learning to reduce the difficulties that students experience with algebra?

Why do many students find algebra difficult? Is it because algebra is meaningless and boring? Should we change the ways we teach algebra? Is it because of the way young students learn arithmetic? Can we change how young children learn about Mathematics before they start to learn algebra? Will this make it easier to move from learning arithmetic to learning algebra?

In many parts of the world, Mathematics education is changing. It is moving away teaching algebra through only learning rules to operate on algebraic symbols. It is moving away from focusing only on numerical answers in the early grades.

[8] Mason, Graham, Pimm, & Gowar. (1985). Routes to / Roots of Algebra. Centre for Mathematics Education. U.K.: Open University Press.

When students work with algebra, they use a number of algebraic laws. These are, in fact, properties of numbers and properties of operations. They are at the heart of the structuring of the number system. This means that students can learn to use these properties before they are introduced to the symbols of formal algebra.

This transforms Mathematics in the primary school from a focus on finding one numerical answer to arithmetic problems to providing opportunities for pattern-building, conjecturing and generalising mathematical facts and relationships. Siemon et al (2011)

At school we learnt, for example, that 7 x 8 = 56, and that 8 x 7 = 56. For many of us these were two unconnected multiplication facts. Instead of learning 7 x 8 = 56, and 8 x 7 = 56 as separate multiplication facts, we can ask students to think about the following sorts of questions:

- What is the same about these two statements?
- What is different about these two statements*?*

Students can work out that the order in which we multiply seven and eight does not matter.

Primary school students can then be asked to investigate, '*will it work for all numbers'?* Students can make general comments like '*The order in which you multiply two numbers does not matter'* or *'We get the same result if we swap the order in which we multiply two numbers.'*

Young students do not need to know that this is called the commutative property. They also do not need to be able to represent this property in formal algebraic notation as $a \times b = b \times a$. They are only expected to know and use the property so that they can change the order of numbers when multiplying.

We can then extend this investigation by ask them: *Will it work for all operations?*

Students can find out that you will get the same answer if you swap the order in which you add two numbers. But this property does not apply to subtraction or division.
The early algebra approach shifts students' attention to underlying patterns and properties of numbers and operations.

The intention of introducing algebraic thinking in the primary years is to focus students' attention on generalising the structure of arithmetic and justifying and explaining solutions to problems.

It can help you students to see patterns if we ask them questions like:

- What is the same?
- What is different?
- Will it work for all numbers?
- Will it work for all operations?

What is algebra?

Table 5.1. What is algebra?

Arithmetic	Algebra
In arithmetic, you calculate with numbers. Numbers are abstract, but they represent real things in the world.	In algebra you generalise about numbers. Algebra is a way of thinking.
In arithmetic you work with specific numbers.	In algebra you reason logically about numbers.

Devlin[9] says:

'… algebra is not "arithmetic with letter". …
algebra is a ***way of thinking*** *…*
…. formulas and equations, involving all those x's and y's, are … a way to represent that thinking on paper.'

What is algebraic thinking? It includes students exploring patterns and learning to generalise from specific examples. Algebra is generalised arithmetic. We examine patterns and from this make general rules. A range of different representations can be used to express generalisations: diagrams, spoken and written words, tables, and a range of symbols. It is not necessary to use letters to express generalisations.

[9] Devlin, K. (2011). 'What is Algebra?'

In algebra, we abstract from calculations and numerical relations to understanding the working of number systems and operations in general.

We use algebra to think about functions, relations and joint variations. Algebra is also about modelling.

Devlin also compares the x's and y's in algebra to musical notes in music. He adds,

'It is possible to do algebra without symbols, just as you can play an instrument without being able to read music. In fact, traders and other people ... used algebra for 3,000 years before the symbolic form (of algebra) was introduced in the 16th century."

Jacobs et al[10]. rite that the focus should shift from learning rules for manipulating algebraic symbols to developing algebraic reasoning'. They add that because of the strong focus on arithmetic in primary school this shift towards algebraic thinking should not be limited to when letters are introduced into Mathematics but should be infused into the teaching of arithmetic from Grade 1.

From the early grades, students can go beyond only computing answers. They can focus more on making sense of the relationships between both numbers and operations and be taught to use the properties of arithmetic. Young students can investigate patterns and make generalisations. It makes more sense for students first to learn to generalise, then to learn to use symbols to represent generality.

Mason et al. (1985)[11] write that algebra is a language which allows you to see and express general statements. At the heart of algebra is the process of generalising. Included in the notion of generalising the structures underlying arithmetic are:

- Providing opportunities for students to work with the properties of operations (commutative, associative and distributive properties) and the properties of numbers (for example, 1 as the identity element of multiplication, 0 as the identity element of addition) while they are learning arithmetic; and

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[10] Jacobs, V.R., Franke, M. L., Carpenter, T.P., Levi, L., & Battey, D. (2007). Professional Development Focused on Children's Algebraic Reasoning in Elementary School. Journal for Research in Mathematics Education, 38 (3).

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- Helping students to develop a deep conceptual understanding of equality (what the equals signs means)

Comprehension (after the first period of this lesson)

After reading the article above, answer the following questions.

1. List four questions that can help students to generalise in Mathematics.
2. How can young students express a general rule without using variables (letters)?
3. Give an example of a computational approach to solving a calculation
4. Give an example of a relational approach to solving a calculation
5. What properties of operations are named in the reading?
6. What properties of numbers are named in the reading?
7. What is Algebra?

List any questions that you have.

In the second period of this lesson, we will explore the ideas covered in the last lesson and the homework reading to develop our understanding of algebra and to think of ways to introduce algebraic thinking to students in the primary grades.

Learning activity 4

What can we do in primary schools to make the transition to algebra easier for students? Develop a flipchart presentation of these points. This discussion will lay down the basis for the lessons on algebra in the early grades that follow.

Learning activity 5

Use what you read and learnt about yesterday to give your new understanding of 'What is Algebra?' Develop a flipchart presentation of these points.

5.1.2. Teaching algebraic thinking in the early grades: the notion of equality

In the next two lessons, we explore how, through helping primary students to grasp the notion of equality, teachers are able to help them develop early algebraic thinking and possibly avoid some of the difficulties which students have when introduced to formal algebra.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Describe that a sound conceptual understanding of the equal sign and equality is an important building block of algebra;
- Describe that some arithmetic activities develop a conceptual understanding of equality; and
- Facilitate activities that develop a relational rather than a computational understanding of the equal sign.

Learning activity 1: Finding the unknown number

1. Imagine you are a Grade 3 student. Set out the following learning aids on a page of your book.
2. Do not change the calculation into written numbers and variables.
3. Each match box contains the same number of matches.
4. You know that the equals sign means that what is on the left-hand side of the equals sign is equal to what is on the right-hand side of the equals sign.
5. In this example, it means that there are the same number of matches on the left-hand side and the right-hand side of the equals sign.
6. Find out how many matches are in each box (do not open the box to look).

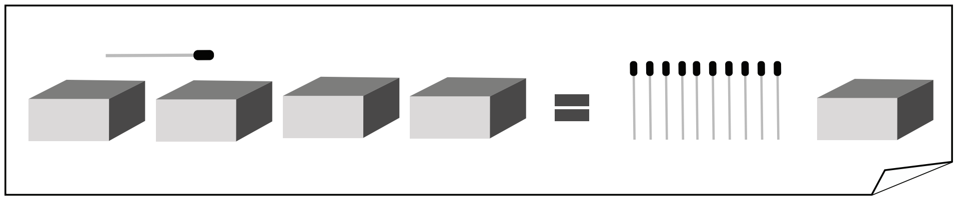


Figure 5.1. Finding out the matches in the box

7. Explain to the rest of the class how you worked out the number of matches in each box.

Learning activity 2: Presentation

Your lecturer will talk with you about the meaning of the equal sign.

Learning activity 3: Puzzle – Multiple unknowns, multiple symbols and varying position of the unknowns

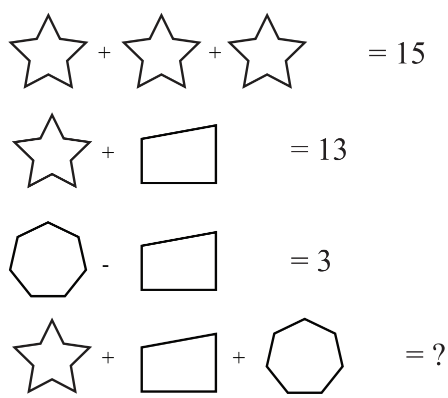


Figure 5.2. Puzzle

Homework reading: The meaning of the equal sign

In primary school, Mathematics students often work with number sentences that look like this:

$$12 + 8 + 6 = \square$$
$$48 – 16 = \square$$
$$25 \text{ x } 32 = \square$$
$$84 \div 21 = \square$$

The calculations that they must do are on the left-hand side of the equal sign, and the space for the answer is on the right-hand side of the equals sign. From this, many students think that the equals sign means students to assume that the equals sign means '*I am working towards the answer*' or '*I am getting closer to the answer'* or *'Here comes the answer*' or simply '*This is the answer*'.

Many students do not understand that the equal sign means what is on one side of the equals sign has the same value (even if it looks different) to what is on the other side. It is important that young students understand that the equal sign represents equality. If they know this before they start to work with formal algebra, it will help them to move from arithmetic to algebra more easily. It is important to provide primary school students with examples where the unknown occurs in different places in the number sentence, for example:

$$10 = \square + 6$$
$$\square = 23 + 6$$
$$26 + \square = 30$$
$$48 - \square = 16$$
$$8 \text{ x } \square = 32$$
$$63 = \square \text{ x } 21$$
$$84 \div \square = 21$$

This helps to stop young students from developing an incorrect view of the meaning of the equal sign.

To make sure that young students understand that the equal sign represent equality, they can be given problems where the 'answer box' is not on its own on one side of the number sentence, for example, $9 + 7 = \square + 6$.

Solve this number sentence before reading further.

Children who think that the equals sign means 'this is the answer' or 'the answer follows' will usually give the answer as 16 for example, they will ignore '+ 6' on the right-hand side of the number sentence. They will write the number sentence 9 + 7 = 16 + 6 which is not true.

Students can use different strategies, to reach the correct answer. They must, however, understand that the left-hand side of the number sentence must be equal to the right-hand side of the number sentence.

Strategy 1: Calculating the left-hand side

Some students will add the left-hand side to get 16.

They will then subtract 6 from both sides, which will give 10 = □

This step is based on the fact that + 6 – 6 = 0 or the use of inverse operations. This strategy starts by calculating or computing the one side of the number sentence. This kind of thinking is called computational (calculating) thinking. When we teach number calculations, we usually focus on computational thinking. When we introduce students to algebra, we tend to continue to encourage computational thinking. Stephens (2006) notes: 'Successful computational thinkers do rely on a correct understanding of equivalence. … This sense of equivalence used here may be called equivalence of results.'[11]

Strategy 2: Using the relative size of numbers on both sides of the equals sign

Some students may notice that six on the right-hand side is one less than seven on the left-hand side, so the unknown must be one more than nine, for example, 10.

A similar approach is to say that seven on the left-hand side is one more than six on the right-hand side, so that the unknown must be 1 more than 9, for example, 10.

Here the question to be asked is: 'What is similar about the two sides of the equation and what is different?' These approaches focus on the relative size of the numbers on either side of the equal sign. The relationship between numbers on the left- and right-hand sides is considered. The focus is on the relationship between the numbers, so this is called **relational thinking**.

12 Stephens, M. (2006). Describing and exploring the power of relational thinking. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), Identities, Cultures and Learning Spaces, Proceeding of the 29th annual conference of the Mathematics Education Research Group of Australasia (pp. 479–486). Camberra: MERGA.

13 Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional Development Focused on Children's Algebraic Reasoning in Elementary School. Journal for Research in Mathematics Education, 38 (3).

Jacobs et al.[12] describe relational thinking as looking at the whole expression or equation and noticing the relationships between numbers.

> *'Relational thinking represents an approach to working with numbers that is different from carry out a computational procedure in a single step-by-step sequence. For example, children can solve the number sentence 25 + 58 + 75 = □ by calculating left to right, but they can solve it more easily if they take advantage of a number combination (25 + 75). They may know or be able to calculate easily ... relational thinking entails an awareness of relations among numbers and the fundamental properties of number operations.'*

Using the relationships between numbers is useful when there is a small variation in the numbers. It is not as helpful when there is a large variation in the number. For example, if one had an example like 109 + 76 = 23 + □, it would be just as easy to add the numbers on the left and get 185 = 23 + □. (Stephens, 2006)

Use the relationships between numbers to solve these number sentences. Do not calculate the one side of the number sentence and then isolate the answer box.

89 – □ = 80 – 39
104 – 45 = □ – 46
□ + 17 = 15 + 24
53 + □ = 58 + 46
546 – 362 + □ = 547

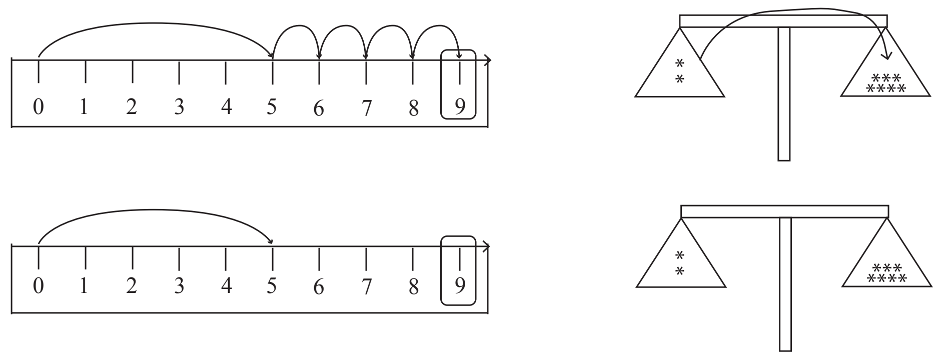
When we write out the steps in relational thinking, it looks complicated but it is relatively easy to do. Stephens (2006) provides examples of Grade 5 to 7 students' relational thinking:

A Grade 5 student commenting on 89 – □ = 80 – 39	*'89 is 9 more than 80, the missing number must be 9 more than 39. Therefore, the answer is 48.'*
A Grade 6 student commenting on 104 – 45 = □ – 46	*'I added 1 to 104. As long as I add the same number to both it, (104 – 45) will stay equivalent. '*
A Grade 6 student commenting on □ + 17 = 15 + 24	*'If I take 2 from 17 and add 2 to 22, it is the same as what is [the expression] after the equal sign.'*
A Grade 7 student commenting on 53 + □ = 58 + 46	*'In 53 + □ = 58 + 46, 53 to 58 is +5, so 51 to 46 is -5. These are equivalent, as you've done the same action to both sides.'*
A Grade 5 student commenting on 546 – 362 + □ = 547	*'546 is 1 less than 547, so 362 is one less than my answer. My answer is 363.'*
A Grade 7 student commenting on 546 + □ – 362 = 547	*'546 is 1 unit less than 547, so if you add 363 and minus 1 unit less than 363,* for example, *362, the equation is equal on both sides. '*

Figure 5.3. Relational thinking approach (1)

If we wanted to develop this kind of thinking to help students from Grade 1 upwards, we can use arrow diagrams to visualise relational thinking. This can be done in several ways (shown below).

