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Ministry of Education



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**Curriculum and Pedagogy Studies:
Mathematics**

Teacher Educator Guide

PREFACE

The Myanmar Ministry of Education developed the four-year Education College curriculum, in line with the pre-service teacher education reform as specified in the National Education Strategic Plan (NESP) 2016–2021.

The Myanmar Education College curriculum consists of several components: the curriculum framework, syllabi, Student Teacher Textbooks, and Teacher Educator Guides. This curriculum for the four-year Education College was designed and structured to align with the Basic Education Curriculum and to equip student teachers with the competencies needed to teach effectively in Myanmar’s primary and middle school classrooms. It is based on a Teacher Competency Standards Framework (TCSF) which articulates the expectations for what a teacher should know and be able to do in the classroom.

The curriculum follows a spiral curriculum approach, which means that throughout the four years student teachers return to familiar concepts, each time deepening their knowledge and understanding. To achieve this, the four-year Education College programme is divided into two cycles. The first cycle (Years 1 and 2) is repeated at a deeper level in the second cycle (Years 3 and 4) to enable student teachers to return to ideas, experiment with them, and share with their peers a wider range of practices in the classroom, with the option to follow up on specific aspects of their teaching at a deeper level.

The curriculum structure provides an integrated approach, where teaching of subject knowledge and understanding educational theories are learned through a supportive learning process of relevant preparation and practical application and experience. The focus is, therefore, not just on subject content, but also on the skills and attitudes needed to effectively apply their knowledge, skills, and attitudes in teaching and learning situations, with specific age groups. As the focus is on all components of a ‘competency’ – knowledge, skills, attitudes and their effective application – it is referred to as a competency-based curriculum.

Accordingly, a competency-based curriculum is learner-centred and adaptive to the changing needs of students, teachers, and society. Where new concepts are learned, they are then applied and reflected on:

1. Learn (plan what and how to teach);
2. Apply (practise teaching and learning behaviours); and
3. Reflect (evaluate teaching practice).

Beyond the Education College coursework, it is intended that student teacher graduates will be able to take and apply this cycle of ‘learn, apply, and reflect’ to their own teaching to effectively facilitate the learning and development of Myanmar’s next generation.

The Myanmar Education College curriculum was developed by a curriculum core team, which is a Ministry of Education-appointed team of Myanmar Education College teacher educators supported by the Ministry of Education, resource persons from the Universities of Education, University for the Development of National Races of the Union and a team of national and international experts. Overall guidance of the work was provided by the Department of Higher Education, Ministry of Education.

The curriculum development was also supported by the Strengthening Pre-Service Teacher Education in Myanmar project, with technical assistance from the United Nations Educational, Scientific and Cultural Organization (UNESCO) and financial contributions from Australia, Finland, and UK Governments. Substantial input to the drafting process was also provided by Japan International Cooperation Agency and the primary education curriculum development team through the Project for Curriculum Reform at Primary Level of Basic Education (CREATE) team.

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HOW TO USE THIS GUIDE

Who will use this Mathematics Teacher Educator Guide?

This Teacher Educator Guide has been designed to help you facilitate student teachers' learning of Year 1 Mathematics. It is addressed to you, as the teacher educator, and should be used in tandem with the Student Teacher Textbook as you teach Mathematics. This Teacher Educator Guide contains step-by-step instructions to help you guide the student teachers in your class towards achieving the learning outcomes for each lesson and unit in the Student Teacher Textbook.

When and where does Mathematics take place?

The learning area of Mathematics has been allotted 72 periods of teaching for each year of the four-year Education College programme. Classes will be held on the Education College campus.

What is included in the Year 1 Mathematics Teacher Educator Guide?

The organization and content of both the Student Teacher Textbook and this Teacher Educator Guide align with the syllabus of the four-year Education College programme on Mathematics.

The Student Teacher Textbook, and accompanying Teacher Educator Guide, for this programme contains the following topics for Year 1 Mathematics:

- Introduction to Mathematics;
- Problem-Solving and Misconceptions;
- Understanding Mathematics;
- How we Learn and How we Teach Mathematics; and
- Mathematical Modelling and Representation.

The Teacher Educator Guide follows the same structure as the Student Teacher Textbook. For each unit and lesson, there are **expected learning outcomes** that indicate what student teachers should know and be able to do by the end of the unit.

For each lesson, the Teacher Educator Guide includes:



Competencies gained: This list of competencies highlights the teacher competencies from the Teacher Competency Standards Framework that are focused on in that lesson.



Time: This is the total teaching minutes and number of 50-minute class periods allocated for the lesson as per the syllabus.



Learning strategies: This is an overview of all the learning strategies used during the suggested lesson learning activities.



Preparation needed: This can include: guidance on what you need to know about the topic and references to subject knowledge resources; technology preparation; links to other subjects; room organisation; time management; reference to expected answers.



Resources needed: This can include: printed media; flipchart paper; coloured paper; marker pens; URLs; video clips; low/no cost resources; practical equipment.



Learning activities: Each lesson includes a variety of suggested learning activities designed to help student teachers achieve the expected learning outcomes within the allotted time. Each lesson should begin by activating student teacher prior knowledge or fostering interest in the subject. Learning activities are varied and in line with competency-based approaches to teaching and learning.



Stop and think: This instruction box is included as an occasional ‘safety net’ at key points during the lesson, reminding you to quickly check that the lesson is flowing in the direction as planned, and to check if there are any points to emphasise to ensure that student teachers are learning effectively before moving forward.



Assessment: This comes at the end of each activity, an explanation or recap as to how each activity can be assessed formatively in order to assess success and inform future teaching. Instructions for facilitating various types of assessment are included in the Toolbox for assessment approaches.



Possible student teacher responses: The responses that you may get from the student teachers for each learning activity’s assessment are recorded here.



Check student teachers’ understanding: This is the lesson plenary. At the end of the lesson, revisit the learning objectives and TCSF competencies, summarise the learning outcomes and briefly assess the extent to which they have been achieved. Summarise the competencies and how they were addressed by the lesson content. Explicitly remind student teachers what they have studied and how they did so.



Extension and differentiation activities: Each lesson in this guide includes ideas on ways to adapt the learning activities to either provide additional stimulus for stronger student teachers (extension), or extra support for student teachers who are struggling or who have different learning needs (differentiation).

For each unit, the Teacher Educator Guide includes:



Review questions: Possible student teacher responses: A box at the end of each unit gives you the answers to the Review questions in the Student Teacher Textbook. This section exists to support your knowledge as a teacher educator, and enables you to support your student teachers by confirming the answers to the questions in their Student Teacher Textbook. It is NOT part of the lesson.

For each unit, the Teacher Educator Guide includes:



Unit summary: This is a summary of the unit, including a reminder of the key points that student teachers should take from the unit.



Unit reflection: This section is part of the student teachers' self-study material and is included in the Student Teacher Textbook. It is duplicated here to inform you of its content. Your only task here is to remind the student teachers to read it. **It does not form part of any lesson.** It provides the student teachers with reflection points or questions relating to the learning in the unit.



Further reading: Suggestions for additional resources are listed according to the relevant unit. You can use these resources to learn more about the topic yourself, or encourage student teachers to look them up in the library, on the internet, or in your Education College's e-library.

Please note that the learning activities in the Student Teacher Textbook are designed for individual self-study. At times, these individual learning activities may be incorporated into the learning activities outlined in this guide. You may also wish to assign the learning activities in the Student Teacher Textbook for homework, or encourage student teachers to do them at their own pace. While this Teacher Educator Guide contains detailed learning activities to help you plan and deliver lessons, the instructions in this guide are only suggestions. The student teachers in your classroom will have different characteristics and learning needs. As their teacher educator, you are encouraged to come up with your own learning activities which suit these needs, interests, and ability levels. You should feel free to change and adapt the lessons as much, or as little, as needed.

What is a competency-based curriculum?

The Student Teacher Textbooks and Teacher Educator Guides for all Education College programmes follow a competency-based approach. This is outlined in the four-year Education College Curriculum Framework and is based on the Myanmar Teacher Competency Standards Framework (TCSF). A competency-based approach means that the Education College does not just focus on subject content. Rather, it emphasises the

development of knowledge, skills, and attitudes and their application in real-life contexts. Competency-based curriculums are learner-centred and adaptive to the evolving needs of learners, teachers, and society.¹

The following elements are integrated throughout this Teacher Educator Guide, in line with a competency-based approach to teacher education:

- **Contextualisation:** The learning content and learning activities are based on the Myanmar context to ensure that student teachers can relate what they learn to daily life.
- **Flipped classroom:** This pedagogical concept and method replace the standard lecture-in-class format with opportunities for student teachers to review, discuss, and investigate course content with the teacher educators in class. Student teachers are typically expected to read the learning materials before class at their own pace. Classroom time is then used to deepen understanding through discussion with peers and problem-solving activities facilitated by you, as teacher educator.
- **Collaborative learning:** This educational approach involves groups of student teachers working together to solve a problem or complete a task. Learning occurs through active engagement among peers, either face-to-face or online. The main characteristics of collaborative learning are: a common task or activity, small group learning, co-operative behaviour; interdependence, and individual responsibility and accountability.²
- **Problem-solving:** This involves the act of defining a problem; determining the cause of the problem; identifying, prioritising and selecting alternatives for a solution; and implementing a solution. The learning content and activities included in this Teacher Educator Guide provide opportunities for student teachers to apply their problem-solving skills, as appropriate.