9 = 5 + □

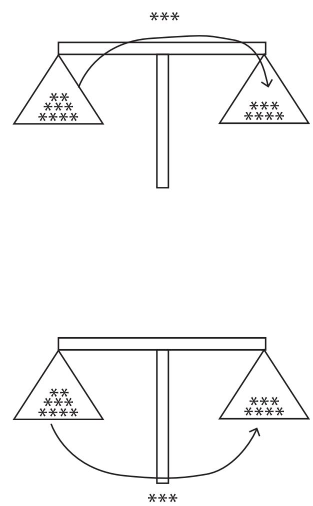


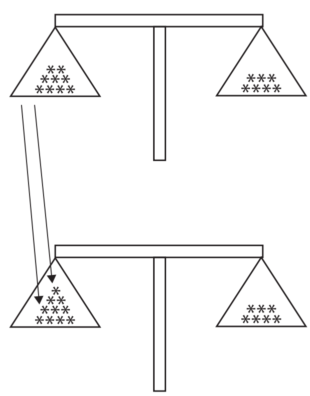
104 - 45 = □ - 46

→ 104 - 45 = □ - 46

→ 104 - 45 = □ - 46

→ □ =105





89 - □ = 80 - 39

80 + 9 - □ = 80 - 39

→ □ = 39 + 9

Figure 5.4. Relational thinking approach (2)

We often introduce algebraic equations with the image of a balance, and explain to students that what is done on the one side of the equals sign needs to be done on the other side of the equals sign. In this way, the value of the expressions on either side of the equals sign remains equal to each other. Often, this notion is only introduced when students are exposed to 'letter-maths' where the unknown is represented by a letter. Students are then asked to solve the equation by isolating the unknown on the one side. By the time students start to study formal algebraic equations, they may have already misunderstood the meaning of the equals sign and this may be difficult to change. Once students understand a little about mass, and are able to compare masses, it may be helpful to use this image to link the notion of balance, equality and the equals sign.

Another important aspect of working algebraically is understanding the relationship between operations, in particular, the inverse relationship between addition and subtraction (subtraction undoes what addition does and vice versa) and the inverse relationship between multiplication and division (division undoes what multiplication does and vice versa).

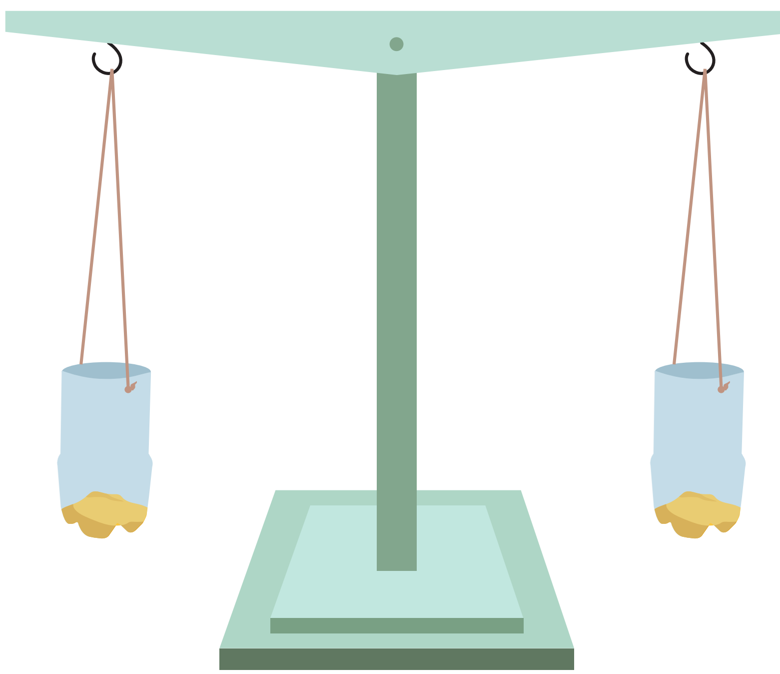


Figure 5.5. A balance scale

Learning activity 4

When people think about algebra, they often think about equations. The notion of equations is often introduced with the image of a balance. Balancing scales are easy to make. Choose locally available materials to make a balance.

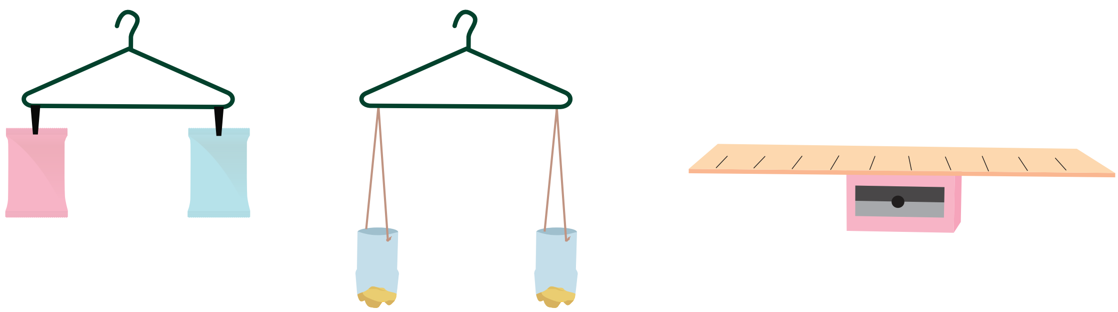


Figure 5.6.

It is useful to initially use the metaphor of mass when introducing the notion of equality through a balance.

The cans all have the same mass. The balls all have the same mass.

1. State the mass of the cans in terms of the mass of the balls.

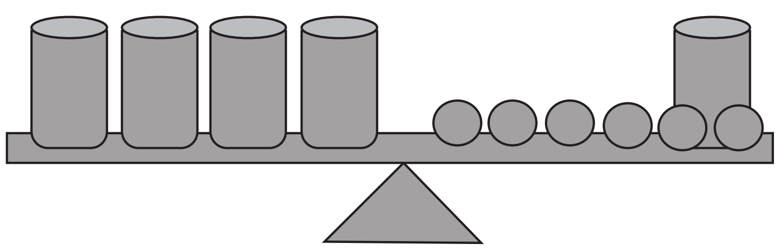


Figure 5.7

2. How many balls are in each small can (assume that if the big can was empty then it would have the same mass as two cans)?

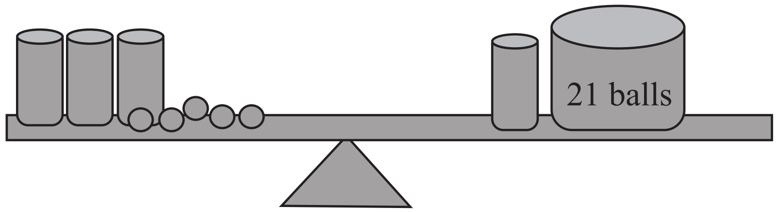


Figure 5.8

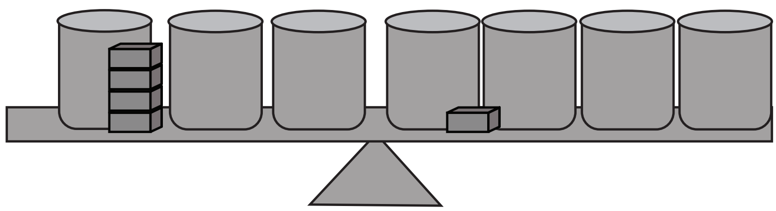


Figure 5.9

3. Answer the questions below as a way of finding how many matchboxes are in each can.

a. What would happen if we put two matchboxes in each can?
b. What would happen if we put five matchboxes in each can?
c. How do you know that there need to be between two and five matchboxes in each can?
d. Why can you three cans off each side of the balance?
e. What else can you do without disturbing the balance?
f. How can thinking about this balance help you to solve: 3x+4=4x+1? and
g. Draw a balance to model another number sentence. Place 'unknowns' on both sides of the balance.

Learning activity 5

Write the digits one to six in the circles in the triangle. The sum of the three circles on each line should be equal.

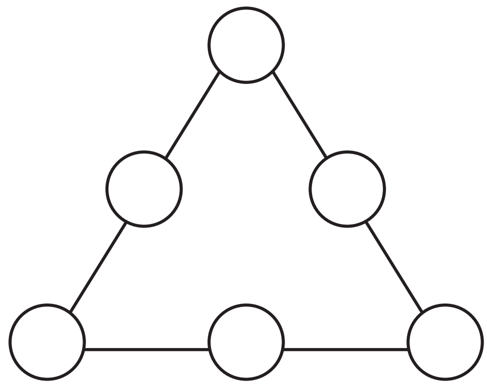


Figure 5.10

Learning activity 6

1. Solve:
   a. $17 + 9 = c + 10$
   b. $46 + 15 = 50 + c$
   c. $27 + 36 = 25 + c$

2. Solve:
   d. $63 – 17 = 61 - c$
   e. $87 - c = 80 - 43$
   f. $101 – 45 = c - 46$

3. Solve:
   g. $342 – 174 + c = 342$
   h. $728 + c - 139 = 729$

5.1.3
Teaching algebraic thinking in the early grades: generalising arithmetic

In the next two lessons, we explore how, through helping primary students to grasp the notion of equality, teachers are able to help them develop early algebraic thinking and possibly avoid some of the difficulties which students have when introduced to formal algebra.

Expected learning outcomes

By the end of the lesson, you will be able to:
- Generalise arithmetic from particular examples;
- Explain that generalised arithmetic is one aspect of algebraic thinking;
- Identify that arithmetic can be generalised without using variables (letters); and
- Explain how primary school students can generalise arithmetic.

Learning activity 1

Imagine that you are a Grade 4 or Grade 5 student who has not yet learnt about variables. A 3 x 3 square is shaded on a number grid.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 5.11

1. What do you get when you add the three numbers in the middle column? Write a number sentence to show this;

2	3	4
12	13	14
22	23	24

Figure 5.12.

2. What do you get when you add the three numbers in the middle row? Write a number sentence to show this;

2	3	4
12	13	14
22	23	24

Figure 5.13.

3. What do you get when you add the three numbers in this lighter diagonal? Write a number sentence to show this;

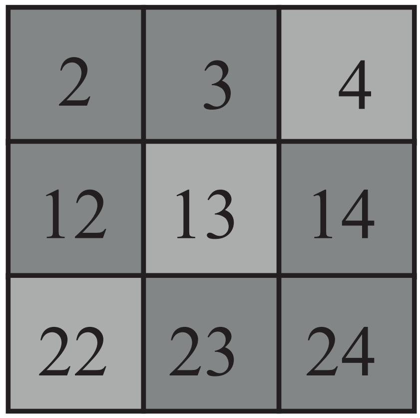


Figure 5.14.

4. What do you get when you add the three numbers in this lighter diagonal? Write a number sentence to show this;

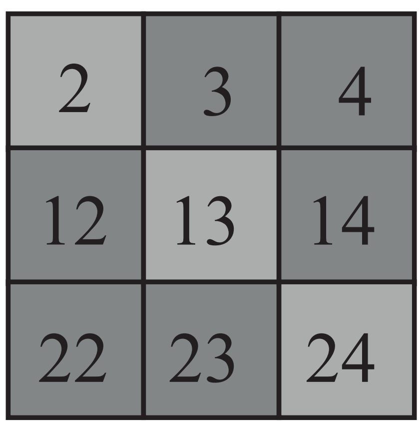


Figure 5.15.

5. What is the relationship between the sum of the numbers you get in questions 1, 2, 3, and 4, and the number in the middle of the shaded square? Write a number sentence to show this;

6. Shade a different 3 x 3 square on the number grid;
    a. Add the three numbers in the middle column.
    b. Add the three numbers in the middle row.
    c. Add the three numbers in the one diagonal.
    d. Add the three numbers in the other diagonal.
    e. What is the relationship between the sum of the numbers you get in questions a, b, c, and d and the number in the middle of the shaded square?
7. What is the same and what is different about the two examples (questions 1-5 and 6)? Juxtapose the number sentences you wrote for question five and question six to help you frame your answer;
8. Will the relationship that is constant in these two examples work if you select a 3x3 square on a calendar?
    a. Convince the teacher educator that this is or is not always true.
    b. Explain why this pattern will always work on a 3 x 3 square.
    c. How can you be sure that this explanation works in all cases?

9. What will happen if you choose a larger square: for example, 5 x 5 square? 7 x 7 square? 9 x 9 square? Write number sentences that show what will happen in each case you select;

10. If you choose a 2 x 2 block of numbers, add them and state the sum: Your partner should be able to tell you which 4 numbers you chose;
11. Can you also do this?
12. Explain how he/she works it out.

Learning activity 2: Generalising from a pattern of number sentences

$$2^2 - 1^2 = 2 + 1$$
$$3^2 - 2^2 = 3 + 2$$
$$4^2 - 3^2 = 4 + 3$$
$$5^2 - 4^2 = 5 + 4$$

1. Calculate each of the number sentences above to check that the statements are true.
2. Try three more different examples. Then compare your results with the results of others.

3. What about $101^2 - 100^2$? Is it equal to 101 + 100?
4. We can show that this works visually, using diagrams;
    a First, let us show why particular examples work.

$2^2 - 1^2 = 2 + 1$;

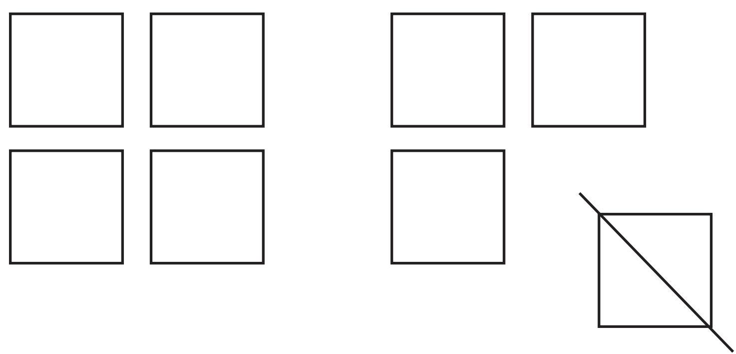


Figure 5.16

b $3^2 - 2^2 = 3 + 2$

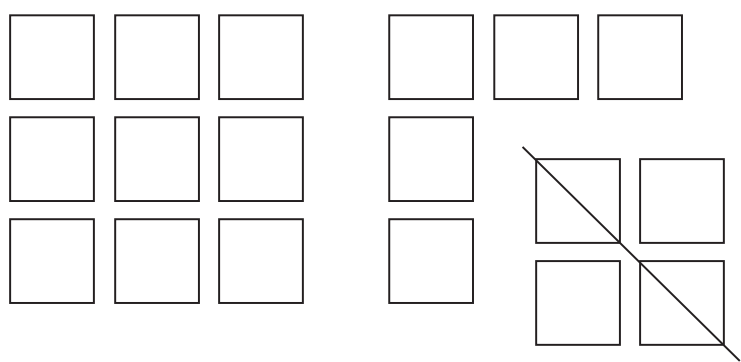


Figure 5.17

c $42 - 32 = 4 + 3$

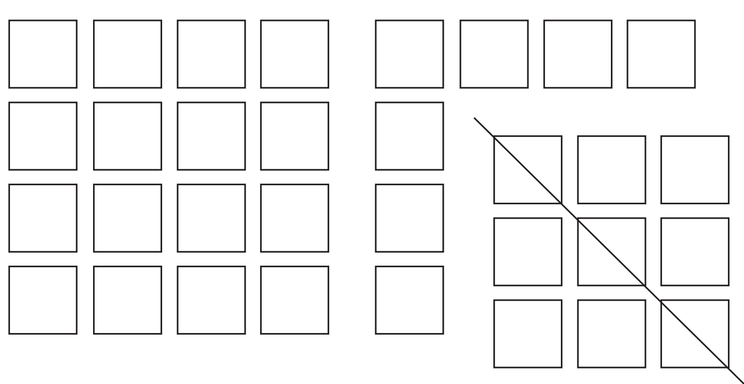


Figure 5.18

Now show this with some of your other examples. You can use square grid paper.