¹ Adapted from the Glossary of curriculum terminology (2013), International Bureau of Education (IBE), UNESCO

² Lejenue's collaborative learning for educational achievement (1999)

Course rationale and description

The overall objective of Mathematics is to ensure that you are prepared to teach the Mathematics curriculum as defined for basic education in Myanmar. You will learn to academic standard equivalent to primary and middle school level in order to ensure a strong subject proficiency foundation for being effective teachers for primary school students (Education College Curriculum Framework, 2018).

Learning objectives for primary school students for Mathematics subject (primary Mathematics syllabus):

- To have basic mathematical knowledge and skills regarding numbers, quantities, geometrical figures and data representation (knowledge and understanding, skills);
- To reason and explain logically in problem-solving (thinking);
- To apply mathematical knowledge and skills to the problems in our daily life as well as in their learning (knowledge and understanding, skills); and
- To appreciate the usefulness of mathematical ideas and approaches (attitude).

This course will prepare student teachers to equip primary students with Mathematics as a life skill for understanding the world we live in. The 21st century citizen needs the skills of Mathematics to solve problems in work and other life situations. Everyone has the potential to be a mathematician and the teacher is key in helping students achieve this. The vision for teacher education in Myanmar is to produce graduates who over the course of their four-year degree have developed deep understanding and skills in collaborative problem-solving. Both academic (what we teach) and methodology (how we teach) content is important and is presented in an integrated approach in this course. In order to become teachers who can adapt their teaching to the needs of different students, student teachers will learn how to develop learning outcomes of subject topics so as to be able to create activities that will assist primary students' deeper learning. They will know how to create their own teaching aids making them appropriate for the age of students they will be teaching. With reference to the Educational College Curriculum Framework in Years 1 and 2, student teachers are expected to develop fundamental knowledge of Mathematics and develop basic pedagogical content knowledge for teaching Mathematics. In Years 3 and 4, they will further develop deeper understanding of Mathematics subject knowledge and to gain a more systematic grasp of primary Mathematics curriculum, instruction and assessment.

Basic Education Curriculum objectives

The aims of the Basic Education Curriculum are as follows:

After the completion of basic education, students will be able to:

- a) Attend the school until the completion of basic education;
- b) Develop ‘union spirit’ and appreciate, maintain, and disseminate languages and literatures, cultures, arts and traditional customs of all national groups;
- c) Become good citizens with well-developed five strengths including critical thinking skills, communication skills and social skills;
- d) Apply their civic and democratic in daily lives, and become good citizens who abide by laws;
- e) Be competent in Myanmar language which is the official language of the Republic of the Union of Myanmar and develop their skills in respective ethnic language and English;
- f) Develop foundational knowledge and skills for higher learning and technical and vocational education;
- g) Develop sound body and sportsmanship through participation in physical education activities and school health activities, and apply health knowledge in daily lives;
- h) Appreciate and maintain natural environment and materialise its sustainability;
- i) Become global citizens with awareness and appreciation of human diversity and abilities to practice basic knowledge of peace in their daily lives; and
- j) Take pride in being a citizen of the Union of Myanmar.

Note: According to the Basic Education Law, the aims of basic education will be mentioned.

In order to realise the above-mentioned aims, thirteen guiding principles as mentioned in the following need be employed in the design and development of Basic Education Curriculum in Myanmar.

Table A. Mathematics teacher competencies in focus

Competency Standard	Minimum Requirement	Indicators
A1: Know how students learn	A1.1. Demonstrate understanding of different theories of how students learn relevant to their age and developmental stage	A1.1.2. Prepare learning activities according to students' level of cognitive, physical, social and emotional development
	A1.2. Demonstrate understanding of how different teaching methods can meet students' individual learning needs	A1.2.1. Identify various teaching methods to help students with different backgrounds (gender, ethnicity, culture) and abilities, including special learning needs, learn better A1.2.2. Prepare focused and sequenced learning activities to assist students to link new concepts with their prior knowledge and experiences
A2: Know appropriate use of educational technologies	A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources	A2.1.1. Plan learning experiences that provide opportunities for student interaction, inquiry, problem - solving and creativity A2.1.2. Use teaching methods, strategies and materials as specified in the textbooks and additional low cost support materials to support student learning
	A2.2. Demonstrate understanding of appropriate use of Information Communication Technology (ICT) in teaching and learning	A2.2.1. Describe the function and purpose of online and offline educational tools and resources to support the teaching and learning process
A4: Know the Curriculum	A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum	A4.1.1. Describe the core principles, education concepts, content and objectives for learning of the Kindergarten curriculum, for example social development and foundational literacy A4.1.2. Prepare lesson plans reflecting the requirements of the curriculum and include relevant teaching and learning activities and materials A4.1.3. Describe the assessment principles of the Kindergarten curriculum
A5: Know the subject content	A5.1. Demonstrate understanding of the subject matter to teach the subject/s for the specified grade level/s	A5.1.1. Describe the key concepts, skills, techniques and applications for the subjects covered in the grade levels taught

Competency Standard	Minimum Requirement	Indicators
	A5.2. Demonstrate understanding of how to vary delivery of subject content to meet students' learning needs and the learning context	A5.2.2. Explain how lessons are contextualised to include localised information and examples related to the subject content, concepts and themes
B1: Teach curriculum content using various teaching strategies	B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly	B1.1.1. Clearly explains the curriculum content and intended learning outcomes
	B1.2. Demonstrate capacity to apply educational technologies and different strategies for teaching and learning	B1.2.1. Use teaching and learning strategies appropriate for the class culture and size B1.2.2. Use knowledge of literacy and numeracy instructional strategies to support students' language and literacy development. B1.2.3. Facilitate children's learning through play and provide visual, auditory, oral examples to introduce and illustrate concepts to be learnt
	B1.3. Demonstrate good lesson planning and preparation in line with students' learning ability and experience	B1.3.1. Plan and structure lesson to ensure all of the lesson time is used effectively B1.3.2. Plan timed and focused learning activities to engage students B1.3.3. Use themes to integrate topics, connected with the local language, culture and environment to learn about self, family and others
B2: Assess, monitor and report on students' learning	B2.1. Demonstrate capacity to monitor and assess student learning	B2.1.1. Use assessment techniques as part of lessons to support students to achieve learning outcomes B2.1.2. Use assessment information to plan lessons B2.1.3. Use questioning and discussion techniques to check students understanding and provide feedback
D2: Engage with colleagues in improving teaching practice	D2.1. Improve own teaching practice through learning from other teachers and professional development opportunities	D2.1.1. Discuss teaching practices with supervisors and, colleagues and willingly seek constructive feedback D2.1.2. Participate in professional development activities related to identified goals for improving practice

Source: Ministry of Education, Teacher Competency Standards Framework (TCSF), Draft version 3.2. (2019, p. 30-36).

Teaching young adult learners

The student teachers in your classroom are young adult learners. As such, evidence suggests that they will learn best when:

- The course content is related to their prior knowledge and experiences;
- There are opportunities for them to be active in their learning, both in and outside the classroom; and
- They are asked to develop their critical thinking and social skills and to take ownership of their own learning.

The different types of content delivery and learning strategies proposed in this Teacher Educator Guide are based on the following ‘good practice’ principles of teaching adult learners:

1. **Keep it relevant.** Adults tend to be goal-oriented and practical. They want to understand how what they are learning will be important in their daily lives. This means that it is important to have clearly defined goals and objectives for what student teachers will accomplish in a lesson, and why. Student teachers need to see the relevance of what they are learning for their future jobs as teachers. You can tell them explicitly what they are learning, or how individual learning activities will be useful to them as teachers.
2. **Recognise your student teachers’ backgrounds.** Your student teachers are coming to you with at least 18 years of life experience. The content of your course should reflect the level of education that they have completed and the realities of their daily lives. Adult learners need to be shown respect by valuing the experience and knowledge that they bring to the class. In your lessons, you can look for places where student teachers can draw on their real-life experiences and prior knowledge to help them understand and connect to a topic.

3. **Encourage exploration.** As adult learners, your student teachers are capable of learning on their own and being self-directed. Activities that require problem-solving and collaboration can help your student teachers to deeply and meaningfully connect with the lesson content. To do this, look for ways to actively involve your student teachers through discussion groups, real-life practice, and opportunities to teach others. It may help to think of yourself as a ‘facilitator’ of learning, rather than a teacher. You can encourage the student teachers in your classes to take ownership of their learning by finding out what is interesting to them and encouraging them to pursue these things.

Gender equality and inclusivity in the classroom

Actively promoting gender equality in the classroom is an essential element of your teaching. Facilitating a safe and positive environment and atmosphere where all student teachers feel that their contributions are equally valued, and have equal access to learning, requires you to be mindful of the teaching and learning strategies and resources you use.

As a teacher educator, it is your responsibility not only to ensure your student teachers have equal access to learning, but also to ensure that they understand and value the importance of gender equality and take that knowledge with them into their own teaching practice. The skills, knowledge, values and attitudes developed in the classroom with regards to gender, either implicitly or explicitly, can have a long-lasting impact on the future behaviour of your student teachers.

Be aware of your own gender biases. Reflect on your actions and the teaching strategies you use. Consider these ways in which you can ensure gender inclusivity in your classroom:

- Ensure that there is equal frequency in the representation of male and female names and characters. When identifying characters whose gender is unknown, use alternating pronouns (he, she).
- When using quotes ensure that both female and male voices are heard.
- Ensure that females and males are represented equally in illustrations and that any existing gender stereotypes are not reinforced.