5. It looks like that we can generalise the result. What if we use ∇ for the first number and □ for the second number, then we will get $\nabla^2 - \square^2 = \nabla + \square$. This does not seem to help us;
6. What if use x for the first number and y for the second number, then we will get $x^2 - y^2 = x + y$? Which you know is not true. In fact, we saw that the statements are only true for consecutive numbers. So, our symbols need to show that these numbers are consecutive. If we let the lower value be x then the higher value must be one more for example, $x + 1$.

   Now we can write $x^2 - (x+1)^2 = x + x + 1$.

   Solve for x to show that this is true;

   Now let us show why it works in general using the visual 'proofs-without-words'. We can use area diagrams to show the general results.

So, a square with side length x + 1, can be shown by the following diagram to have an area $x^2 + 2x + 1$.

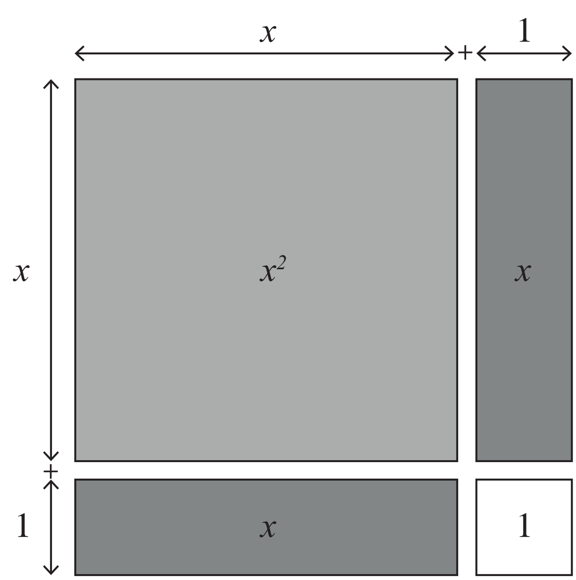


Figure 5.19

If we take away x^2, we will be left with x + 1 + x, which we can write as 2x + 1.

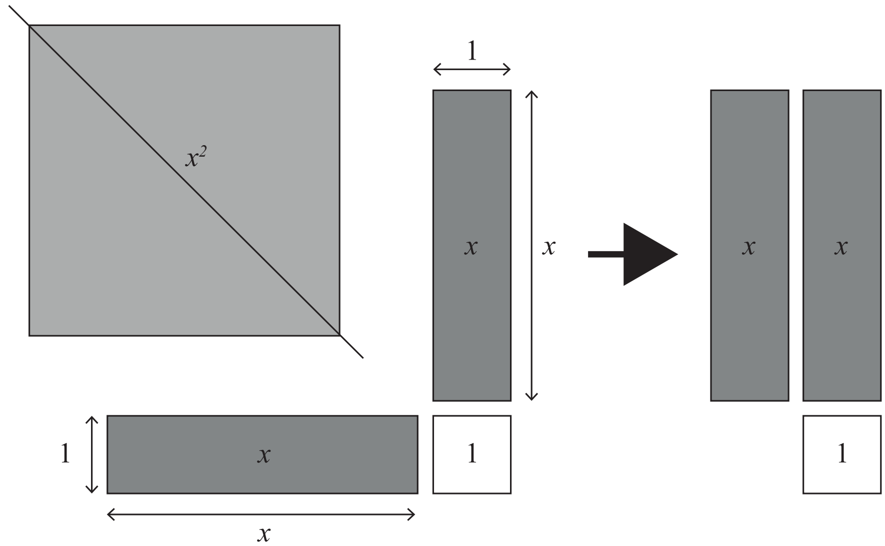


Figure 5.20

7. Watch the Vital Maths video: *Difference of two squares*
   http://vitalmaths.com/videos/all-videos/item/177-difference-of-two-squares

Assessment and follow-up homework: Adding two-digit numbers

1. Take any two-digit number whose digits sum to nine or less. Make a second number using the same digits but reverse the order. Add the two two-digit numbers.
   What do you notice?
2. Repeat what you did in Question 1, but use different two-digit numbers each time;
3. Can you make a general rule?
4. Does it work if the sum of the digits in your chosen number is more than nine, for example 76?
5. Now use symbols to show that this works when using a general case (not specific numbers).

5.1.4. Algebra in the early years: generalising towards the idea of a function

Expected learning outcomes

By the end of the lesson, you will be able to:

- Generalise patterns with shapes;
- Identify that patterns can be represented in different ways;
- Demonstrate that different representations can make different aspects of a pattern visible;
- Represent patterns in pictures or diagrams, in words, in function tables and function diagrams and in equations and graphs; and
- Teach children how to generalise from patterns with shapes.

Learning activity 1: Pair Work

1. Use toothpicks or matches to copy this pattern.

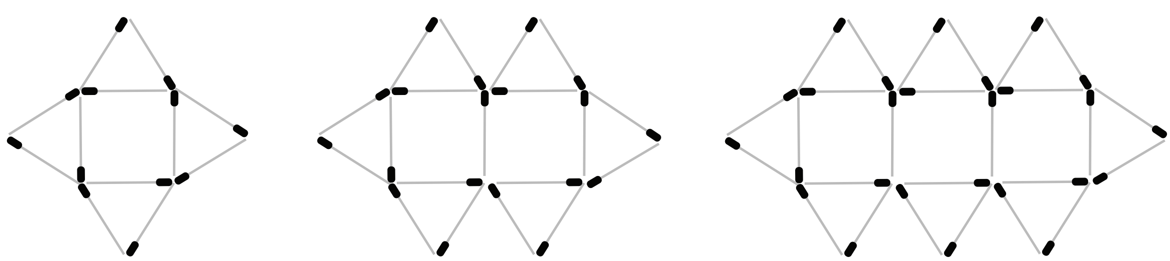


Figure 5.21.

a. Extend the pattern;
b. Describe the pattern in words;
c. Say how each shape is similar to the one that comes before it;
d. Say how each shape is visually different to the shape that comes before it;
e. Where do you add the extra toothpicks of matchsticks?
f. How many do you add each time?
g. Show the pattern in a numbers sequence: 12; 19; 26 …;
h. What do you do each time to get to the next number in the sequence?
i. If you went back to Shape 0, what number would you get?
j. How would Shape 0 be represented with matchsticks?
k. Show this pattern in a function table;

Shape number	1	2	3	4	5	6	7	8	9	10	20	100
Number of matches	12	19	26	33								

Figure 5.22.

2. Show the pattern in a function diagram.
Input: Shape number; Output: Number of matches.

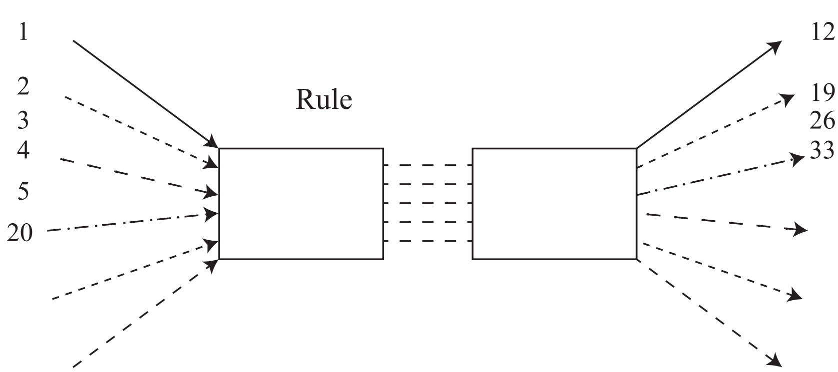


Figure 5.23

a. Write an equation which describes the function represented above; and
b. Draw a graph of the function above.

Learning activity 2

1. Use counters or bottle tops to copy this pattern.
2.

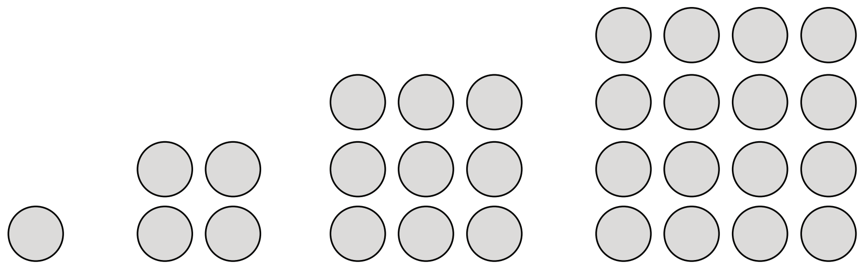


Figure 5.24.

a. Extend the pattern;
b. Copy this pattern onto square grid paper;
c. Describe the pattern in words;
d. What is the same about all of the shapes you built?
e. What did you change each time you built another shape?

3. Different recordings help you to generalise the pattern.
Show this pattern on the multiplication chart below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Figure 5.25.

Show this pattern in the function table below.

Shape number	1	2	3	4	5	6	7	8	9	10	20	50
Number of counters	1	4	9	16								

Figure 5.26.

a. What is the relationship between the shape number and the number of counters for example?

- What are the possible relationships between 3 and 9?
- What are the possible relationships between 2 and 4?
- What are the possible relationships between 4 and 16?
- Is there one rule that can link all of the above pairs of numbers?
- Will this rule also describe the relationship between 1 and 1?

Show this pattern in a numbers sequence: 1; 4; 9; 16; ___; ___; ___; ___; ___; ___

Each person in the class gets matches to build a square;

- The person who gets 4 matches builds the 2nd square;
- The person who gets 9 matches builds the 3rd square; and
- You get 144 matches; what square number do you build?

In the pattern above, the number of counters in each shape has increased but the pattern is neither of repeated addition (or subtraction), nor multiplication nor a combination. Write an equation which describes the function represented above.

Learning activity 3

Watch the TED Talk by Stanford Mathematics professor Jo Boaler entitled '*How you can be good at Maths and other surprising facts about learning.'*

https://www.youtube.com/watch?v=3icoSeGqQtY

Homework

Investigating and re-describing geometric patterns: Learning to generalise beyond the few diagrams provided

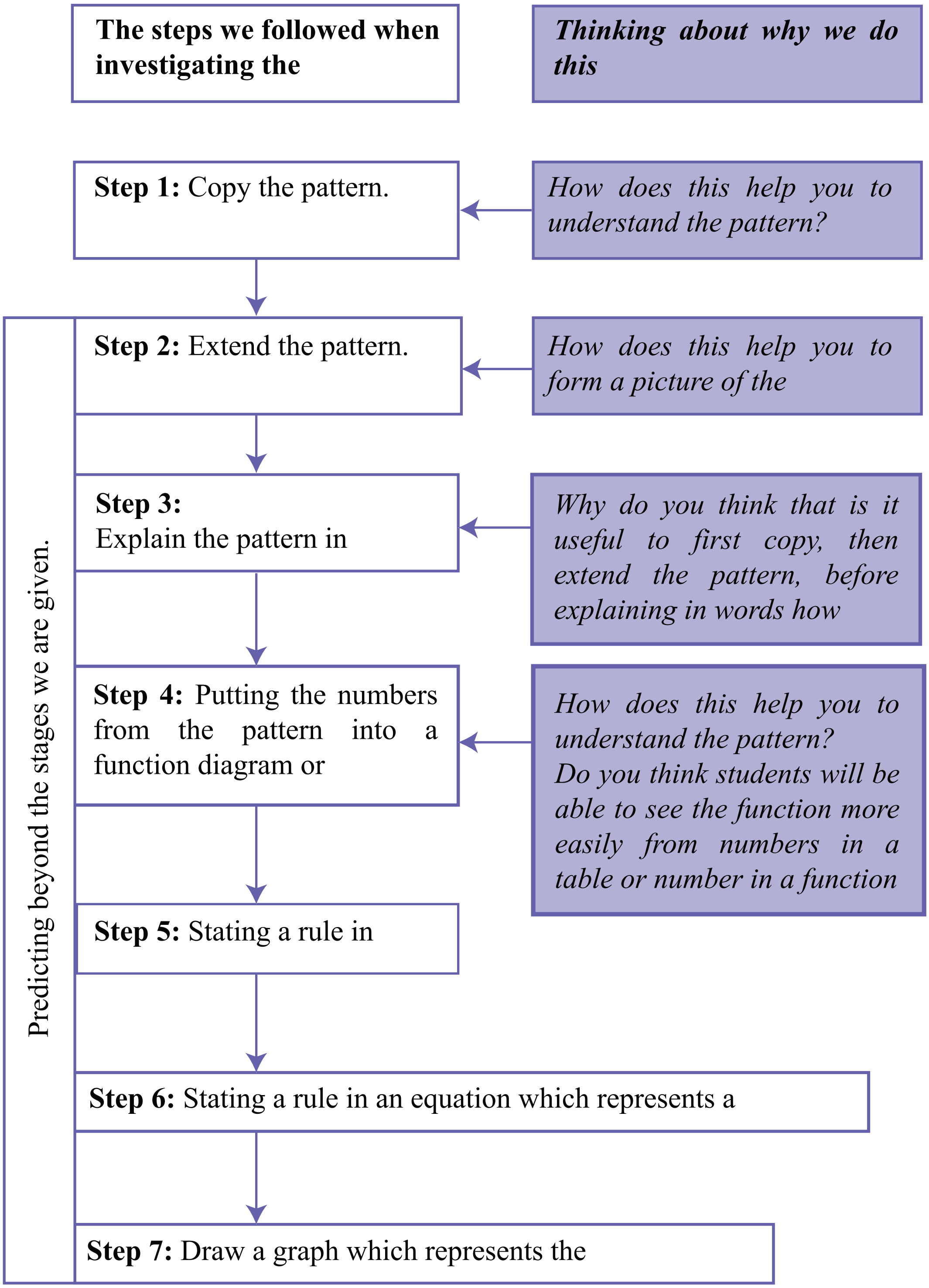

The steps we followed when investigating the
Thinking about why we do this
Step 1: Copy the pattern.
How does this help you to understand the pattern?
Step 2: Extend the pattern.
How does this help you to form a picture of the
Step 3: Explain the pattern in
Why do you think that is it useful to first copy, then extend the pattern, before explaining in words how
Step 4: Putting the numbers from the pattern into a function diagram or
How does this help you to understand the pattern? Do you think students will be able to see the function more easily from numbers in a table or number in a function
Step 5: Stating a rule in
Step 6: Stating a rule in an equation which represents a
Step 7: Draw a graph which represents the
Predicting beyond the stages we are given.

Figure 5.27.

There are many different ways to solve a problem. There are many different ways to approach a mathematical question. Sometimes, you need to approach a single question in lots of different ways. Each way gives you a little bit more information. Sometimes, you can just see the answer, because you brain just skips through all the steps. But you can teach your brain to do this.