- Use equitable and gender-inclusive language in the classroom and ensure that your student teachers do likewise.
- Help and encourage your student teachers to be gender-aware, highlight any perceived gender-biased attitudes and encourage your student teachers to reflect on their own actions.
- Ensure that you interact equally with male and female student teachers, addressing and engaging them both to the same degree in your teaching, across different subjects, for example, when asking questions, asking for volunteers, selecting activity leaders, giving complements, giving eye contacts, or even remembering the names of student teachers.
- Encourage and support the participation of quieter student teachers, regardless of gender.
- Use teaching and learning strategies and assessment approaches that support equal participation from both genders, for example, group work, role plays and group discussions. Manage the activities in a flexible manner addressing different needs and learning styles of all student teachers, to ensure that both female and male student teachers have the opportunity to participate actively and that individuals do not dominate activities.
- Ensure to set an equal expectation for both female and male student teachers on their performance across different subjects.
- Arrange the classroom setting in a gender-sensitive and equal manner, in terms of classroom decorations, seating arrangement, or group formation/division.

Gender stereotypes are often inadvertently reinforced in the classroom through the use of language, pedagogical approaches and resources that support the preconceived culturally expected norms, roles, and responsibilities of women and men. By promoting a gender-inclusive environment in the classroom, you can support both male and female student teachers in building a healthy understanding of gender equality and further mainstreaming of this gender-sensitive and inclusive practice into basic education classrooms.

Toolbox for teaching and learning strategies

This Teacher Educator Guide includes suggested learning activities for each lesson in the Student Teacher Textbook. These learning activities are intended to help support you as you plan your lessons, but they do not dictate what you must do to help student teachers develop the desired knowledge, skills, and attitudes for each lesson. On the contrary, you are encouraged to come up with the lesson activities that will best help the student teachers in your classroom to learn, given their unique backgrounds and needs.

Many of the learning activities listed below are used in this Teacher Educator Guide. You can also use this list to help you plan, or further adapt, your lessons. This is not an exhaustive list of teaching and learning strategies. You may wish to brainstorm additional teaching strategies by visiting <http://www.theteachertoolkit.com/index.php/tool/all-tools> or similar websites.

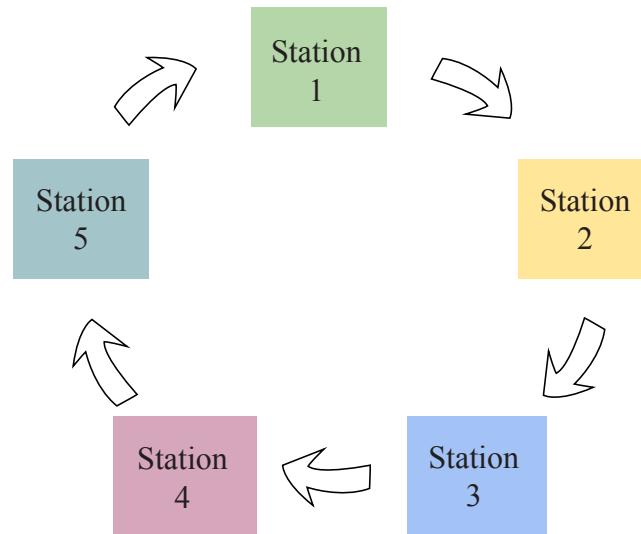
Assignments: The assignments that you give to student teachers might include formal written essays, portfolios, and reflection journals. They also might be smaller, developmental tasks – for example, a short homework assignment answering questions about a reading. Assignments can help student teachers to review previously taught materials. They can also help student teachers prepare for future learning – for example, you might assign student teachers to read the Student Teacher Textbook content in advance of the next lesson.

Case studies: Working through case studies can help student teachers to develop their problem-solving and critical thinking skills as they must apply what they are learning to a scenario or story (the ‘case’). To complete a case study, student teachers first read the scenario and then discuss and answer one or more open-ended questions about the scenario. Case studies often require student teachers to propose solutions to the problem presented in the scenario.

Directed activities: These are activities set by you, as the teacher educator, but carried out by the student teacher independently. For example, a directed activity might be for a student teacher to interview a basic education teacher during their Practicum school placement, or to independently research a specific teaching strategy. Directed activities are typically followed up in tutorials, seminars, or workshops, which provide an opportunity for student teachers to share about what they have learned and to learn from their peers.

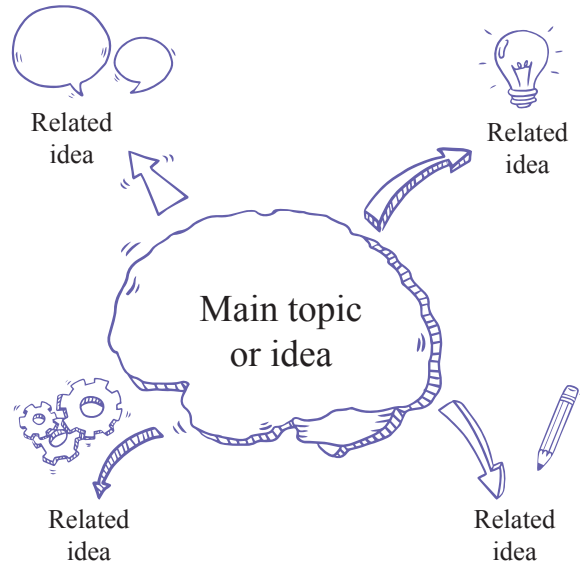
Gallery walk: In a gallery walk, student teachers work in groups to answer questions or complete a task on poster paper at various stations. They then rotate stations and add comments, questions, or further content to the poster at that station.

You can also use a version of the gallery walk to display student teachers' work. In this type of gallery walk, posters created during individual or group work are displayed around the room. Student teachers then circulate at their own pace to either simply view the posters, or to add their questions or comments to the poster.

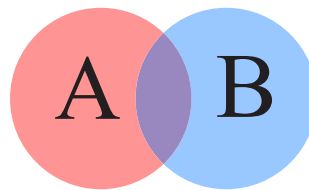


Graphic organisers: Graphic organisers are a simple and effective tool to help student teachers brainstorm and organise their thoughts and ideas in a way that makes it easier for them to understand. Graphic organisers can be used in any lesson for brainstorming, planning, problem-solving, or decision-making. Some of the most popular graphic organisers that you will see in your Teacher Educator Guides include:

Concept map (also called a mind map): Concept maps, or mind maps, can be used to visually show the relationships between concepts, or ideas. They are useful for brainstorming and also organising information. Concept maps can be organised in different ways and with different levels of complexity, but most start with broad topics first, connected to sub-topics (or more specific concepts) to form a web of connecting ideas. The diagram below shows a very simple concept map.



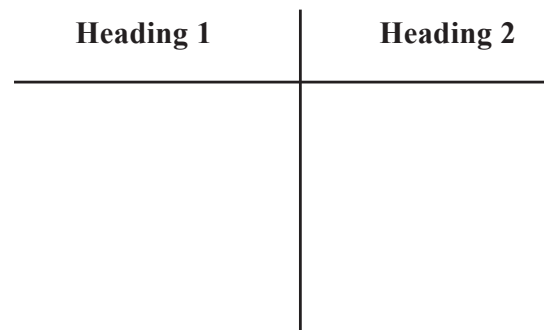
Venn diagram: Venn diagrams can be used to compare and contrast at least two different things or ideas (A and B). In the Venn diagram below, the overlapped area represents the characteristics belonging to both A and B, and the two areas without overlap are for listing the characteristics that belong only to A and those that belong only to B.



KWL chart: KWL charts can help student teachers organise information before, during, and after a unit or a lesson. They can be used to engage students in a new topic, activate prior knowledge, share unit objectives, and monitor student teachers’ learning. KWL charts can be completed as a small group, whole class, or by an individual. Before the lesson or unit, student teachers should fill in the first two columns about what they already know and what they want to know. After the lesson or unit, they can fill in the column about what they have learned.

K What I <u>K</u> now	W What I <u>W</u> ant to know	L What I <u>L</u> earned

T-chart: T-charts can help student teachers examine two facets of a topic; for example, the advantages and disadvantages, or facts versus opinions.



Group work: Group work refers to any time you ask student teachers to cooperatively work together in groups on a task (for example, see the Jigsaw activity below). Group work can help motivate student teachers and encourage active learning. It requires student teachers to practise key critical thinking, communication, and decision-making skills. Student teachers can work in groups to answer questions, create a presentation, write a lesson plan, analyse a case study, conduct a role-play, and many more learning activities. You may wish to assign roles to group members – for example, recorder, presenter, and team leader – to make sure that everyone is involved in the task.

Jigsaw: In a jigsaw activity, small groups of student teachers become experts on one component of a topic and then ‘teach’ that component to their peers. This gives student teachers the opportunity to work with others and to see different points of view. The jigsaw technique is especially effective because each student teacher is responsible for another’s learning, and student teachers come to realise that each group member has something important to contribute to the group. In a jigsaw, student teachers must practise using many important skills, including communication, problem-solving, and critical thinking.

Lecture: Lectures are largely one-way communication between you, as teacher educator, and a group of student teachers. They can be useful for delivering straightforward new content. Even when giving a lecture, you can involve student teachers more actively by pausing to ask and respond to questions, or by asking a student teacher to reflect or comment on the topic.

Micro-teaching: During a micro-teaching experience, a student teacher, or a small group of student teachers, teaches their peers all or part of a lesson. They then receive feedback on the mini-lesson and reflect on the experience in order to develop practical skills and apply their learning. Micro-teaching is an important opportunity to prepare for the Practicum Lesson Study and school placements. It can also provide a chance to focus on specific core teacher practices; for example, asking open-ended questions, or giving students positive feedback.

Modelling: Modelling is an instructional strategy in which the teacher demonstrates a new concept or approach and students learn by observing.³ As a teacher educator, you may choose to demonstrate a learning activity or teaching strategy, rather than simply telling the student teachers about it – this is modelling.

Modelling may also be followed by a discussion about how you presented the activity or strategy and what impact that had on the student teachers as learners. This can highlight the role of modelling in teaching and encourage student teachers to reflect on how they might use modelling in their own teaching in the future.

Observation: Student teachers can observe a peer or expert teacher teaching, then participate in structured, reflective discussion to make sense of what was observed. You may also observe a student teacher teaching all or part of a lesson and then follow this with a discussion to explore and develop the student teachers' thinking and practice. This strategy is an excellent opportunity to make links between theory and practice, and to support student teachers in making accurate assessments of their progress.

Practicals: Practical can include demonstrations by you as teacher educator (for example, showing how to conduct a science experiment) and those led by, or involving, student teachers (for example, having student teachers complete a mathematical investigation and associated worksheet). This strategy can help student teachers to understand how different activities can help students learn. Practical can also encourage student teachers to connect theory to their developing practice as teachers.

³ Eggen and Kauchak, *Strategies and models for teachers: Teaching content and thinking skills*, (2001)

Reading groups: A reading group is a small group session focused on the analysis and interpretation of a text, most commonly an academic paper. The paper is usually issued in advance and student teachers are expected to be familiar with its contents before attending the reading group. One student teacher may be asked to present the paper to the group, followed by a discussion to which all student teachers contribute. This strategy helps to familiarise students with academic writing as well as with the ideas within papers. Discussions may focus on the content, presentation, or the methodology of the papers presented.

Role-playing: Role-play is a technique that allows student teachers to explore realistic situations as they interact with people and scenarios in a simulated way to try different strategies. This can allow student teachers to work through common challenges, or specific aspects of teaching, in a safe and supported environment.

Self-study: In a self-study, student teachers must take responsibility for their own learning, with you as a guide. This strategy can supplement face-to-face and Education College-based learning and is important to help frame, supplement, and consolidate new learning. Self-study can take a number of forms, such as reading around topic areas and action planning. Self-study includes time to think about specific areas of education.

Seminars: Seminars are small group sessions where questions can be explored and views debated and analysed. Student teachers usually complete preparatory work or reading before the seminar. While you would lead the seminar as teacher educator, all student teachers are expected to contribute to discussions. Seminars can be good for developing student teachers' deeper thinking about content with which they are already familiar.

Think-pair-share: Think-pair-share is a simple and collaborative strategy where learners work together to solve a problem or answer a question. To use think-pair-share in your class, you can follow these three steps:

1. **Think:** Begin by asking a specific question about the text. Ask student teachers to 'think' about what they know or have learned about the topic.
2. **Pair:** Each student teacher should pair up with a classmate, or with a small group.
3. **Share:** With their partner or small group, student teachers should share and discuss their thinking about the question. You can then expand this time of sharing into a whole class discussion about the topic.

Tutorials: Tutorials are one-on-one or small group sessions between you and a student teacher. Tutorials allow for personalised, detailed discussion and exploration of ideas. They may have a pastoral or academic focus and may be used to support student teachers who are struggling with specific academic content, or who have missed out on an in-class learning experience.

Virtual Learning Environment (VLE): This widely-used tool is a teaching strategy to supplement and support learning and self-study. In VLE, activities, study skills, and links to websites are shared with student teachers and different tools are used to explore understanding, such as wikis, forums, and blogs. An e-library is available for student teachers to access teaching and learning resources.

Workshops: Workshops are group sessions in which student teachers engage with new content and skills in order to develop their understanding and practice. This strategy often incorporates a great deal of collaboration and discussion as well as more lecture ‘teaching’ by you, as teacher educator. Workshops allow for detailed discussions about a topic and for student teachers to practise applying what they are learning.

Toolbox for assessment approaches

There are many different ways you can monitor student teachers’ learning before, during, and after a lesson. This Teacher Educator Guide includes many of these assessment approaches. Remember that providing feedback, either written or verbally, is an important part of formative assessment. Your feedback is what will help student teachers to learn and improve on future tasks. You can think of formative assessment as a chance for student teachers to practise before the summative assessment, where they will be asked to show what they have learned through a larger test, exam, or project.

Some of the most popular assessment methods you will see in this Teacher Educator Guide include:

Demonstration: In a demonstration, you may ask a student teacher to show you – or demonstrate – a skill that they have been learning. For example, you may ask a student teacher to demonstrate a dance technique, a step in a science experiment, or a movement in Physical Education. By observing the demonstration, you can monitor student teacher progress and provide suggestions for improvement. As with all formative assessment approaches, the feedback you provide on the student teacher’s demonstration is what will help him or her to improve.

Homework assignments: Checking student teachers’ homework assignments, which may include tasks such as reading and answering questions or looking up additional information, is a good way to monitor if they are on the right track. Depending on the homework assignment, you may wish to discuss answers as a class, check for completion, or collect and provide written feedback.

Journal log/reflection papers: These are a detailed log of student teachers’ thoughts and feelings about their professional development and growth. The journal log and reflection papers are intended to help student teachers think deeply about their own learning by reflecting on their progress towards becoming a teacher. The process of consciously reflecting on their learning will help student teachers make connections between the content they learnt in a subject and other subjects, solve problems that come up, and learn from their experiences. Teacher educators may provide advice to student teachers on the areas to focus on when preparing the journal logs and reflection papers.

Observation: Informal observation – by circulating the room, listening to groups discuss, and making eye contact – is a good way to get a general sense of whether student teachers are understanding the material. More formal observation would involve using a checklist or criteria that you are looking for in a student teacher’s answers or presentation. You can then provide feedback on the basis of what you have observed.

Peer assessment: If you ask student teachers to evaluate, or judge, the work of their peers, this is called peer assessment. You will need to have the appropriate peer assessment tools – either a rubric or a checklist – so that student teachers can provide feedback to their classmates based on established criteria. When student teachers observe each other during micro-teaching and complete an observation sheet, this is a form of peer assessment.

Presentation: A presentation may be similar to a demonstration, but often involves more preparation on the part of the student teachers. Asking groups or individuals to present their work – perhaps at the end of the lesson – is an excellent opportunity to check for understanding, correct any misconceptions, and provide feedback.

Projects: Projects are completed by each student teacher, either individually or collaboratively in a group. This is to demonstrate their understanding in the subject content knowledge and their competencies gained through designing, planning and developing projects. Student teachers work on a project over a certain period of time to investigate a topic or a real-life issue. Teacher educators are requested to provide instructions on completing the projects, including the rubrics of the assessment.

Question and answer: Asking student teachers both closed-ended and open-ended questions is a good way to monitor if student teachers are understanding the material. During question and answer sessions, be sure to call on a variety of student teachers for their responses. While you may want to use some closed-questions (with one correct answer) to check understanding, you will be able to foster better and deeper discussions through open-ended questions, which have more than one right answer and generally require more thinking on the part of the student teachers.

Quiz: You may wish to use a short quiz to test the knowledge of your student teachers. Quizzes can be graded in class as a whole class activity, or you may wish to collect and check the quizzes outside of class. Quizzes can also be seen as a way to ‘practise’ for a summative test or exam.

Self-assessment: In a self-assessment, student teachers evaluate their own strengths and weaknesses. This process can help them to understand their own gaps in skills or knowledge and to create a plan to address these gaps. Self-assessments are good ways to encourage student teachers take ownership of their own learning and development. As in peer assessment, student teachers will need some coaching to understand the assessment criteria and how to apply them to their own work or skill sets.

Written examinations: Written examinations are conducted usually at the end of each semester to test the basic subject content specific knowledge and reflection of related pedagogy discussed during the course.

General tips for facilitating a lesson

Some of the teaching and learning strategies suggested here and throughout this Teacher Educator Guide may be new to you. If so, it is recommended that you spend some time carefully planning out how you will use them in your lessons so that student teachers can achieve the desired learning outcomes.

The following are some additional general tips that you can implement to help your student teachers learn.

Before teaching a class, you may wish to do the following:

- Choose a small amount of content to deliver. Keep in mind that in a given 50-minute class period, you generally do not want more than one-third of the class period should be focused on content delivery. This will enable there to be enough time for student teachers to practise their skills and deepen their understanding of the topic.
- Note down the key points you think are most important for your student teachers to learn from the lesson content. You can refer to these as you deliver the content to the class to make sure you discuss these key points.
- Make sure you are clear on how you will carry out the content delivery and the learning activities. Refer to the suggestions in this guide and discuss with other teacher educators, if needed. Always feel free to change the suggested steps so that the lesson activities work well for your specific classroom situation.
- For each learning activity, prepare clear written instructions for your student teachers describing, step-by-step, how to do the activity. The instructions could be displayed on a presentation slide, printed on a handout or written on the board. Make sure the instructions are large enough to be read by all student teachers.
- You may want to practise explaining the instructions verbally, going slowly and step-by-step. This will help you be ready to explain the instructions to your student teachers before the activity, so they will understand what to do. You can practise the explanation with a friend or colleague ahead of time and then ask them what needs to be explained more clearly.

- If time allows, prepare to model what student teachers are expected to do during the activity. This might involve one or two teacher educators doing a short role-play, pretending they are the student teachers doing the activity. This will enable you to see exactly what they should be doing.
- If student teachers are expected to produce something at the end of an activity, you may wish to prepare an example, or ‘end product,’ to show student teachers what they should be aiming to create during the activity.