1. **We copy the pattern**

   Working physically with our hands on the apparatus helps us to see a lot more than just looking at the diagrams on the page. When we actually build each shape, we see what the changes are;

2. **We extend the pattern**

   This helps us to apply what we have seen from building the stages of the pattern. In some ways, even if we are not aware of it, we may be forming ideas about how the pattern works. If we can state our ideas about how we think the pattern works, we call this a hypothesis in Mathematics. This is the start of a rule, but we need to test it before it we can be sure, and state it as a rule. We can check if our ideas about the pattern are correct by fiddling with the objects (counters / matches / boxes etc.) until the next set(s) of objects start to fit the pattern. This gives us more chances of seeing how the pattern works; and

3. **We explain the pattern in words**

   When you as student teachers explained your pattern, did everyone see the pattern the same way? Did everyone explain the pattern the same way? Did you understand other student teachers' explanations?

Explaining the pattern in words can sometimes be difficult. It is often easier to see what the pattern does, than to find the words to say how it works. Students may use hand signals a lot to explain themselves. Explaining what you see in words takes practice. Students need to be given time and support before they are able to state the pattern in words. You can ask them the following sorts of questions to help them explain their thoughts:

- 'How is the 3rd shape different to the 2nd shape?'
- 'How is the 2nd shape different to the 1st shape?'
- 'When you built the 4th shape: How did you start? What did you do next? Why did you do that?'
- 'When you built the 5th shape: How did you start? What did you do next? Why did you do that?'
- 'What if you move backwards from the 3rd shape, what would you take away to make the second shape?'
- 'What would you take away from the 2nd shape to make the 1st shape?'
- 'If you were to repeat this and take away items in the same way from the 1st shape, what would you be left with?'
- 'What is the same about each shape?'
- 'How does each shape differ from the one before?'

You can use colours on the diagrams to show where:

- The 1st shape is contained in the 2nd shape
- The 2nd shape is contained in the 3rd shape
- They added the extra matches / counters / dots / lines and so on

You could leave spaces between the matches or counters to show where the preceding shape is contained in the current shape.

Help to build students' mathematical language so that it is easier for them to explain their mathematical thinking.

One of the important skills of teaching is to learn to listen for the logic in other people's thinking. Not everyone thinks the same way. Not everyone sees pictures in the same way. People will see many of the patterns above in different ways; the process that people follow when they build or extend the pattern will be different. Because of this, people's explanations will differ. Just because an explanation is not what you expected, it does not necessarily make it incorrect. Train students to listen to each other even if they see the pattern differently. Focus students both on what they see, for example, the shapes in the pattern, and on how they built/drew the shape, not only the shape itself. Examples of the sorts of description students could give are:

Table 5.2 Describing patterns

Describing what they see in the pattern in their own words	Describing how they made / built the pattern or answering the question 'How do I get from one stage to the next'
'It is a pattern of triangles / squares/ hexagons'	'I added one more match to each side of each triangle/square/hexagon'
'Each triangle/square/ hexagon is bigger than the one before'	'Each triangle/square/hexagon has one more match in each side than the triangle/square/hexagon on its left'
'Each shape has one more triangle / square / hexagon than the one before it'	'Each shape has one/two/three more match/matches than the shape on its left'

Do not encourage students to just put in words the arithmetic rule that they may see from the function diagram. Only after students have made sense of the function are, they asked to write an algebraic rule.

4. **We put the pattern numbers in a table or a function diagram**

   Once students can talk about a pattern, the next stage is recording the pattern. This recording often allows us to see new things which we can then talk about and record in a different way, and so on and so on. Another way to explain these processes is in the diagram of the helix below:

Not everyone thinks the same way: Some people think more easily in pictures. Some people think more easily in words. Some people think more easily in sounds. Some people think more easily in symbols.

For some, it is easy to see the pattern from the pictures; for others, it is easy to see the pattern from the function diagram (reflect: what is more clearly visible in a function diagram than in a table?); for others, it is easy to see the pattern from the tables.

There are many other different ways to think. Many people think in more than one way. Looking at a pattern in different ways gives us different clues to finding the rule, or solving the problem.

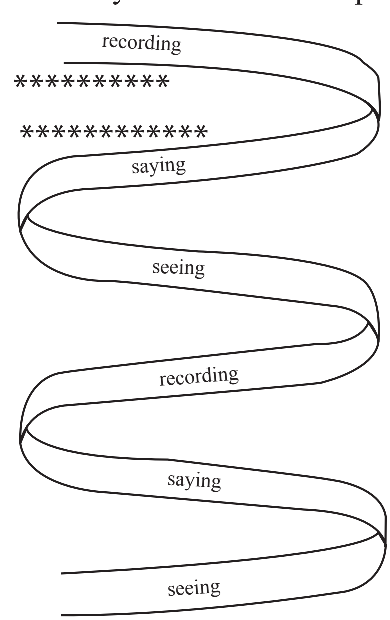


Figure 5.28.

But it is not only people who are different – the patterns themselves are different. For some patterns, certain ways of recording allow you to see best how the patterns works. For other patterns, a different way of recording will reveal the way the pattern works better.

The function diagram or table allows us to see the pattern differently. They are another aid to understanding how the pattern works. The numbers can show us information about the pattern that was difficult to see in the pictures of the pattern. Putting the numbers in a function diagram or function table, is a way of recording: recording is an aid to seeing generality more clearly. This helps us to see more of the pattern. Then we could say more about the pattern, and record what we had said.

When young students look at a pattern in a table, their first response is often to look at the difference between numbers in the bottom row of the table. For example, in the table below, students may focus exclusively on the bottom row.

Number of triangles	1	2	3	4	5	6
Number of matches	6	10	14			

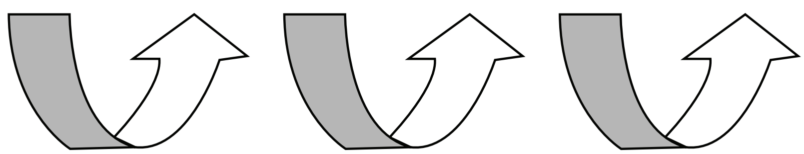


Figure 5.29.

Looking only at this row of numbers draws on the work that students have done with additive thinking, counting in groups and number sequences. So in some ways it is logical that students read the question as 6, 10, 14, ____, ____, ____, ______.

They may then see that they are adding four each time. This allows them to generate the next number in the pattern through counting. However, it does not allow them to generate any term or stage in the pattern. For example, this strategy is not useful if they are asked for the 200th term. Encourage students to see the relationship between the stage and the number of objects. Let us consider the following pattern:

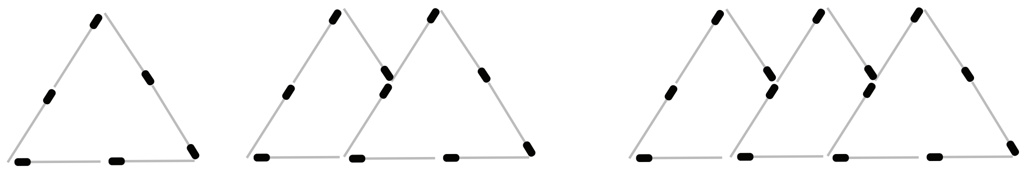


Figure 5.30.

One is trying to establish the relationship between the stage and the number of matches used, or the number of triangles and the number of matches. The relationship between these two numbers is created by the function or rule. If students want to predict beyond the given stages, they need the rule. In order to access this rule, they need to look at the relationship between the numbers in any column.

Number of triangles	1 ⇩	2 ⇩	3 ⇩	4	5	6
Number of matches	6	10	14			

Figure 5.31

- To build 4 triangles you use ___ matches;
- To build 5 triangles you use ___ matches; and
- To build 6 triangles you use ___ matches.

Where can you start to see the relationship between the number of triangles and the number of matches? Let us start with two triangles, then we can look at the bigger shapes. What must you do to two to get ten? You could add 8.

- Does this pattern work for three triangles? Is 3 + 8 = 14?
- No, what else could you do to two to get ten? You could multiply two by two and then add two.
- Does this pattern work for two triangles? Is 3 x 4 + 2 = 14? Yes.
- Does this rule work to get from one to six? 1 x 4 + 2 = 6. Yes.
- Then we can check that it works for the other stages you have built in the pattern.

5. **Stating the rule in words and in an equation that represents the function**
   It may well be easy for you with your experience of algebra to start with this step, but do not assume that this is the starting place for students or that the other steps are less important. The main aim is for students to develop an understanding of generalising towards the idea of a function.

6. **Draw a graph of the function.**

7. **Predicting beyond the values given**
   Being able to use the rule of the pattern, predicting shapes we have not built is important. Predicting is one of the power keys in Mathematics. From looking at the pattern in different ways, we had a good idea about how the pattern worked. We could use this to predict the pattern for shapes we had not yet built.

You may find that if students work too much with a number line or number grid and too little with tables and function diagrams, they may struggle to see the general rule or to be able to predict beyond the numbers they are given.

Review questions

1. What is relational thinking in Mathematics?
2. Why is the image of a balance often used when introducing algebraic equations?
3. How do Mason et al. (1985) describe algebra?

5.2. Geometry

In the Education College primary school Mathematics syllabus, this sub-strand of mathematical Modelling and Representation is called Geometry and Trigonometry. However, in Year 1, the focus is on learning to teach at Primary School level, so the focus of this sub-unit is exclusively on Geometry.

This sequence of lessons focuses on Geometry in the primary curriculum. The lessons are designed to explore methods and approaches to teaching the knowledge and the skills that are part of this curriculum. The lessons cover lines, circles, triangles, quadrilaterals, pentagons and hexagons, and the interior angles of polygons.

5.2.1. Lines

In this lesson, we explore some methods to teach primary students about right angles, perpendicular and parallel lines, how to fold them and how to draw them.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Teach primary school students about right angles, perpendicular lines and parallel lines;
- Teach primary students to fold right angles, perpendicular lines and parallel lines; and
- Teach primary students to draw parallel and perpendicular lines.

Learning activity 1

You will go outside the classroom and your lecturer will instruct you to position yourselves so that your bodies together make lines and shapes.

Learning activity 2: Right angles

1. Look around the classroom. Give examples of objects and shapes with right angles;
2. In the section on angle measure, you folded a right angle;
3. Fold a right angle;
4. Then open up the folded paper: the fold lines will cross each other at right angles;
5. The fold lines will be perpendicular to each other; and
6. A line is perpendicular to another if it meets or crosses it at right angles (90 degrees).

Right Angles

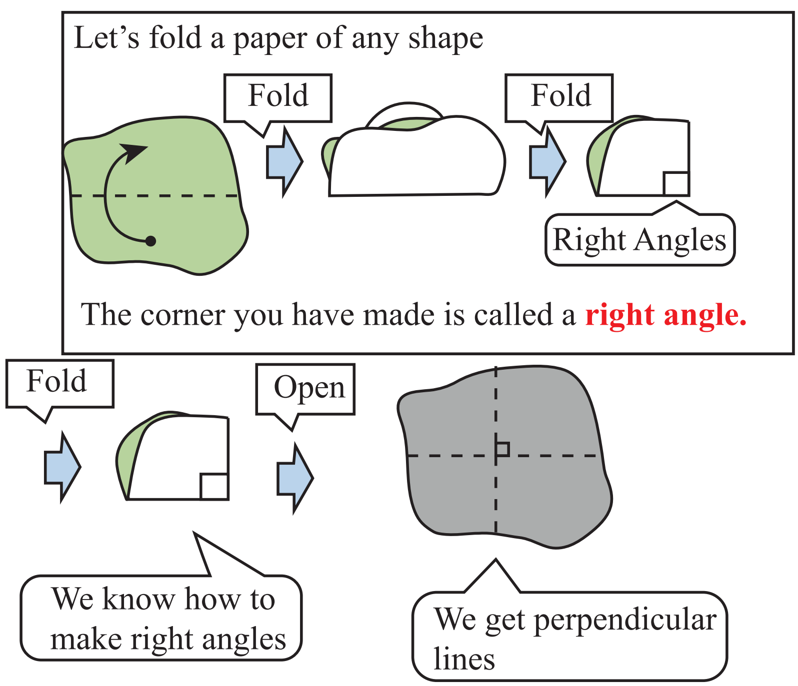


Figure 5.32

Learning activity 3

Most textbooks and Mathematics education courses show how students can draw perpendicular and parallel lines with a set square and ruler. They show how to construct perpendicular lines and to drop a perpendicular using a pair of compasses. This is difficult for students who do not have access to a mathematical set. Sometimes, these sections of the syllabus are left out simply because students do not have the specialist equipment. In this activity, we model ways in which students can draw parallel lines with other more readily available equipment.

Take a book (or anything with a straight edge such as a ruler, CD cover, mobile phone and so on). Hold the object static with one hand while you draw a line along the straight edge of the object.

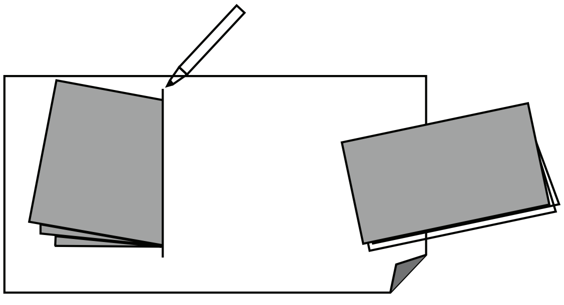


Figure 5.33.

Take another book (or any other object whose straight sides are at right angles to each other for example, CD cover, mobile phone).

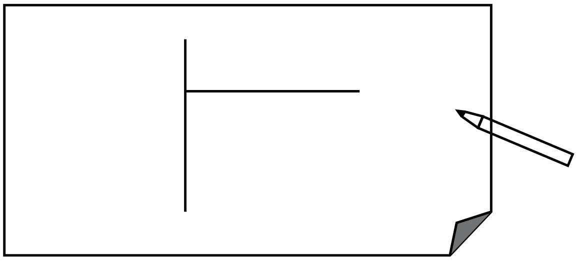


Figure 5.34.

1. Position the second book so that one edge touches the first book. Hold both books in place as you draw a line along the edge of the second book.
2. Remove both books: you will have drawn two lines that meet at right angles. The two lines are perpendicular to each other.

Learning activity 4

In the old primary curriculum, 'parallel lines' were defined as 'two lines are parallel if the distance between them is always the same.' But in the new primary curriculum, it is defined as 'two lines are parallel if both lines are perpendicular to another line'. This is because we need to use perpendicular lines to find the distance between parallel lines.

Use the diagram below to fold a set of parallel lines.

Make parallel lines by folding paper. Recall the definition first.

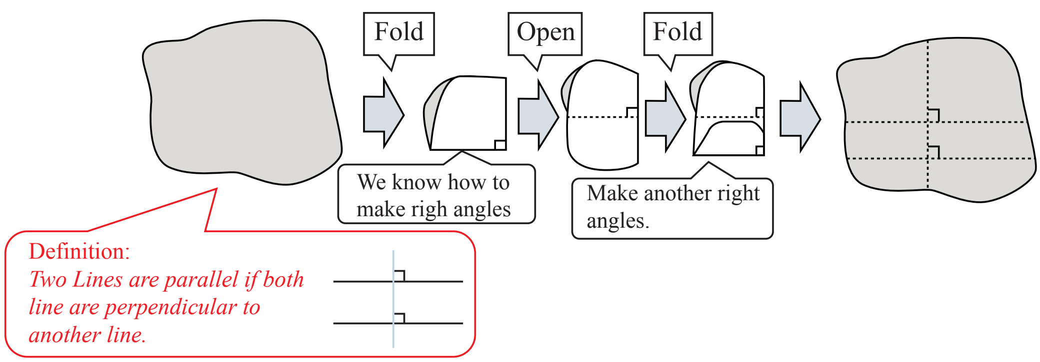


Figure 5.35.