During class, just before the content delivery or any learning activity, if applicable, it may be helpful to:

- Distribute any materials or learning supplies that student teachers will need to carry out for the tasks you will ask them to do. Make good use of the e-library to request student teachers to access necessary teaching and learning materials online as appropriate.
- Provide clear verbal and written instructions to student teachers about any task you would like them to do as you deliver the content.
- Model what the student teachers should do using a short role-play.
- Show the example end product to student teachers that you prepared before class.
- Ask one or more student teachers to repeat back to the class how to do the activity, using their own words, to make sure they understand the instructions.
- Tell student teachers how long they have to complete the activity.

Throughout the class, it may be helpful to:

- Look for any signs that suggest whether the student teachers understand the content you are delivering or the task they are working on. If you suspect certain points may be difficult for student teachers to understand, consider explaining the information in a different way or breaking down the information into smaller, more manageable pieces.
- Walk around to all parts of the classroom to:
 - Ensure all student teachers are on task.;
 - Answer questions student teachers have;
 - Ensure student teachers have all the materials needed to do the activity; and

- Assess student teachers' understanding by observing whether they are carrying out the activity as instructed.
- Encourage student teachers to ask questions.
- If you detect a misunderstanding, either talk directly to the student teacher to clarify, or if the whole class may benefit from the clarification, call the attention of all student teachers and explain to everyone.
- Check for **Stop and think** instruction boxes for points to emphasise and to ensure that student teachers are learning effectively before moving forward.

At the end of class, it may be helpful to:

- Consider following the suggested ways to 'Check student teachers' understanding' at the end of each lesson. This is an opportunity to summarise the lesson and to briefly assess the student teachers' achievement of the learning outcomes and understanding of how the lesson addressed the Teacher Competency Standards Framework (TCSF).
- Assess student teachers' understanding by asking them to share a point from the content you delivered that they thought was particularly interesting, or that surprised them.
- Encourage student teachers to ask questions and provide comments on what you have just taught them.
- Ask one or two student teachers to share what they produced during the activity. If the activity was not designed to produce an end product, ask one or two student teachers to describe what they learned from the activity.
- After student teachers share their work or their thoughts, choose one or two aspects of what they shared to emphasise to the class. The point you choose to emphasise should be key points that you would like all student teachers to learn and remember from the activity.

As a teacher educator, you have an important role to play in creating a classroom where all student teachers feel free to ask questions, share their reflections, and practise teaching in a safe, supportive environment. It is your feedback and support that will help them grow into teachers who can foster the holistic development and learning of Myanmar's children and youth.

Table B. Year 1, semester 1, Mathematics content map

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
1. Introduction to Mathematics	1.1. The Nature and Objectives of Mathematics	1.1.1. Patterns and Relationships	<ul style="list-style-type: none"> • Explain that Mathematics is partly about patterns (and the rules patterns follow) and about relationships • Classify that patterns can be concrete and abstract • Develop ideas about how they could teach primary school children about the Mathematics in patterns in the world. 	A4.1.	1
		1.1.2. Mathematics, Science and Technology	<ul style="list-style-type: none"> • Explain the strong connection between Mathematics, Science, and Technology • Identify the connections between new technologies and the Mathematics that supports them • Develop ideas about how to teach primary school children to understand the connection between Mathematics, Modern Science, and Technology. 	A4.1.	1
		1.1.3. Mathematical Thinking	<ul style="list-style-type: none"> • Identify that teachers should be encouraging problem-solving in their classes that is stimulating and creative • Reflect on the thinking process and algorithms used to solve mathematical problems. 	B1.2.	1
		1.1.4. On Being a Mathematician	<ul style="list-style-type: none"> • Demonstrate ideas on how to motivate students to be inspired and interested in Mathematics • Explore some of the ways in which people are 'Mathematicians' in their everyday lives • Explore some of the careers in Mathematics and have ideas to inspire primary students • Explore the schooling of some famous Mathematicians to highlight the idea that 'Everyone can be a Mathematician'. 	A2.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
	1.2. Mathematics in the Basic Education Curriculum	1.2.1. Mathematics in Primary Education	<ul style="list-style-type: none"> Explain why Mathematics is taught to primary students and how they use Mathematics in real life 	A4.1.	1
		1.2.2. Mathematics in the New Primary Curriculum	<ul style="list-style-type: none"> Explain the linkage between the goals of primary Mathematics, principles of basic education and the features of grade-wise content of primary Mathematics. 	A4.1.	1
	1.3. Course Overview of Education College Mathematics Curriculum	1.3.1. The Mathematics Curriculum Year 1	<ul style="list-style-type: none"> Identify the EC Mathematics curriculum in Year 1 and the syllabus and its component parts Recognise the connections between the strands in the EC Mathematics syllabus and the Primary Mathematic syllabus 	A4.1.	1
2. Problem-Solving and Misconceptions	2.1. Enquiry-Based Learning	2.1.1. Developing Thinking Skills for Primary School Learners	<ul style="list-style-type: none"> Describe the current problems of teaching Mathematics in our schools in Myanmar Reflect on the issues and propose some solutions to them 	A2.1. B1.3.	1
		2.1.2. Teaching Thinking through Enquiry	<ul style="list-style-type: none"> Explain problems of the conventional teaching of Mathematics Explain the new teaching strategy which improves children's mathematical thinking skills 	A2.1. B1.3.	1
		2.1.3. Polya's Four Phases for Solving Problems	<ul style="list-style-type: none"> Explain Polya's four phases for solving problems Identify how teachers can promote the mathematical thinking of primary students by understanding Polya's four phases. 	A2.1.	1
		2.1.4. Introduction to Heuristics	<ul style="list-style-type: none"> Understand the concept of heuristics 	A2.1.	1
		2.1.5. Heuristics for Problem-Solving	<ul style="list-style-type: none"> Explain how to use 'Back to Definition', 'Visualise by Drawing' and 'Finding a Pattern' heuristics in problem-solving. 	A2.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
	2.2. Research in Mathematics	2.2.1. Common Errors and Misconceptions in Addition and Subtraction	<ul style="list-style-type: none"> Describe some of the common errors and misconceptions that students have in adding and subtracting in columns, Recognise examples of how to avoid the development of these errors and misconceptions in the Grade 2 Mathematics textbooks. Identify the errors or misconceptions that a student has made and help him/her correct them 	A1.2. A2.1.	1
	2.3. Inclusive Approach to Teaching Mathematics	2.3.1. The Challenges for Mathematics Teachers in Developing an Inclusive Classroom	<ul style="list-style-type: none"> Explain how all children are different and have potential to be mathematician Describe the advantages of developing an inclusive approach to teaching and learning in schools, Demonstrate how to promote and develop inclusive approaches in schools and in the classroom, Prepare as Mathematics teachers for the challenges in promoting inclusive education in Myanmar 	A1.2. A5.2.	1
3. Understanding Mathematics	3.1. Numbers	3.1.1. The History of Numbers and Learning about a Different Place Value Number System	<ul style="list-style-type: none"> Identify different aspects when primary students learn about numbers Explain the central aspects of a positional number system , in other word, a number system like ours which is based on place value. Explain the basic properties of addition, namely <ul style="list-style-type: none"> - commutative property - associative property - distributive property - 0 as the additive identity 	A4.1. B1.1.	1
		3.1.2. Adding and Subtracting in Base 5	<ul style="list-style-type: none"> Identify the notion of grouping and regrouping through working with a base5 positional number system. Explain how addition and subtraction are inverse operations 	A4.1. B1.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		3.1.3. Understanding multiplication in a positional number system by working in a base 5 system	<ul style="list-style-type: none"> Explain the basic properties of multiplication, namely <ul style="list-style-type: none"> - commutative property - associative property - distributive property - 1 as the multiplicative identity - multiplication and division are inverse operations 	A4.1. B1.1.	1
		3.1.4. The Concept of Fractions (1)	<ul style="list-style-type: none"> Compare different meaning of fractions Work practically with different models for representing fractions. Use these models to help students to both identify fractions and to understand the concept of fractions. Identify common student misconceptions around part-whole relationships Develop students' proportional thinking skills whilst teaching fraction concept 	A4.1. B1.1.	1
		3.1.5. The Concept of Fractions (2) Equivalence	<ul style="list-style-type: none"> Use various models to find equivalent fractions. Identify different kinds of errors, in particular the differences between mistakes and misconceptions Explain the benefits to both teaching and learning of working with student's misconceptions Identify student misconceptions around fractions on a number line Explain that there are an infinite number of fractions between any two fractions. 	A4.1. B1.1.	1
		3.1.6. Adding and Subtracting Fractions with the same denominator	<ul style="list-style-type: none"> Recognise the common student misconception that they can add numerators and denominators 	A4.1. B1.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		3.1.7. Adding and Subtracting Fractions with different denominators	<ul style="list-style-type: none"> Recognise that in order to add or subtract fractions we make equivalent fractions with common denominators. Use paper folding to visually demonstrate the logic of making common denominators. Design follow up parts of lessons that work with pictorial and abstract ways to add and subtract fractions with different denominators 	A4.1. B1.1.	1
		3.1.8. Multiplying Fractions	<ul style="list-style-type: none"> Multiply fractions and whole numbers Multiply fractions with fractions Simplify fractions by factorising and re-arranging the factors (using the commutative or associative properties) to form fractions equivalent to 1. Use 1 as the multiplicative identity to simplify fractions. Generalise towards a procedure for multiplying fractions from examining patterns in several calculations. Explain that when we make common denominators, we are really making equivalent fractions by multiplying by a fraction that is equivalent to 1. 	A4.1. B1.1.	1
	3.2. Measurement	3.2.1. Teaching Measurement	<ul style="list-style-type: none"> Describe informal and formal measurement systems that can be used to teach measurement concepts Demonstrate practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students have are addressed Explain how to teach standard units in the Myanmar context using metric, imperial and Myanmar units 	A4.1. B1.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		3.2.2. Length, Height and Distance	<ul style="list-style-type: none"> Demonstrate practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students have are addressed Explain how to teach standard units in the Myanmar context using metric unit 	A4.1. B1.1.	1
		3.2.3. Weight	<ul style="list-style-type: none"> Explain what are the different measurement systems for weight Demonstrate practical ideas to teach measurement of weight Develop practical tools for primary students to establish standard weights, to measure and to compare Propose ideas about how to convey the concept of density of materials 	A4.1. B1.1.	1
		3.2.4. Volume	<ul style="list-style-type: none"> Explain the measurement systems for volume that are used in Myanmar Explain the approaches to teach the measurement of volume of liquids Describe practical tools to make and mark out for measurement. Explain how best to teach volume of three-dimensional spaces 	A4.1. B1.1.	1
		3.2.5. Measuring Length in Taung and Htwa and Weight in Peithar and Kyattha	<ul style="list-style-type: none"> Identified in what contexts these Myanmar units of length and weight are used. Adapt a number of the work station lessons from the previous lessons and developed some others to be used to teach measurement with these units. 	A4.1. B1.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		3.2.6. Areas quadrilaterals	<ul style="list-style-type: none"> Explain how to derive the area of parallelograms, rhombuses and trapezia from the area of rectangles Teach Grade 5 students to find the area of parallelograms, Rhombuses and Trapezia from the area of rectangles Explain the importance of linking topics and formulae when teaching Mathematics Explain the value of visual images when teaching Mathematics 	A4.1. B1.1.	1
		3.2.7. Angles Measurement Part 1	<ul style="list-style-type: none"> Teach the dynamic concept of angle as an amount of turning Plan lessons on angle measurement that minimise common student misconceptions about angles 	A4.1. B1.1.	1
		3.2.8. Angles Measurement Part 2	<ul style="list-style-type: none"> Name angles Link dynamic and static notions of angles Teach Grade 4 students how to use a protractor 	A4.1. B1.1.	1
	3.3. The Language of Mathematics	3.3.1. Language of Mathematics, Learning of Mathematics and Problem-Solving in the Early Grades	<ul style="list-style-type: none"> Describe the importance of considering the language of Mathematics particularly when teaching early grade students Explain the levels through which Mathematical learning develops for young students through the early grades Explain the problem types that are useful to help early grade students develop problem-solving skills 	A1.1. B1.1.	1
		3.3.2. Problem-Solving up to Grade 3	<ul style="list-style-type: none"> Describe the kinds of problem-solving questions that are useful to extend mathematical thinking for students to Grade 3 Categorise problem-types and develop similar examples for Grade 3 students Explain the level Grade 3 students should be able to attain 	A1.1. B1.1.	1