Look around the classroom, find pairs of parallel lines.

Learning activity 5

Take two books. Position them at right angles to each other, with their spines touching, as shown in the picture below**.** (You could also use other objects such as CD covers, books, pieces of hardboard, mobile phones.) Draw a line along the spine of the one book, as shown below.

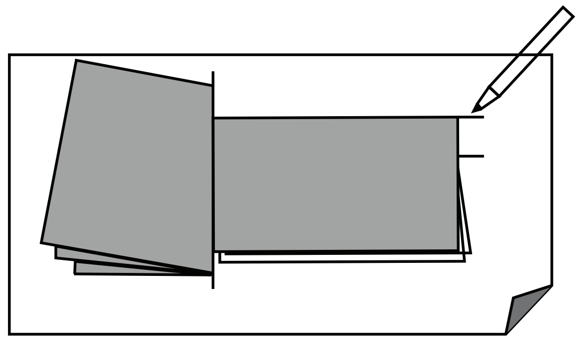


Figure 5.36

1. Slide this book along the edge of the other book, as shown below. Draw another line along the spine of the book as shown below. Both of the lines that you drew make right angles with the stationary book on the left.
2. Remove the book. You now have a pair of parallel lines.

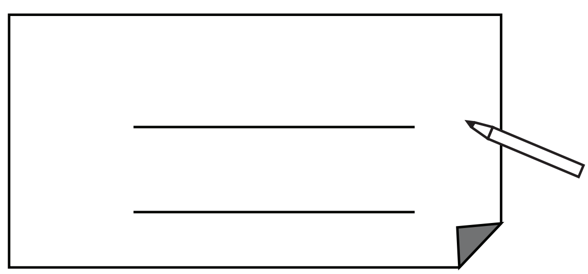


Figure 5.37.

5.2.2 Circles

In this lesson, we explore the definition of a circle and the names of its parts. We explore how to teach primary school students to draw circles and to make patterns with circles.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Explain how a circle is defined, and name the parts of a circle; and
- Draw circles with or without pairs of compasses and make patterns by drawing circles.

Learning activity 1

A circle is the set of points in a plane that are the same distance from the centre of the circle. The distance from the centre of the circle is called the radius. Twice the radius is known as the diameter.

For this activity, you need a drawing pin and a strip of card.

1. Cut a cardboard strip about 2 cm wide and 8 cm long. Make a hole 1 cm in from each end. Make more holes at 1 cm distances from each other;
2. Pin the strip of card to a page in your books with a drawing pin;
3. Draw a circle by placing your pencil in one of the holes, use the pencil to move/ turn the card strip a full revolution. What shape have you drawn? We will call this strip of card our circle-maker;
4. Keep the drawing pin in the same place, but this time place your pencil in another of the holes;
5. Repeat step four above; and
6. What is different between the circles you draw when you use a hole that is close to the drawing pin and when you use a hole further from the drawing pin?

Learning activity 2

Work in pairs. Name each part of the circle:

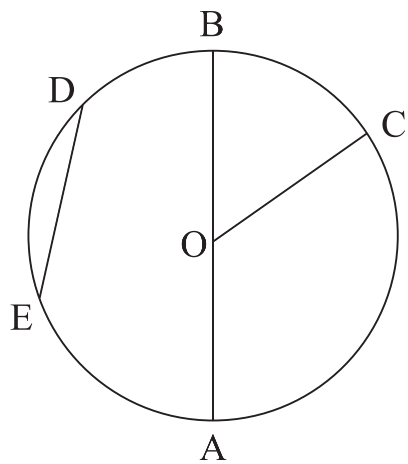


Figure 5.38.

- The outer edge of the circle;
- The line AB through the centre (O) of the circle;
- The line OC from the centre to the edge of the circle;
- The straight line DE; and
- The curved part of the circle DE.

Learning activity 3

Use your circle maker to make the following patterns:

1. Intersecting circles: Circles whose circumferences touch.

2. Concentric circle: Circles with the same centre.

3. Circles that touch in one place on the circumference.

Learning activity 4

1. Draw a line across your page.
2. Draw a circle. Position the centre of the circle on the line.
3. Draw a second circle by placing the drawing pin (or point of your pair of compasses) on the circumference of the first circle and your pencil in the centre of the first circle.
4. Draw a line connecting the two points where the circles intersect.
5. Your two lines are perpendicular.

Self-study

Do an internet search about drawing patterns with circles. Make an illustrating poster with step-by-step instructions on how to draw more complex patterns of intersecting circles.

5.2.3. Triangles

In this lesson, we classify triangles based on their angles and on the lengths of their sides, and we explore how to teach students to draw triangles using grid paper.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Classify and name triangles according to the sizes of their vertices;
- Classify and name triangles according to the relative lengths of their sides;
- Draw specified triangles on grid paper or with a pair of compasses or a circle-maker; and
- Draw lines that connect dots or point where grid lines intersect, and understand that slanting lines are longer than vertical or horizontal lines between the same number of dots or points of intersection.

Learning activity 1

Below we have dotty paper.

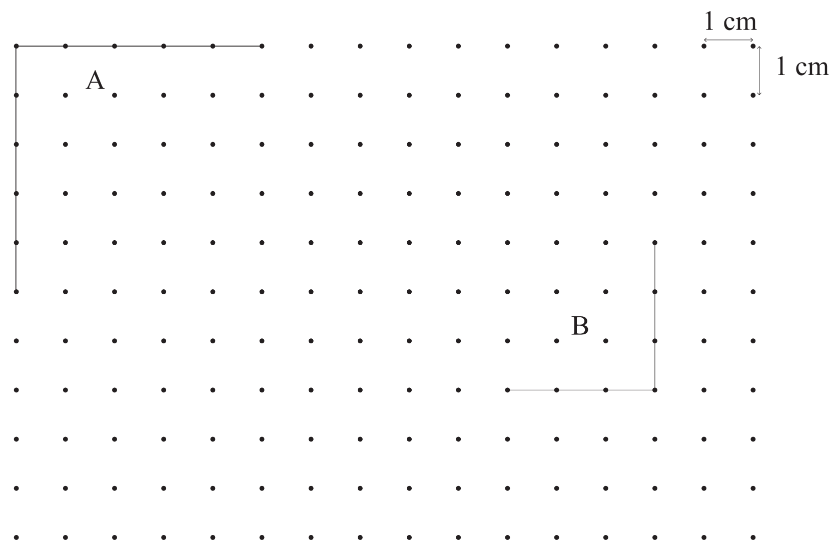


Figure 5.39.

1. Draw Triangle A on the dotty paper above. Two sides of 5 cm are already provided. Draw in the third side;
    a. How many dots does the third side connect?
    b. Measure the length of the third side.
    c. Is it 5 cm, less than 5 cm or more than 5 cm?
2. Draw Triangle B on the dotty paper above. Two sides of 3 cm have already been provided. Draw in the third side.
3. Draw a square on each side of Triangle B.
    a. Are all the squares the same size?
    b. Are any of the squares the same size? If so, which squares are the same sizes and which square is a different size?

Reading: horizontal, vertical and slanting lines on square grid paper

When we have a grid with parallel vertical and horizontal lines, it makes grid squares.

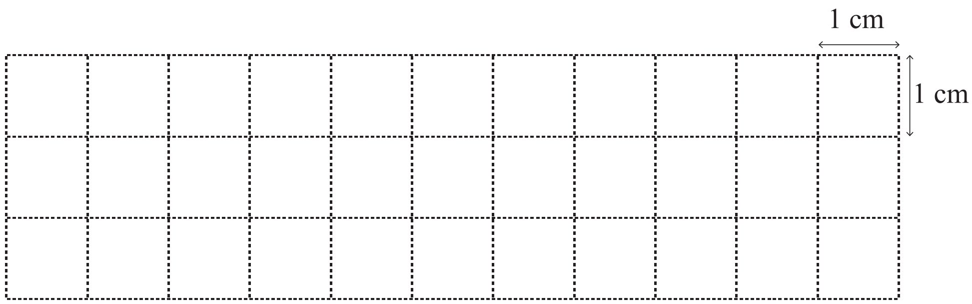


Figure 5.40.

When we have lines of dots arranged in vertical and horizontal parallel lines, it makes ‘invisible’ grid squares.

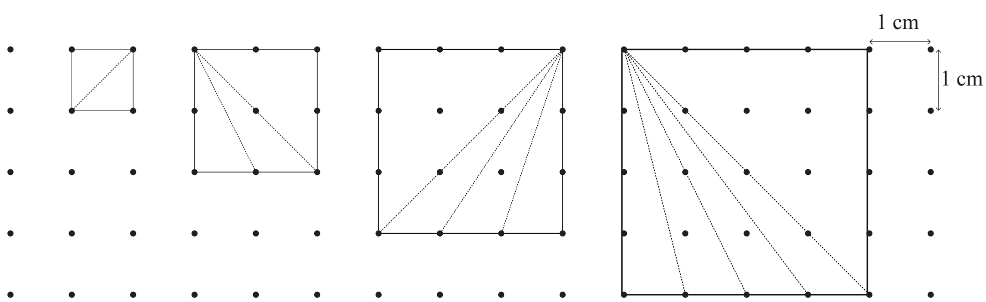


Figure 5.41.

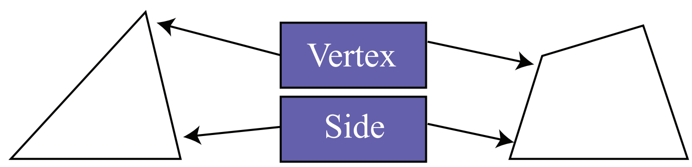
In any square grid, the distance between adjacent horizontal or vertical square lines is always the same. This is also true if the grid squares are shown by dots: the distance between adjacent vertically or horizontally adjacent dots is always the same. In the examples above, this distance is 1 cm. However, if we draw a sloping line between two dots, the distance is always greater than the distance between two vertically or horizontally adjacent dots.

If you are using a square grid, you cannot say that the distance of a sloping line between two dots is the same as the horizontal or vertical distance between the same number of dots.

Learning activity 2

Revising terminology: Read the extract from page 79 of the Grade 2 Mathematics textbook.

A straight line in a triangle or quadrangle is called a **side.**
The corner point of a triangle or quadrangle is called a **vertex.**



- A triangle has ______ sides and ______ vertices.
- A quadrilateral has ______ sides and ______ vertices.

Figure 5.42.

Draw triangles on the grid paper provided by your teacher educator.

1. Draw right-angled triangles on grid paper.
   a. Draw Right-angled Triangle A whose sides are all different lengths.
   b. Draw a Right-angled Triangle B which has two sides the same length.
   c. Can you draw a right-angled triangle with all of its sides the same length?
   d. Can you draw a triangle that has two right angles?

2. Draw triangles with only acute angles on grid paper.
   a. Draw Triangle C with only acute angles whose sides are all different lengths.
   b. Draw a Triangle D with only acute angles that has two sides the same length.
   c. Can you draw a triangle with only acute angles with all of its sides the same length, by connecting the dots on the grid above?

3. Draw obtuse-angled triangles on grid paper.
   a. Draw obtuse-angled Triangle E whose sides are all different lengths.
   b. Draw obtuse-angled Triangle F which has two sides the same length.
   c. Can you draw a triangle that has two obtuse angles?
   d. Can you draw an obtuse-angled triangle with all of its sides the same length?

Learning activity 3

We have seen that we cannot use the grid lines on a square grid to draw an equilateral triangle. Instead, we can use our card circle maker or a pair of compasses to draw a circle. What do we know about circles? A circle is a set of point equidistant from the centre of the circle. We can use the equidistance of the points that make up a circle to draw an equilateral triangle.

1. Draw a line.
2. Draw a circle, with for example a radius of 3 cm.
3. Position the centre of the circle on the line.
4. Draw a second circle by placing the drawing pin (or point of your pair of compasses) on the circumference of the first circle and your pencil in the centre of the first circle. This will ensure that the second circle is the same size as the first circle.
5. Draw two lines connecting the point the circles intersect either above or below the line. The one line should touch the point where one of the circles intersects with the line, and then to where the other circle;

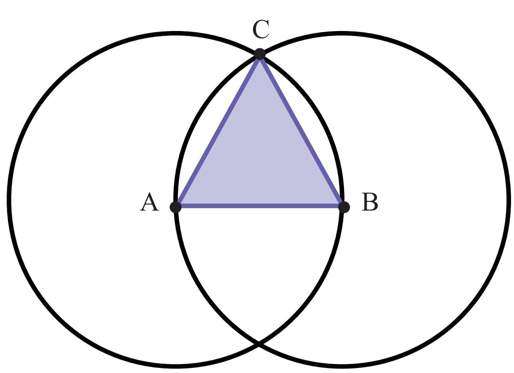


Figure 5.43.

6. You now have an equilateral triangle. All the sides are equal because the two circles are the same size, so the radii are all the same length, in this case 3 cm. Each side of the triangle is a radius of one or both circles. This is shown in the diagram alongside; and
7. Sometimes, people just draw arcs where the circles would intersect with the line and with each other.

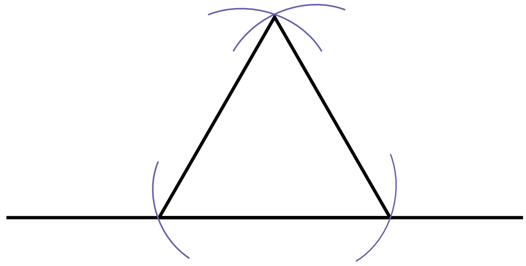
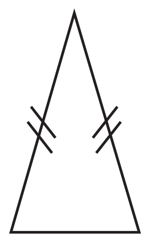


Figure 5.44.

Summarising classification of triangles

We can classify triangles according to the lengths of their sides.

Table 5.3 Classifying triangles by side length

Classification by side	None of the sides have the same length. Scalene triangle	Two of the sides have the same length  Isosceles triangle	All three sides have the same length Equilateral triangle

We can classify triangles according to the size of their angles.

Table 5.4 Classifying triangles by angles

Classification by angle	All of the angles are acute. Acute-angled triangle	One of the angles is a right angle. Right-angled triangle	One of the angles is an obtuse angle. Obtuse-angled triangle

- A triangle can have only one right angle. The reason for this will become clear in Lesson 5.2.6.
- A triangle can have only one obtuse angle. The reason for this will become clear in Lesson 5.2.6.
- A scalene triangle can be acute-angled, right-angled or obtuse-angled.
- An isosceles triangle can be acute-angled or right-angled or obtuse-angled.
- An equilateral triangle can only be acute-angled.
- An acute-angled triangle can be equilateral, isosceles or scalene.
- A right-angled triangle can be isosceles or scalene.
- An obtuse-angled triangle can be isosceles or scalene.

5.2.4. Quadrilaterals

This lesson is designed to explore the relationship between different quadrilaterals, the characteristics of these different quadrilaterals, and how to teach about quadrilaterals.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Classify and name quadrilaterals;
- Explain the relationship between different quadrilaterals, in other words, some are subsets of each other; and
- Facilitate students about quadrilaterals, trapezia, parallelograms, rectangles, rhombuses and squares.

Learning activity 1

Read the definitions of each quadrilateral. Then draw each quadrilateral on the 1 cm dotty paper provided.