Units	Sub-units	Lessons	Learning Outcomes	TCSF	Periods
		3.3.3. Problem-Solving up to Grade 6	<ul style="list-style-type: none"> Discuss the differences between problem-solving and routine calculations Examine different processes for solving problems. Describe the kinds of problem-solving questions that are useful to extend mathematical thinking for students from Grade 4 to Grade 6 Categorise problem-types and develop similar examples for Grade 4 to Grade 6 students Understand the level Grade 6 students should be able to attain 	A1.1. B1.1.	1
	3.4. Mathematics in Everyday Life	3.4.1. Planning an Agriculture Project	<ul style="list-style-type: none"> Use the skills with measurement of length, weight, volume, time and area and graphs to design a lesson around an agricultural or food project Develop similar practical lessons to demonstrate how numeracy skills and Mathematical thinking are used in real life situations 	A2.1. B1.1.	1
		3.4.2. Planning a Class Excursion	<ul style="list-style-type: none"> Use the skills with measurement of length, weight, volume, time and area to plan a class excursion. Develop similar lessons to demonstrate how numeracy skills and Mathematical thinking is used in real life situations 	A2.1. B1.1.	1
		3.4.3. Graphing Climate	<ul style="list-style-type: none"> Use skills with measurement, data and graphs to design a lesson around climate in Myanmar Develop graphic posters as a teaching and learning resources for classroom display Develop similar practical lessons to demonstrate how numeracy skills and Mathematical thinking are used in real life situations 	B1.1.	1
Total number of periods					36

Unit 1

Introduction to Mathematics

Unit 1 introduces the nature and objectives of Mathematics, Mathematics in the primary curriculum, and an overview of the rest of this course. During the first sub-unit of four lessons, student teachers explore the role of patterns and problem-solving in understanding and performing Mathematics. In the second sub-unit of two lessons, students are introduced to frameworks that will guide their teaching of primary Mathematics, including the goals of primary Mathematics education and the grade-wise Mathematics curriculum. The final sub-unit contains only one lesson, in which student teachers are introduced to the syllabus of this course and the syllabus for primary Mathematics. By the end of the unit, student teachers will understand how to motivate primary students to participate in Mathematics, as well as some of the tools that will guide them as teachers.

Expected learning outcomes



By the end of this unit, student teachers will be able to:

- Explain that Mathematics is partly about patterns (and the rules patterns follow) and about relationships;
- Classify that patterns can be concrete and abstract;
- Develop ideas about how they could teach primary school children about the Mathematics in patterns in the world;
- Explain the strong connection between Mathematics, Science, and Technology;
- Identify the connections between new technologies and the Mathematics that supports them;
- Develop ideas about how to teach primary school children to understand the connection between Mathematics, Modern Science, and Technology;
- Identify that teachers should be encouraging problem-solving in their classes that is stimulating and creative;
- Reflect on the thinking process and algorithms used to solve mathematical problems;
- Demonstrate ideas on how to motivate students to be inspired and interested in Mathematics;
- Explore some of the ways in which people are ‘Mathematicians’ in their everyday lives;
- Explore some of the careers in Mathematics and have ideas to inspire primary-school students;
- Explore the schooling of some famous Mathematicians to highlight the idea that ‘Everyone can be a Mathematician’;
- Explain why Mathematics is taught to primary students and how they use Mathematics in real life;
- Explain the linkage between the goals of primary Mathematics, principles of basic education and the features of grade-wise content of primary Mathematics;
- Identify the EC Mathematics curriculum in Year 1 and its component parts; and
- Recognise the connections between the strands in the EC Mathematics syllabus and the Primary Mathematics syllabus.

1.1. The Nature and Objectives of Mathematics

Mathematics is the exploration and use of patterns and relationships in quantities, space and time. Mathematics equips students with effective means for investigating, explaining and making sense of the world in which they live. Furthermore, mathematical problem-solving ability lies at the heart of Mathematics and helps student teachers tackle the problems in their lives with confidence.

1.1.1. Patterns and relationships

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain that Mathematics is partly about patterns (and the rules patterns follow) and about relationships;
- Classify that patterns can be concrete and abstract; and
- Develop ideas about how they could teach primary school children about the Mathematics in patterns in the world.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum.



Time: One period of 50 minutes



Learning strategies: Reading; discussion; group work



Preparation needed: Suggestions where to access materials for the poster assignment



Resources needed: Flipchart paper; pens



Learning activity 1: Introducing the lesson (15 minutes)

1. Provide an overview of the objectives and what will be covered in this lesson.
2. Form the class of student teachers into groups of four which will work together throughout the lessons in this unit.
3. Supply each group with flipchart paper and ask them to capture the ideas of their discussion on the kinds of patterns in the natural and physical world which the children they will teach would know and be familiar with.
4. Allow for short presentation. Review and reflect.



Assessment

Listen to their presentations and check their flipcharts to assess how well student teachers can understand and identify patterns that are relevant to children.



Possible student teacher responses

Student teacher ideas could be concrete or abstract patterns. These are examples:

- Concrete patterns:
 - Patterns in buildings, such as the pattern formed by bricks;
 - Patterns in leaves or flowers; and
 - Patterns of numbers (such as the even numbers: 2, 4, 6,...).
- Abstract patterns:
 - Drumming or other music;
 - The cycle of day and night; and
 - Changing seasons or weather patterns.



Learning activity 2: Group work (20 minutes)

1. Introduce the next part of the lesson. Explain that one volunteer member of each group using flipchart paper should explain Pythagoras' Theorem to the rest of the group and show some of its applications or calculations.
2. Request that the groups then explore possible ways of teaching the theorem.



Assessment

During the activity, walk around to check if student teachers can do the following:

- Write Pythagoras' theorem clearly on the flipchart.
- Explain Pythagoras' theorem correctly in a clear way and show some applications or calculations using it.
- Contribute to the group's discussion of possible ways of teaching the theorem.



Possible student teacher responses

The flipchart paper should have Pythagoras' Theorem ($a^2 + b^2 = c^2$). It should also have a diagram of a triangle with the sides labeled a, b, and c. It may have other information, such as a solved example problem.

Calculations with Pythagoras' Theorem may include finding unknown sides of a triangle. Groups may discuss real-life applications, such as calculating distance or elevation (such as the height of a building). The applications should be relevant to children's lives and understanding of the world.



Learning activity 3: Assignment (10 minutes)

1. Provide an overview an assignment for this lesson.
2. Give some ideas on what they should do to gather information in the library on the internet (pictures, and so on.).
3. Explain that the poster needs to be instructive, ordered, and have some writing and explanation. Also explain that they should not be just a collage of pictures.