Quadrilateral

A quadrilateral is a closed two-dimensional shape with four straight sides.

- Draw and label Quadrilateral A: make one of its sides the 2 cm line shown on the dotty paper.

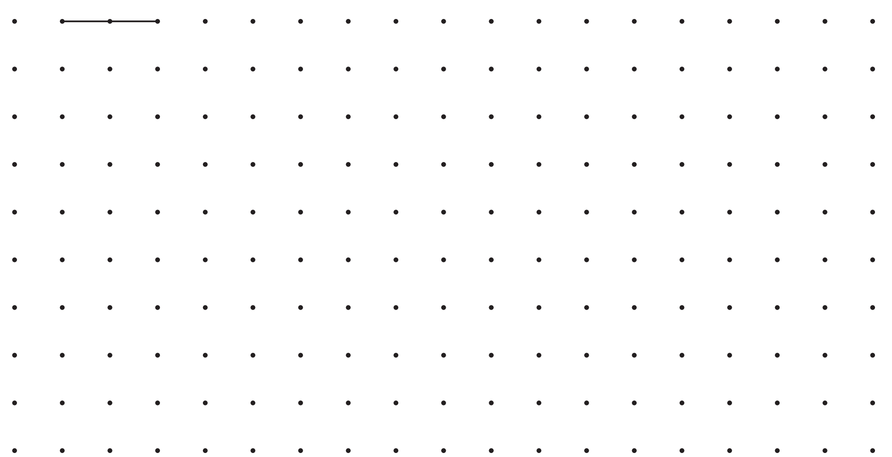


Figure 5.45.

- This particular quadrilateral should have.
  a. No sides parallel to each other;
  b. All sides a different length; and
  c. No right angles.

Trapezium

A trapezium is a closed two-dimensional shape with four straight sides, with one pair of opposite sides parallel to each other.

- Draw and label Trapezium B.
- This particular trapezium should have one side length 2 cm.
- Draw and label Trapezium C.
- This particular trapezium should have one side length 2cm, and one right angle.

Parallelogram

A parallelogram is a closed two-dimensional shape with four straight sides, whose opposite sides parallel to each other.

- Draw and label Parallelogram D.
- This particular parallelogram should have one pair of sides with length 2 cm.

Rectangle

A rectangle is a closed two-dimensional shape with four straight sides, whose opposite sides parallel to each other, and whose interior angles are right angles.

- Draw and label Rectangle E.
- This particular rectangle should have one pair of sides with length 2 cm.

Rhombus

A rhombus is a closed two-dimensional shape with four straight sides, whose opposite sides are parallel to each other. All its sides are the same length.

- Draw and label Rhombus F.
- This particular rhombus should have no right angles.

Square

A square is a closed two-dimensional shape with four straight sides, whose opposite sides parallel to each other. All its sides are the same length and whose interior angles are right angles.

- Draw and label Square G, with sides 2 cm long.

Reading: Summarising the relationship between quadrilaterals

As you read down the table, you will see that the figure in each line has additional criteria added to the one in the line above. This implies that each figure is more specific than the one in the row above it. This also implies that each figure is a special example of the figure in the row (s) above it.

A closed two-dimensional shape with four straight sides.	**Quadrilateral**		
A closed two-dimensional shape with four straight sides of which **one pair of opposite sides is parallel**	**Trapezium**		
A closed two-dimensional shape with four straight sides of which **two pairs of opposite sides are parallel**	Parallelogram		
A closed two-dimensional shape with four straight sides of which two pairs of opposite sides are parallel **and with interior right angles**	Rectangle	Rhombus	A closed two-dimensional shape with four straight sides of which two pairs of opposite sides are parallel **and all four sides have the same length**
A closed two-dimensional shape with four straight sides of which two pairs of opposite sides are parallel: **all four sides have the same length and with interior right angles**	Square		

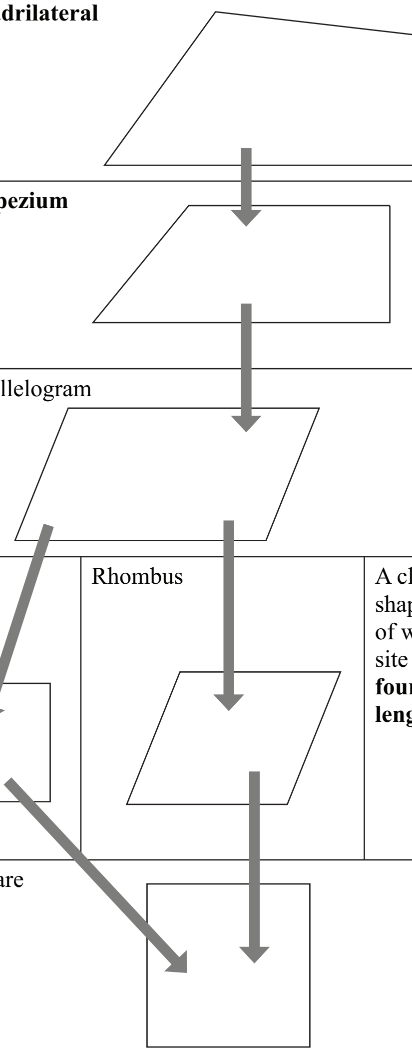


Figure 5.46.

Learning activity 2

1. Work in groups of four.
2. You will need;
    - A pair of scissors;
    - Rulers;
    - A piece of thick cardboard to cut into strips; and
    - Four split pins, or paper clips or pieces of cotton or string.
3. One pair makes a model that can change from a rhombus to a square. This pair needs to cut four cardboard strips 1 cm wide and 10 cm long;

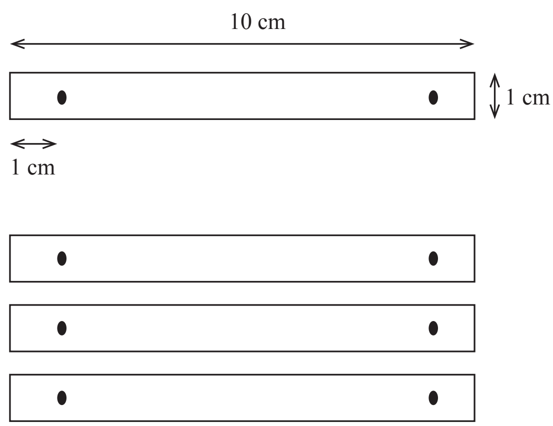


Figure 5.47.

4. Make holes 1 cm in from each end.
5. Join the strips with split pins to make a quadrilateral.

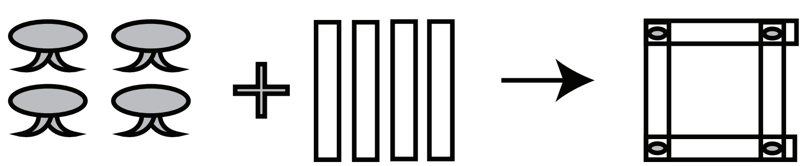


Figure 5.48.

6. Push on one side of the quadrilateral so that the sizes of the angles change.

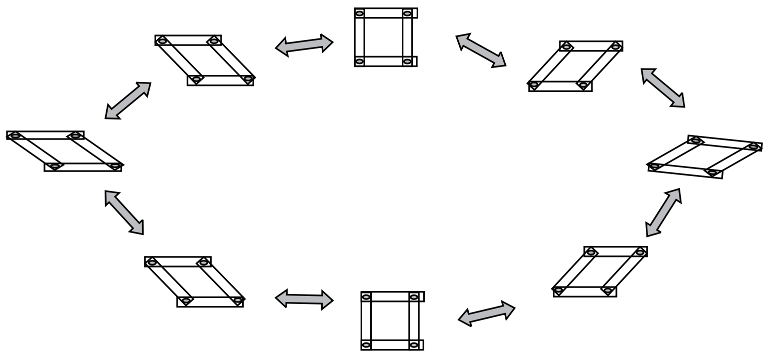


Figure 5.49.

7. One partner moves the shape. The other person says when it is a rhombus and when it is a square. The length of the strips of cardboard does not change. So, the sides of the shape keep the same length, and the sides also stay parallel to each other. Only the sizes of the angles change.
8. The other pair makes a model of a shape that can change from a rectangle into a parallelogram.
9. Cut two cardboard strips 1 cm wide and 8 cm long.
10. Cut two cardboard strips 1 cm wide and 12 cm long.
11. Make holes 1 cm in from each end.
12. Join the strips with split pins to make a quadrilateral.
13. You can move this quadrilateral to make either a rectangle or a parallelogram.
14. One person moves the shape. The other person says when it is a rhombus and when it is a square. The length of the strips of cardboard does not change. So, the sides of the shape keep the same length, and the sides also stay parallel to each other. Only the sizes of the angles change.

Learning activity 3

Groups and sub-groups of quadrilaterals

In everyday life, some categories of objects are mutually exclusive. They do not share features. For example, no sheep is a goat, no goat is a cow, no cat is a dog, no dog is a sheep, and so on. However, some categories of objects contain some or all the objects from other categories, for example, the category mammals contain sheep, goats, cattle, dogs, cats; the category domesticated animals also contains sheep, goats, cattle, dogs and some cats. Some domestic animals, for example, ducks and chickens, are not mammals. Some mammals are not domesticated animals.

We can use a Venn diagram to show how:

- Some groups of animals are contained within other groups of animals
- Some groups of animals overlap with other groups of animals
- Some groups of animals are distinct from other groups of animals

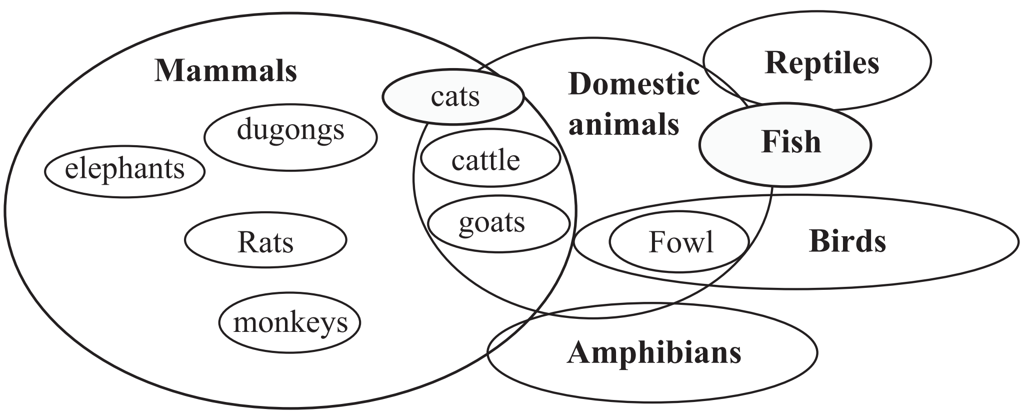


Figure 5.50.

Mathematics also has examples of categories that are mutually exclusive: No quadrilateral is a triangle but all quadrilaterals and triangles are polygons because they are all closed figures with straight sides. All squares, rhombi, rectangles, parallelograms and trapezia are quadrilaterals because they are all closed figures with four straight sides. In the diagram below, we show subsets of quadrilaterals.

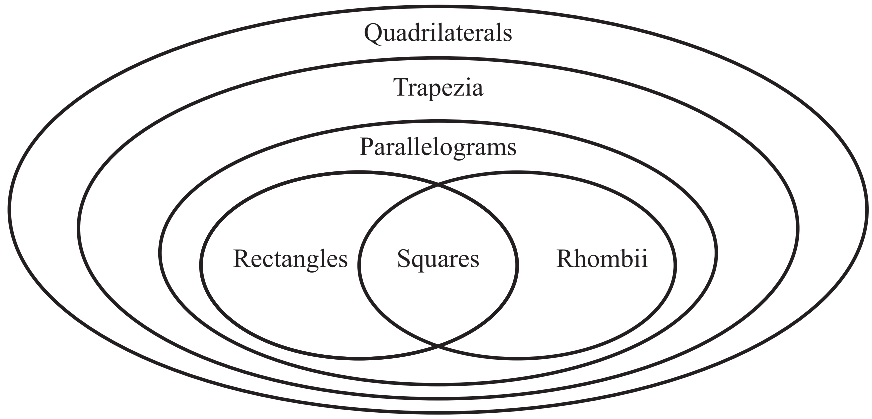


Figure 5.51.

1. Explain what the diagram above means.
2. Consider similarities and differences by answering the following questions
    a. How is a square similar to a rhombus?
    b. How is a square different from more general rhombuses?
    c. How is a rectangle similar to a square?
    d. How is a square different from more general rectangles?
    e. How is a rectangle similar to a parallelogram?
    f. How is a rectangle different from more general parallelograms?
    g. How is a parallelogram similar to a rhombus?
    h. What is different about a rhombus and more general parallelograms?
    i. How is a trapezium similar to a parallelogram?
    j. What is different about a parallelogram and more general trapezia?

Assessment: The relationship between quadrilaterals

1. Look back at the definitions of a parallelogram and a rectangle.
    a. Does a rectangle satisfy all of the conditions of a parallelogram?
    b. Does a rectangle have additional conditions that a parallelogram does not? If so, what are these conditions?
    c. Does a parallelogram satisfy all of the conditions of a rectangle?
    d. Explain why we can say that a rectangle is a special kind of parallelogram;

2. Look back at the definitions of a rhombus and a square.
    a. Does a square satisfy all of the conditions of a rhombus?
    b. Does a square have additional conditions that a rhombus does not? If so, what are these conditions?
    c. Does a rhombus satisfy all of the conditions of a square?
    d. Explain why we can say that a square is a special kind of rhombus.

5.2.5. Petntagons and hexagons

This lesson is designed to explore ways of teaching Grade 5 students about pentagons and hexagons. We explore paper folding methods to construct these polygons as well as using a pair of compasses.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Identify both regular and irregular pentagons and hexagons;
- Use paper folding to make regular pentagons and hexagons;
- Construct hexagons and pentagons using a variety of methods (For example, Paper folding, a pair of compasses or a circle-maker card strip); and
- Teach Grade 5 students about pentagons and hexagons.

Learning activity 1

A hexagon is a closed two-dimensional shape with six straight sides. It also has six vertices.

1. On the grid paper below, draw five different hexagons.

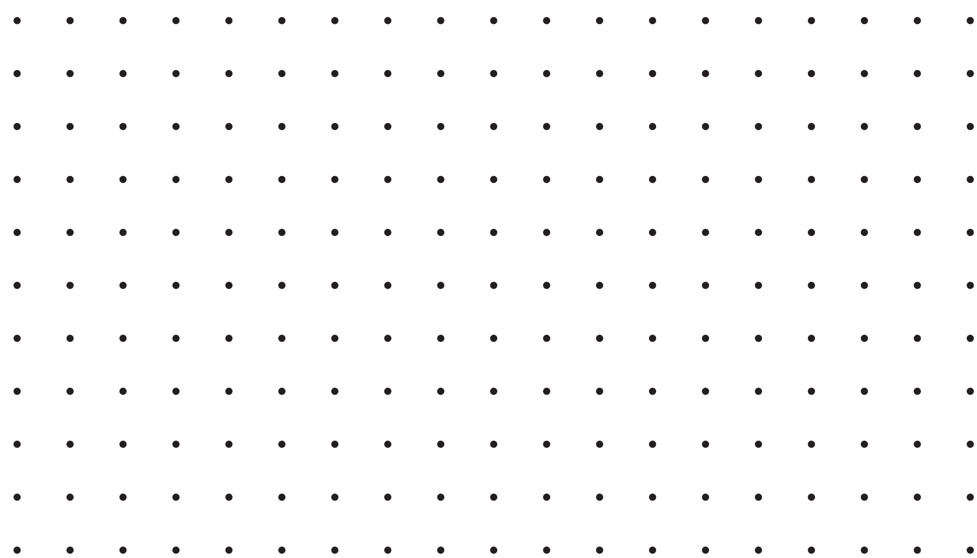


Figure 5.52.