4. Set the deadline for the assignment.
5. Ask for questions of clarification.



Assessment

The assignment forms part of the assessment of the learning outcomes of this lesson. You will assess students after they complete the poster.

The poster should show an understanding of patterns, and it should be appropriate for primary school students. It should be well laid out and should have some explanation (headings, descriptions), and not just be a collection of pictures.



Possible student teacher responses

Posters could have any type of pattern, including abstract or concrete patterns from everyday life. They could also be more mathematical, such as Pythagoras' Theorem or other topics in Geometry.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcomes of this lesson:
 - Understand that Mathematics is partly about patterns (and the rules patterns follow) and about relationships;
 - Understand that patterns can be concrete and abstract; and
 - Develop ideas about how they could teach primary school children about the Mathematics in patterns in the world.
2. Ask student teachers to turn to their partner and to explain how patterns are relevant to Mathematics, and some ways of teaching patterns to primary school students.
3. Remind student teachers that they looked at how patterns are related to Mathematics, and identified some ways of teaching them.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty identifying patterns, share some patterns in the classroom with them, and encourage them to look around them for more patterns.

Learning activity 2: If student teachers have difficulty recalling or applying Pythagoras' Theorem, you may share the theorem and some examples with the class (or let strong student teachers share it.).

Learning activity 3: Support less confident student teachers by encouraging them to read the section of the book on patterns again before doing the assignment. Guide them to resources in the library or on the internet.

1.1.2. Mathematics, Science and Technology

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain the strong connection between Mathematics, Science, and Technology;
- Identify the connections between new technologies and the Mathematics that supports them; and
- Develop ideas about how to teach primary school children to understand the connection between Mathematics, Modern Science, and Technology.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum.



Time: One period of 50 minutes



Learning strategies: Individual reading; group discussion



Preparation needed: Access to the internet for student teachers to do research on technologies of the future



Resources needed: Flipchart paper; pens



Learning activity 1: Introducing the lesson (10 minutes)

1. Introduce the idea that this lesson will explore the strong connection between Mathematics, Science, and Technology.
2. Refer back to the previous lesson on the Mathematical foundation of patterns and rules.
3. Now suggest that Mathematics is the foundation for all areas of Science and Technology.



Assessment

Assess understanding of the previous lesson by asking a few student teachers to explain the role of patterns in Mathematics.

Ask a few student teachers to explain in their own words the expected learning outcomes of this lesson.



Possible student teacher responses

On the previous lesson:

- Patterns have a role in Mathematics theorems such as Pythagoras' theorem;
- Number patterns play a role in Mathematics by building primary students' number sense and enhancing their understanding of operations; and
- There are many real-world examples of concrete and abstract patterns that can be used in a Mathematics classroom.

On the expected learning outcomes of this lesson:

- Identify how Mathematics relates to the fields of Science and Technology; and
- Identify some applications of Mathematics that could be used by professionals working in the fields of Science and Technology.



Learning activity 2: Coding as a subject in schools? (25 minutes)

1. If they have not already, request that the student teachers read the section in their text titled 'Codes and Computing'.
2. Ask them to work in their groups of four and brainstorm creative answers to the question set.⁴
3. Also ask them to discuss the pros and cons of introducing coding as a subject in schools, and to develop their arguments for the position they take.

⁴ The code cracking team allowed the Germans to sink some ships by randomly not passing on the information from a cracked signal.



Assessment

After student teachers discuss ‘Codes and Computing’ in their groups, call on two or three groups to share their pros and cons with the class. Allow others to share their thoughts or ask the groups questions.



Possible student teacher responses

Pros of teaching coding in schools:

- It is a relevant skill for the modern age.
- Students with concrete and relevant skills can get jobs after finishing school.
- It is an application of Mathematics that will build students’ general Mathematics skills.

Cons of teaching coding in schools:

- Technology changes quickly and it is difficult for schools to maintain courses with up-to-date information and skills.
- It is better to focus on core subjects such as Mathematics and allow students to engage with coding on their own time or when they reach a higher level of education.



Learning activity 3: Research assignment (10 minutes)

1. If they have not already, request that the student teachers read the section in their text titled ‘Mathematics, 3D bio-printing, the James Webb Space Telescope, and the 5G Network’.
2. Request the groups choose one of the three future technologies to research (3D bio-printing, the James Webb Space Telescope, or the 5G Network).
3. Clarify that each member should do an investigation on the internet by searching for and reading articles on the chosen topic. Suggest that they read just two pieces that give them a full picture. Ask them to specifically see if they can find some Mathematics behind the technology.

4. Explain that they should each make notes on the key points that they learned. Also explain that they should then meet as a group. One member should volunteer to compile the notes to make sure they are clear and that the information is not duplicated.
5. Set the deadline for the assignment.
6. Ask for questions of clarification.



Assessment

You will assess students based on the set of notes they submit as a group. Assess whether their notes show a good understanding of the future technology they selected and explains it clearly. Make sure the explanation is clear and that the information is not duplicated. You may reward students for showing how the technology is linked to Mathematics.



Possible student teacher responses

These responses are examples adapted from the Wikipedia pages on these topics.

3D bio-printing

- Bioprinting uses biological materials such as cells to print material that resembles body tissue.
- The biological material is printed in layers.
- Tissue must be printed precisely, in order to fit onto an organ, for example. Achieving this precision requires a knowledge of Mathematics.

James Webb Space Telescope

- The telescope is being developed by NASA, and its launch is scheduled for 2021.
- It will study the formation and evolution of galaxies.
- Building and launching the telescope requires an understanding of physics and engineering, which are based on Mathematics.

5G Network

- 5G is the fifth-generation cellular network technology that was launched in 2019.
- Developing new telecommunications technology requires Mathematics and developing the infrastructure to support the new technology in a location does as well.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcomes of this lesson:
 - Understand the strong connection between Mathematics, Science, and Technology
 - Understand the connections between new technologies and the Mathematics that support them
 - Develop ideas about how to teach primary school children to understand the connection between Mathematics, Modern Science, and Technology.
2. Ask student teachers to turn to their partner and to explain how Mathematics is relevant to the fields of Science and Technology, and how these fields can be connected in primary schools.
3. Remind student teachers that they looked at how Mathematics is related to Science and Technology.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty identifying how Mathematics relates to Science and Technology, spend some time discussing the textbook reading for this lesson. Ask volunteers to give concrete examples of applications of Mathematics to these fields.

Learning activity 2: If student teachers have difficulty identifying pros and cons of teaching coding in schools, identify a pro and a con as a class before asking them to continue in groups.

Learning activity 3: Support less confident student teachers by encouraging them to read the section of the book on technology again before doing the assignment. Guide them to resources in the library or on the internet.

1.1.3. Mathematical thinking

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Identify that teachers should be encouraging problem-solving in their classes that is stimulating and creative; and
- Reflect on the thinking process and algorithms used to solve mathematical problems.



Competencies gained: B1.2. Demonstrate capacity to apply different strategies for teaching and learning.



Time: One period of 50 minutes



Learning strategies: Group work; presentation; discussion



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; pens



Learning activity 1: Introducing the lesson (10 minutes)

1. Provide an overview of the lesson. Highlight that the key objective in the lesson is observing mathematical thinking.
2. Describe that student teachers will get to solve a problem and then reflect on the various approaches that were used to solve the problem, and on logical thinking and the use of algorithms.⁵

⁵ An algorithm is a process or a set of rules used to solve a problem.



Assessment

Ask a few student teachers to explain the expected learning outcomes in their own words.



Possible student teacher responses

- Understand that problem-solving skills are important and should be encouraged in a Mathematics class.
- Identify that there are different ways to solve problems and students should be encouraged to apply their problem-solving skills and try different methods.



Learning activity 2: A problem of perspective (20 minutes)

1. The student teachers should work in their groups of four. Request the groups to appoint a member who will be the observer. Ensure that the member knows and understands his/her task.
2. Now read the problem to the class. Ask that groups show all their working and write up their solutions.
3. Move between the groups and encourage them with ideas for a strategy if they are stuck. For the purposes of the lesson, it is best if they follow the long route of repeated division by two. (If, however, they find a quicker route try to ensure that they keep this to themselves until every group has finished.)



Assessment

Walk around to observe groups and check for understanding. Ensure that group members understand the problem and are participating. You may assess their understanding and push them in the right direction by asking questions, for example:

- There are 150 teams in the first round. How many will be in the second round?
- If half of the 150 teams lose, how many will there be in the next round?

Check that the observer of the group is recording the group's ideas and process.



Possible student teacher responses

- The most straightforward way to solve this problem is through repeated division by 2. Note that there are 150 teams at the start of the tournament. Sometimes for a round, there is an odd number of teams, and one team goes to the next round without playing.
- The table below is an example of a process that a group could use to find this answer. Groups will document their problem-solving processes differently, and that is a good thing because they will observe that there are many ways to solve a problem.

Table 1.1. The problem-solving process

Round	Number of teams	Games
1	150	$150 \div 2 = 75$
2	75	$74 \div 2 = 37$ (1 team does not play)
3	38	$38 \div 2 = 19$
4	19	$18 \div 2 = 9$ (1 team does not play)
5	10	$10 \div 2 = 5$
6	5	$4 \div 2 = 2$ (1 team does not play)
7	3	$2 \div 2 = 1$ (1 team does not play)
8	2	$2 \div 2 = 1$

Total games: $75 + 37 + 19 + 9 + 5 + 2 + 1 + 1 = 149$



Learning activity 3: Observing the thinking (15 minutes)

1. Allow each of the observers to present their observations of their groups thinking and strategy for solving the problem. Encourage discussion and questions after each presentation.
2. Make notes on a board of the key and common findings from these observations.
3. Throughout the lesson, avoid concluding an answer. Now in concluding the lesson, ask the question of the student teachers: 'If in the tournament of 150 teams there is one winning team, how many losers must there be? How many games must be played?'