2. Compare your hexagons with those drawn by the person next to you. Say which hexagons are the same and which different to each other.
3. Form a group of four students. Compare your hexagons: say which hexagons are different to each other and which are the same.
4. Use the dotty grid paper provided by your lecturer. Copy those hexagons that are different onto the dotty grid paper. You will use these to make a poster for homework.
5. Discuss: Why is it important for students to see different examples of hexagons?

Learning activity 2

1. Watch the video of how to fold a hexagon - https://www.youtube.com/watch?v=PQAc733KVEc
2. Now fold your own hexagon. Attach this hexagon to your hexagon poster; and
3. Step by step illustrated note of this process are provided at https://www.origami-resource-center.com/hexagon-from-a-square.html.

Learning activity 3

You will need a pencil, ruler (or straight edge) and a card circle maker or a pair of compasses.

1. Watch the video of how to construct a regular hexagon at: https://www.youtube.com/watch?v=NECjlQYjo4s;
2. Now construct your own regular hexagon.
3. Discuss with a partner: Why does this method of constructing a regular hexagon work?

Learning activity 4

A pentagon is a closed five-sided figure.

1. On the grid paper below, draw five different pentagons;

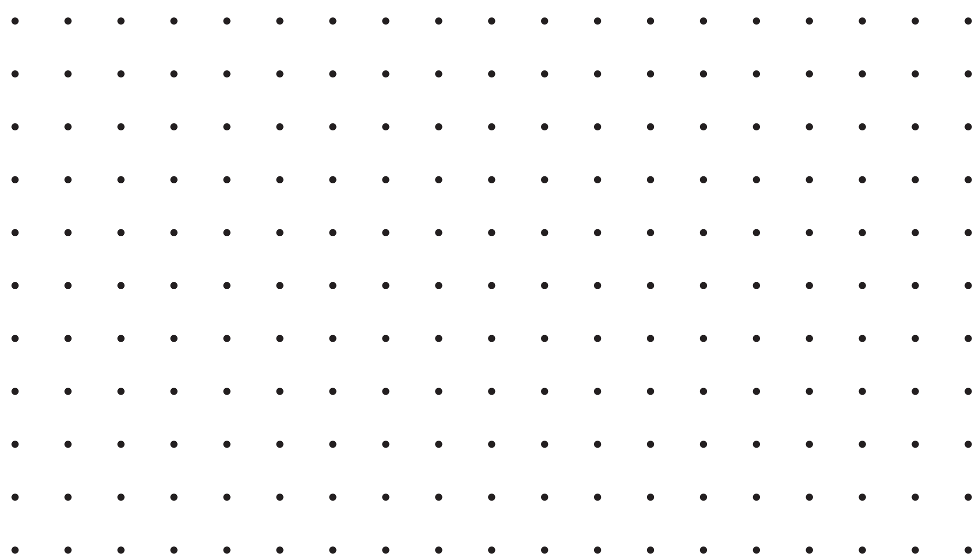


Figure 5.53.

2. Compare your pentagons with those drawn by the person next to you. Say which are the same and which different to each other.
3. Form a group of four students. Compare your pentagons: say which pentagons are different to each other and which are the same.
4. Use the dotty grid paper provided by your lecturer. Copy those pentagons that are different onto the dotty grid paper. You will use these to make a poster for homework.

Learning activity 5

1. Watch the video https://www.youtube.com/watch?v=83nKgFUiu-M.
2. Follow the video and fold a pentagon.
3. You can attach this pentagon to your pentagon poster.
4. Step by step illustrated note of this process are provided at http://www.inthekitchenwithmum.com/2015/01/how-to-fold-an-origami-pentagon/.

Homework

As the group of four that worked together in class, you should make two posters.

1. Make a poster to show different hexagons.
2. Cut out each hexagon that you drew that was different to the other hexagons including the regular hexagon that you folded.
3. Write a definition of a hexagon.
4. Make a poster to show different pentagons.
5. Cut out each pentagon that you drew that was different to the other pentagons.
6. Make a poster to show different pentagons including the regular pentagon that you folded.
7. Write a definition of a pentagon.

5.2.6.
Sum of interior angles of polygons

In this last lesson on Geometry, we investigate the sum of the interior angles of triangles and use this to explore the interior angles of other polygons.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Find the sum of the interior angles of a triangle;
- Teach primary students to find the sum of the interior angles of a triangle; and
- Explain the importance in Mathematics of using known facts to find unknown facts.

Learning activity 1

Interior angles of a polygon refer to the angles at the vertices of that polygon. If we add the size of the interior angles (the angles at the vertices), we get the sum of the interior angles of that polygon.

1. Draw at least three different triangles. Label the angles of one triangle a, b and c, or in some other way. Cut (or tear) out the triangle with the angles labelled.

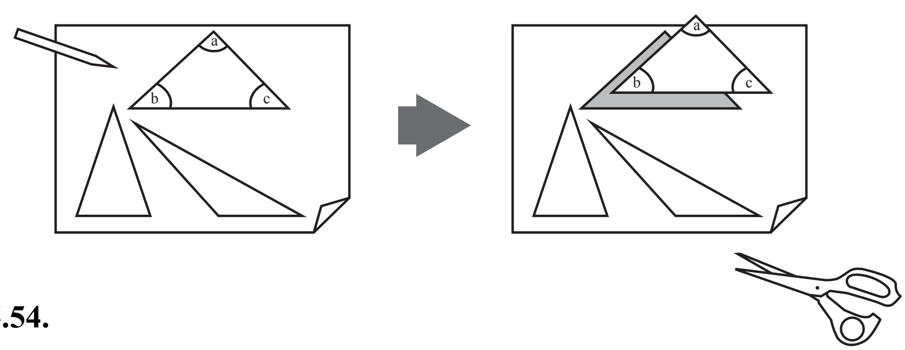


Figure 5.54.

2. Trace around this triangle on a clean sheet of paper, so that you have a record of it before you transform it.

3. Mark each angle of the traced triangle.
4. Cut or tear each corner off as shown below.
5. Place the three corners together along the straight edges.
6. What do you notice? What is the sum of the angles of your triangle?
7. What does this tell you about the interior angles of a triangle?
8. Did you start with the same kind of triangles as other students in the class? Did you get the same result?
9. Will you get the same result if you start with a different kind of triangle? Try it out.

Learning activity 2

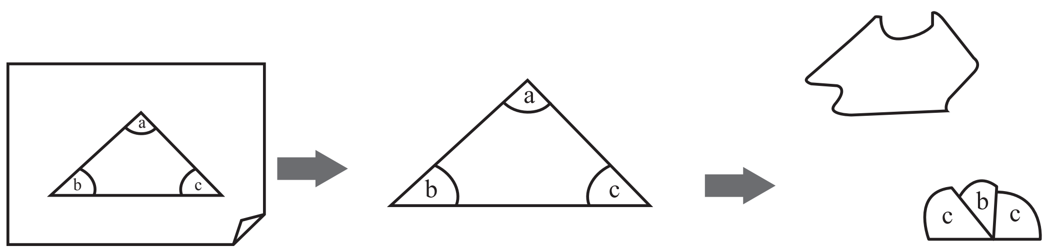
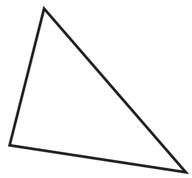
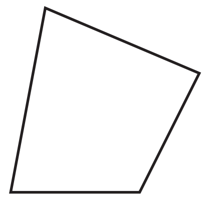
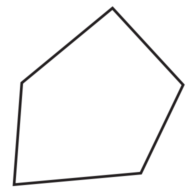
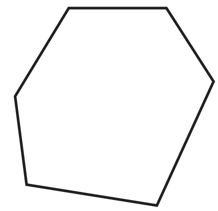


Figure 5.55.

10. The first question to ask is: ‘What do I already know that can help me here?’ You know from the activity above that the interior angles of a triangle add up to 180°. So, can we make triangles inside each of the shapes below? But remember that;

    - We want all of the interior angles of the triangles that we make to be part of the interior angles of the polygon we are investigating. For example, in the drawing alongside, four vertices of internal triangles are not part of the vertices of the quadrilateral.
    - The triangles must not overlap. Talk to the person next to you about why the triangles should not overlap.

Table 5.5. Shape types and angles

Polygons	Triangle	Quadrilateral	Pentagon	Hexagon
Drawing				
Number of internal triangles				
Sum of interior angles				

Diagonal

1. A diagonal is a line segment connecting two non-adjacent vertices of a polygon.
2. Draw diagonals from one vertex in each of the polygons above.
3. Count the internal triangles in each polygon.
4. Multiply the number of internal triangles by the sum of the degrees in the interior angles of a triangle.

Review questions

1. What do you know about circles that can help you draw equilateral triangles?
2. What is same and what is different between a trapezium and a parallelogram?
3. What does ‘The sum of interior angles of a polygon’ mean?

5.3. Time and Space

There are two lessons in this sub-unit. The first lesson focuses on physical maths: young students using their bodies and the space around them to learn Mathematics. The second lesson focuses on teaching the passing of time; this is also called duration of time.

Young children learn well if their bodies are involved. Physical Mathematics gets children outside and using the bodies and their heads doing fun Mathematics. The next two lessons explore how we can get young students to use their bodies to count and make geometric shapes, and then how we can teach time.

5.3.1. Using physical Mathematics (1)

This lesson explores ideas for teaching young students to use their bodies and do physical activities to assist them learning to count and investigate Geometry.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Understand how to use physical Mathematics to teach various aspects of the primary curriculum; and
- Develop some hands-on physical exercises/ lessons to help students practise counting and making various polygons.

Learning activity 1

This can be done outside with physical action. Young students learn well when they use their bodies. (Practise these in your groups).

Counting in 2s

1. Remember that this exercise is about counting.
2. You can use hands, feet or skipping. Hands can be used by clapping with a partner; Feet can be stamped one to the left and the two to the right.
3. When first practising, say both numbers and emphasise the multiples of two.
4. With practice, only say the even numbers.

Counting in 3s

1. Here, you can stamp two feet left and right and then clap.
2. When first practising, say all numbers and emphasise the multiples of three.
3. Then practise only saying the multiples of three.
4. You can also do this in pairs and clap hands together.

Counting in 4s

1. Stamp left and right feet, cross arms and then clap.
2. Count all the numbers and emphasise the multiples of four.
3. Then count, but only sound the multiples of four.
4. In pairs, you can touch your knees, cross your arms, touch hands and then clap.

Counting in 5s

1. Left hand-left knee, right hand-right knee, left hand-right shoulder, right hand-left shoulder, and clap. Count all the numbers and emphasise the multiples of five. Then count, but only sound the multiples of four.
2. In a circle of five students, count by raising left hands (one hand has five fingers) in sequence to the partner on the left. Count only multiples of five. Change direction.
3. In groups, devise routines to count in sixes, eights and tens. Practise your routines and present to the class.

Learning activity 2

In these exercises, the learning objective is to see how many shapes a group of four students can make with a 4 m piece of rope or string. The objective is for the students to make many different-looking shapes. In doing this, they will begin to understand what features of the shape can change and what properties stay the same. In this way, students will learn to recognise shapes and understand the definitions in a concrete way.

You will need a piece of string/rope/twine 4 m long. To make these shapes, a member of the group should stand at the corner. If there are too many people, the remaining person/people must stand in the middle of the shape.

1. Make a triangle: How many sides does a triangle have? How many corners does a triangle have? Now try to make a triangle that looks different. What is different between your two triangles? What makes the two triangles both triangles?
2. Make a quadrilateral: How many sides? How many corners?
3. Make a shape with straight sides that is neither a triangle nor a quadrilateral.
4. Make a square: How many sides? How many corners? Make a square that looks different. What is different between the two squares? What is the same? What makes a square?

Learning activity 3

Refer to the grade-wise curriculum. In groups, list the polygons that are the focus for each grade. Now complete a flipchart diagram that explains with shapes and questions what ‘physical Mathematics’ you could do for geometry in each grade.

5.3.2. Using physical Mathematics (2)

In this lesson, we explore how we could teach time across the grades using the textbook, and going outside to get students to use their bodies and heads at the same time.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Have experience of learning in a co-constructive way; and
- Develop some strategies for teaching the concept of time to young children.

Learning activity 1

Recap the learning from the previous lesson about physical Mathematics. What learning activities did you do? Discuss with a partner how the activities helped you to learn about the concepts of physical Mathematics.

Learning activity 2

In your groups, review the unit on 'Time and Duration' in the Grade 2 textbook. Capture your observations and answers to the questions that follow on flipchart paper.

In first example 2(a), where Su Su and her mother go shopping, what is the learning objective?

The big scales are multiples of five and there are 60 scales in all, including the small scales, on the clock.

In example 2(b), the learning objective is to recognise that there are 60 minutes in an hour and to do calculations in minutes.

Can you think of a way to teach this using physical Mathematics?

What do you think the learning objective is for the next example discussing 'Su Su and her mother having lunch'? Notice that students are always given an opportunity to practise.

What is the next example covering? What are the key concepts here?

In the last example describing Khine Khine's day, what is example five trying to teach?

What are the key operations students need to know in order to do the calculations in the exercises that follow?

Now devise some ideas using physical Mathematics to teach and test the concepts that have been covered in the textbook.

How could you create a clock where students are participating? What preparation would you need? In exploring this idea, how would you teach and test each of the component parts that are included in the text and any other to ensure that students are able to grasp telling the time and calculating duration?

Review questions

1. What is 'physical Mathematics'?
2. Can you give an example on how to use physical Mathematics for counting?

5.4. Data Handling

There are two lessons in this sub-unit. In the first lesson, we explore how Mathematics is used across subjects in depicting data. In the second lesson, we use the ideas in the textbook to devise a series of exercises to teach Grade 2 students about tables and graphs.

5.4.1.
Using bar graphs and pie charts across subjects

The topic of how Mathematics is used in everyone's world and how it is used in other subjects is covered quite frequently across this first year course. It is an interesting challenge for Mathematics teachers to constantly show this to students and get them to explore the Mathematics they would use in different settings. This lesson seeks to demonstrate one of the more frequently used applications of Mathematics across most subjects: using graphs to depict data.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Understand the number and operations skills that are required to develop graphic pictures used in other subjects;
- Develop skills to read and analyse graphical representations in informative articles and reports; and
- Develop some strategies for teaching the concept of analysing data to young children.

Mathematics across subjects

Mathematics is used in most other subjects in research and for presenting data. Most often, data is collected, measured, compared and analysed for research across subjects. In Life Sciences, Physics, Engineering, Economics, Geography, and the other Social Sciences, research that requires analysis of data is core to the scientific method that underpins our way of developing knowledge.

The skills to collect, collate, analyse and present data in various ways are needed in other subjects. Mathematics skills are also needed in languages to read and comprehend and to follow detailed instructions that require measurement and calculation and problem-solving. There is mathematical skill and comprehension in giving and following directions and in following the instructions of a recipe.

Mathematics is used to explain principles and theory and to model, design and undertake complex calculations.

The more abstract Mathematics in algebra, geometry and trigonometry, and calculus are used in subjects like Physics, Engineering, Chemistry and Astronomy to explain principles and theory and to model, design and undertake complex calculations.

Learning activity 1

Skills for collecting, organising and presenting data that are developed at primary school level are useful. Teachers can use this area to explain to students how Mathematics is important and used in the other subjects.

Frequency tables

Denpo and his student team were surveying the popularity of the various colours of cars. They observed and recorded the colours of 30 cars moving through a junction near their college.

These were his findings:

R	G	W	R	B	B	W	R	G	G
G	R	S	R	S	B	B	R	W	S
B	G	B	B	S	G	W	W	B	B

Figure 5.56.

The frequency table is a method to capture and organise the raw data. Denpo is able to capture the number of cars (the frequency that they are observed) of a particular colour in the table.

Car Colour	Tally	Frequency
Red	卌 |	6
Green	卌 |	6
White	卌	5
Blue	卌 ||||	9
Silver	||||	4
	Total	**30**

Figure 5.57.

In the case of the car colour, research the variable was discrete – a colour. Often the variable is continuous. In the case below, the weights of a class of girls ranging from 40kg to 60kg are measured. Categories of 5kg groups (for example, 40-45kg) are created for the frequency table.

Groupings	Tally	Frequency
56 - 60 kg	||||	4
61 - 65 kg	卌 |	6
66 - 70 kg	卌卌卌||||	19
71 - 75 kg	卌卌|||	13
76 - 80 kg	卌	5
81 - 85 kg	|||	3
	Total	50

Figure 5.58.

Now the frequency table can be translated into a picture so that the data can be read with ease and the variables and their frequency seen in a graphical form. This simple graph is a bar graph or histogram.

Grade	Girls	Boys
8	55	50
9	43	57
10	58	39
11	48	52
12	42	56

Figure 5.59.

The frequency table below is translated into a bar graph.

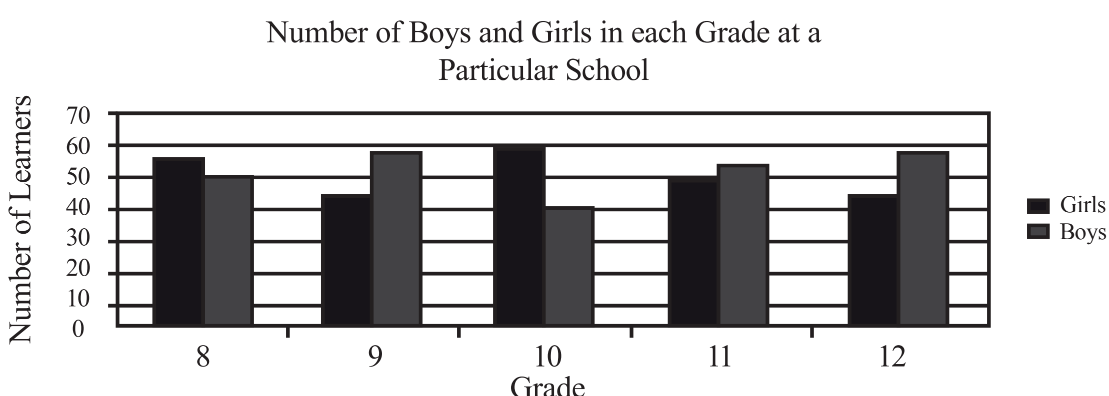


Figure 5.60.

You can now use the graph to do answer questions and do some analysis.

In what year, are there the most girls? In what year, are the most boys?

Pie charts

In a survey of students' means of transport to college, the following data is captured.

Forms of Transport	Frequency
Bicycly	5
Motorbike	2
Walk	6
Car	10
Taxi	7
Total	**30**

Figure 5.61.

Since the pie graph is a circular graph, the frequencies must be expressed as a fraction of 360 degrees. The frequency must be expressed as a fraction of the total sample (30) and then as a fraction of 360 degrees.

Bicycle: $\frac{5}{30} \times 360 = 60°$ *Motorbike:* $\frac{2}{30} \times 360 = 20°$ *Walk:* $\frac{6}{30} \times 360 = 72°$

Car: $\frac{10}{30} \times 360 = 360 = 120°$ *Taxi:* $\frac{7}{30} \times 360 = 84°$.

Figure 5.62.

Now the pie chart can be developed where the frequencies are shown as a fraction of the pie.

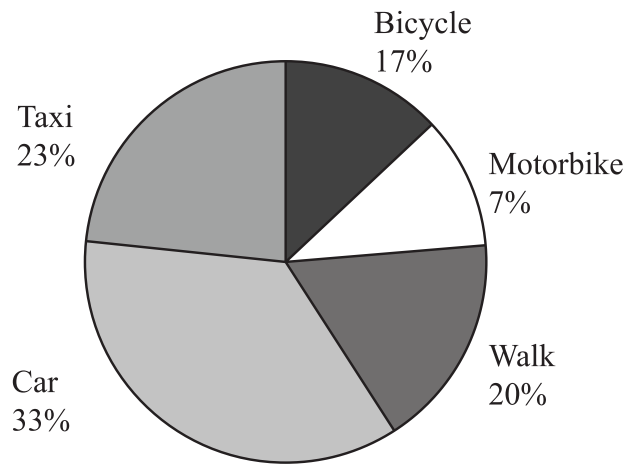


Figure 5.63.

Line graphs

It is useful to use a line graph if the graph is likely to show trends. Are things going up or down or staying the same? In the example below, enrolment in a primary school and a high school are tracked from 2005 to 2010.

Year	2005	2006	2007	2008	2009	2010
Primary School	564	587	601	594	610	623
High School	642	655	680	692	695	700

Figure 5.64.

The line graph that plots these frequencies indicates a steady increase in the enrolment of students in high school. There is an increase over the five years in enrolment in the primary school with an unusual decline in enrolment in 2008.

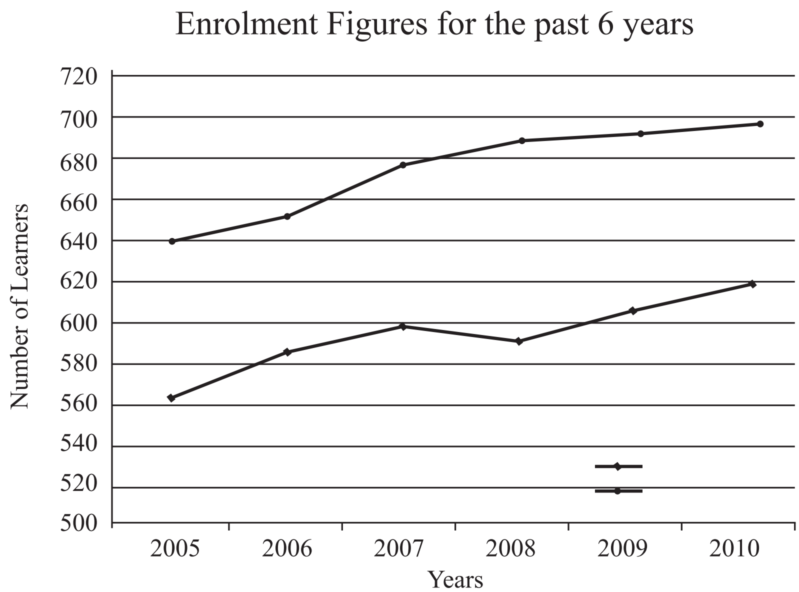


Figure 5.65.

Now a school is surveying the distances that students travel each day to get to and from school. The frequency table below captures the data from a questionnaire that was given to the students. Distance is a continuous variable and so the responses have been placed in groups of distances (0-5km, 6-10km and so on). The last group was for responses over 31km.

Distance Travelled	Frequency
0-5 km	123
6 - 10 km	97
11 - 15 km	208
16 - 20 km	51
21 - 25 km	49
26 - 30 km	17
31 + km	15
	560

Figure 5.66.

In your groups, discuss the best way to present this data graphically. Develop this graph on a flipchart.

Learning activity 2

Your teacher educator will give you a copy of the executive summary of Myanmar: Analysis of Farm Production Economics (The International Bank of Reconstruction and Development / The World Bank, 2016) to read.

In groups, discuss:

- What are the key lessons for you in this reading? (Five points)
- What did the graphs inserted tell you? (Five points)
- Collect your points and nominate a representative to report for the group.

5.4.2. Teaching tables and graphs in Grade 2

In this lesson, we will again review the Grade 2 textbook, this time on Tables and Graphs. We will devise a lesson or series of lessons using the ideas in this material to develop our own worksheet.

Expected learning outcomes

By the end of the lesson, you will be able to:

- Explain what primary students have to learn in the areas of 'data collection and arrangement' and 'data analysis'; and
- Develop some strategies for teaching the concept to young children.

Learning activity 1: Reviewing data in the Grade 2 textbook

In your groups, you should read and review the unit on tables and graphs. Observe how the textbook writers construct the example and stage the learning. What is the first part of the example on pages 1 and 2 demonstrating? What is the extended example demonstrating on page 3? On page 4, the example now gets students to depict the sample with a zero. Why do you think this is? What are the students meant to discover here? On page 5, there is a new example, this time using fruits. What is the learning objective here?

Capture the points of your discussion. Now get a representative to present your discussion to the class. The teacher educator will facilitate the discussion.

Developing lessons and designing a worksheet

In your group, develop two lessons to cover the topic for Grade 2. Describe the learning objectives for the lessons. Then, using the ideas set out in the textbook, design a worksheet that students will complete over the lessons. Find other useful, relevant things to measure that may interest the students. Develop you own examples. Each student should develop a worksheet for homework.

Review questions

1. In what subjects, other than Mathematics, would you need data handling and data analysis skills?
2. Can you name some data handling tools that are in the Mathematics curriculum for students to learn?

Unit Summary

Key message

- This unit covers topics in the primary Mathematics curriculum.
- The unit was seeking to build on the skills and knowledge developed in the previous unit on how we teach and learn Mathematics and focused on ways to teach various topics.
- In the six lessons on algebra, we examined the question 'What is Algebra?' and sought to dispel some unhelpful misconceptions that have an impact on the way algebra is taught. Then, we look at how teaching the notion of equality in the primary grades is a good introduction to algebraic thinking. Then, we looked at how we can develop algebraic thinking further by getting students to generalise their thinking from arithmetic to algebra. Lastly, we looked at how we can teach students in primary grades to generalise thinking to the notion of a function so that they are able to grasp the concept of functions easily in later grades.
- In the sub-unit covering Geometry, the lessons focused on practical ideas to teach lines, circles, triangles, quadrilaterals, pentagons and hexagons and the interior angles of polygons.
- Two lessons have been included to explore ways in which we can teach Mathematics outside, playing games to get students to use their bodies (and their heads) to count, make shapes and calculate time.
- To cover the introduction to Data Handling and Graphs, we had one lesson to cover the introductory concepts and one lesson practicing the use of the Grade 2 Mathematics textbook.

Unit reflection

What were the key lessons you learnt in the 'What is algebra?' reading and discussion. What do you remember learning about the notion of equality and why it was important to teach this to primary learners? What were the key lessons you learnt when considering teaching primary learners how to generalise from their understanding of arithmetic? How would you go about teaching primary learners to generalise their thinking to understanding functions?

Do you remember how to teach drawing lines and perpendiculars without mathematical equipment? What would you use dotter paper to teach? Can you remember ways to teach quadrilaterals and their characteristics? What ways would you use to demonstrate how to calculate the interior angles of polygons?

What is useful about physical Mathematics when teaching ideas to young students? Investigate how you could teach various mathematical concepts using the ideas of physical Mathematics. How could you incorporate Mathematics into other subjects using these ideas (Physical Education is an obvious possibility)? This leads us towards investigating the idea of a joined-up curriculum, in which learning opportunities are presented across subjects, which promotes inclusivity by engaging students in their learning through different approaches.

Finally, review the final sub-unit and plan how you would introduce data handling and graphs to primary school students at different grades.

Further reading

5.1

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5.4

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Glossary

Terms	Elaborations
Charts	Charts are mounted on the wall and are usually large enough for students to see from anywhere in the classroom. They are generally useful for teaching purposes. Charts for Mathematics may summarise information on a variety of topics: Numbers, fractions, decimals, measurement, geometry, algebra, and data handling. They can be used for teaching or revision of key points of a topic.
Community of practice	A community of practice comprises a group of people who do the same job. The group generally shares experience, resources, and knowledge. The group serves to motivate and encourage its members.
Exemplar	A person or thing that serves as a typical example or appropriate model
Formative assessment	Formative assessment is part of the teaching in class. It provides the information needed to adjust teaching and learning while they are happening. In this sense, formative assessment informs both teachers and students about student understanding at a point when timely adjustments can be made.
Grade-wise curriculum	A view of the curriculum that helps us see the scope and sequencing of a topic by grades.
Grade-wise objective	Shows what children should be able to do by the end of each grade.
Handouts	Handouts are printed additions to the textbook on a topic not covered in the text or on a topic for enrichment.
Learning outcome	What specifically a student should learn as the result of a learning programme or lesson.
Learning strand	A strand is a thread, so a learning strand in the primary Mathematics curriculum is a part of the curriculum, a thread, which together with other strands makes up the curriculum.

Terms	Elaborations
Motivation	Motivation is 'a general desire, need or want that generates the energy required for someone to behave in a particular way. This means that there's actually a motive behind the action, which is really an impulse that will trigger the motivational cognitive process which will lead an individual to act on that impulse' (Leadership-central , 2018).
Physical Mathematics	In physical Mathematics students use their bodies and the space around them to learn Mathematics. For example, a class of students stands so they form a circle.
Posters	Posters are generally used to represent information of interest and can form part of a display. There could be a poster display on interesting Mathematicians.
Relational thinking	Relational thinking in Mathematics can be described as looking at the whole mathematical expression or equation, and noticing the relationships between numbers.
Scaffolding	This is a concept phrased by Vygotsky (1978). 'Scaffolding' is similar to its name sake - a scaffolding on a building which is slowly withdrawn as the structure goes up, so a skilled teacher gives scaffolding support to the student and then slowly withdraws this when the support is no longer needed.
Scope of the curriculum	In the Mathematics curriculum, the scope would indicate where we start and where we finish. (A good example would be Number. Where do we start and finish with Number in the primary curriculum?)
Sequence of the curriculum	In the Mathematics curriculum, this would describe the order in which we teach a topic.
Sum of interior angles of a polygon	Interior angles of a polygon refer to the angles at the vertices of that polygon. If we add the size of the interior angles (the angles at the vertices) we get the sum of the interior angles of that polygon.

Terms	Elaborations
Summative assessments	Summative assessments are given periodically to determine at a particular point in time what students know and do not know. They occur after instruction every few weeks, months, or once a year and do not give information to inform on any changes required of the learning process or the class teaching.
Worksheet	A worksheet is an assignment of sorts. It contains description and explanation and then a series of exercises for the student to complete. For Mathematics, a worksheet would usually cover aspects of a topic and can be used as an assessment.
Zone of proximal development (ZPD)	This is a concept phrased by Vygotsky (1978). ZPD is the difference (space or gap) between what a student can do without help and what this student can do with help.

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Notes

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The Government of the Republic of the Union of Myanmar

Ministry of Education