Assessment

Listen to each observer's presentation to see if their group's problem-solving process is logical and they present their process clearly.

Ask the questions and allow groups to discuss the questions briefly before sharing with the class: If, in the tournament of 150 teams, there is one winning team, how many losers must there be? How many games must be played?



Possible student teacher responses

Presentations:

- Student teachers may use any logical process. Encourage them to share their problem-solving strategy even if it is different from the processes used by other groups.

Responses to the questions:

- If in the tournament of 150 teams there is one winning team, how many losers must there be? There are 149 losing teams: $150 - 1 = 149$; and
- How many games must be played? 149 games must be played.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcomes of this lesson:
 - Understand that teachers should be encouraging problem-solving in their classes that is stimulating and creative; and
 - Reflect on the thinking process and algorithms used to solve mathematical problems.
2. Ask student teachers to turn to their partner and to explain actions they will take to ensure that students are comfortable using creativity to solve problems in their classroom.
3. Remind student teachers that they explored different ways to solve a single problem, and considered the thinking processes that are involved in solving Mathematics problems.



Extension and differentiation activities

Learning activity 1: If student teachers do not understand the learning outcomes, discuss them as a class. It may help to define difficult words (such as algorithm, which is simply defined as the process or steps used to solve a problem).

Learning activity 2: If student teachers have difficulty working through the problem, ask questions or give them clues to get started (for example, you may remind them that half of the teams lose after each round and are out of the tournament).

Learning activity 3: Encourage groups during the presentation and remind them that there are no incorrect answers. The important point here is not the answer to the problem but the thinking process.

1.1.4. On being a mathematician

Expected learning outcomes



By the end of the lesson, the student teachers will be able to:

- Demonstrate ideas on how to motivate students to be inspired and interested in Mathematics;
- Explore some of the ways in which people are ‘mathematicians’ in their everyday lives;
- Explore some of the careers in Mathematics and have ideas to inspire primary-school students; and
- Explore the schooling of some famous mathematicians to highlight the idea that ‘everyone can be a mathematician’.



Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources.



Time: One period of 50 minutes



Learning strategies: Individual thinking, group project producing classroom posters and discussion



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; pens



Learning activity 1: Introducing the lesson (10 minutes)

1. Provide an overview of the lesson. Highlight that the key objective in the lesson is motivating students by highlighting the fact that everybody uses Mathematics.
2. Discuss ways that people use Mathematics in everyday life. Ask student teachers to give examples of ways that they use Mathematics in their everyday lives.



Assessment

Ask a few student teachers to explain how they use Mathematics in their everyday lives.



Possible student teacher responses

- Deciding which product to buy.
- Planning a personal budget of how to spend money.
- Planning how to spend time.
- Measuring ingredients to follow a recipe.
- Doing handicrafts, art or other hobbies.



Learning activity 2: Group work and presentation (25 minutes)

1. Introduce the lesson by explaining that, in the lesson, we will explore the topic 'being a mathematician.' Student teachers should think individually who in their jobs use Mathematics. Explain that the purpose of the lesson is to explore the multiple occupations that require Mathematical skills to think and calculate.
2. Divide student teachers into their groups. Each group should choose one of the occupations outlined in the notes (a carpenter, a clothes manufacturer, a textile manufacturer). They should discuss all the Mathematics that each of these occupations use. They should also consider mathematical thinking when examining this question. Then they should develop a visual presentation in the way of a poster and present these ideas.



Assessment

Listen to their presentations and check their flipcharts to assess how well student teachers understand the applications of Mathematics to the occupation they chose.



Possible student teacher responses

- Carpenter
 - Calculating how much wood or other materials to buy;
 - Measuring and cutting pieces of wood; and
 - Budgeting (calculating cost, income and profit).
- Clothes manufacturer
 - Calculating the amount of fabric and other materials needed;
 - Measuring and cutting pieces of fabric; and
 - Budgeting (calculating cost, income and profit).
- Textile manufacturer
 - Designing patterned fabric;
 - Calculating the amount of raw materials needed; and
 - Budgeting (calculating cost, income and profit).



Learning activity 3: Pair work assignment (10 minutes)

Student teachers should work in pairs to identify a famous mathematician or scientist. They should research this person's history and develop a poster presentation for homework to be presented in the next class.



Assessment

Assess student teachers' understanding of the idea that everyone can be a mathematician by the quality and detail of the poster presentations to the class.



Possible student teacher responses

Student teachers may highlight any aspects of the person's history. They may note that some very famous Mathematicians and scientists were not very successful at school or had difficulties in their lives.

Here is one example:

Stephen William Hawking (8 January 1942 – 14 March 2018)⁶ was an English theoretical physicist, cosmologist and author. He was the director of research at the Centre for Theoretical Cosmology at the University of Cambridge at the time of his death and he was professor of Mathematics at the University of Cambridge between 1979 and 2009. He worked on gravitational singularity theorems. He used this theory to predict that black holes emit radiation. This radiation is often called Hawking radiation. Hawking was the first to set out a theory of cosmology explained by combining the general theories of relativity and quantum mechanics.

Hawking had a form of motor neurone disease. The disease paralysed him over the decades. After he lost his speech, he communicated through a speech-generating device, initially through use of a hand-held switch, and eventually by using a single cheek muscle. He died on 14 March 2018 at the age of 76.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcomes of this lesson:
 - Have ideas on how to motivate students to be inspired and interested in Mathematics;
 - Have explored some of the ways in which people are 'mathematicians' in their everyday lives;
 - Have explored some of the careers in Mathematics and have ideas to inspire primary-school students; and
 - Have explored the schooling of some famous mathematicians to highlight the idea that 'everyone can be a mathematician'.

⁶ Stephen Hawking (adapted from Wikipedia)

2. Ask student teachers to turn to their partner and to explain actions they will take to ensure that students understand that everyone is a mathematician.
3. Remind student teachers that they explored how people working in different fields must use Mathematics to do their jobs well.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty identifying ways that they use Mathematics in their daily lives, give them some ideas before asking them to share.

Learning activity 2: You may allow students to choose another profession that is not described in the textbook, and come up with their own ideas of how Mathematics might be used in that profession.

Learning activity 3: Support less confident student teachers by encouraging them to read the section of the book on being a mathematician again before doing the assignment. There are several careers listed, and the Stephen Hawking example is included. Guide them to resources in the library or on the internet.



Review questions: Possible student teacher responses

Question 1: How are patterns relevant to the study of Mathematics?

Answer: Many mathematical concepts and theorems follow patterns (for example, the pattern of even numbers or Pythagoras' Theorem), and understanding patterns builds students' understanding of mathematical concepts.

Question 2: Why is it important to encourage creativity in Mathematics?

Answer: There are often many different ways to solve a problem and students build their problem-solving skills by being creative and trying different methods during the problem-solving process.

Question 3: What are some methods of motivating students to engage with Mathematics?

Answer: Help students understand how the topic is relevant to their life through examples from the real world, including instances when people use Mathematics in their everyday lives or in specific occupations.

1.2. Mathematics in the Basic Education Curriculum

In this sub-unit, we will explore why Mathematics is taught to primary students. The linkages between the goals of primary Mathematics, the principles of basic education and the features of the grade-wise contents of primary Mathematics will be explained. We will develop a basic understanding of the features in the curriculum for Mathematics.

1.2.1. Mathematics in primary education

Expected learning outcomes

By the end of the lesson, student teachers will be able to:

- Explain why Mathematics is taught to primary students and how they use Mathematics in real life.





Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum.



Time: One period of 50 minutes



Learning activity 1: Group discussion and presentation (10 minutes)

1. Divide student teachers into small groups to discuss the question: 'Why should children study Mathematics in primary school?'
2. Invite a representative of each group to present the group's ideas.



Assessment

Listen to each group's presentation and assess how well-organised and logical their responses are.



Possible student teacher responses

There are no correct/incorrect answers, so you should not make comment on any opinions from student teachers. These are some examples:

- To know the numbers to apply;
- To be able to calculate;
- To be able to buy things by understanding the price;
- To be able to measure all matters;
- To know the units for measurements;
- To be able to draw Geometry figures;
- To be able to apply their mathematical skills to everyday problems; and
- To develop critical thinking skills.



Learning activity 2: Group work and presentation (10 minutes)

1. Student teachers discuss the question in groups: 'What are the goals of Mathematics education in primary education?'
2. Student teachers study the goals of Primary Mathematics Education and discuss the differences and similarities with the ideas collected in their groups.
3. Representative student teachers make presentation on their findings.



Assessment

Listen to each group's presentation and assess how well they understand the four goals of primary Mathematics education. Make note of how well they describe similarities and differences to their own responses to the first question.



Possible student teacher responses

As there are no correct/incorrect answers, student teachers should discuss this freely. Student teachers' responses will depend on the ideas they provided for the previous question. These are examples of similarities and differences for the example ideas above:

- Similarities:
 - Both lists mention the importance of mathematical and problem-solving skills in daily life.
 - Both lists mention basic mathematical knowledge and skills regarding numbers, quantities, geometrical figures.
- Differences:
 - The goals of primary Mathematics education also mention that students should appreciate the usefulness of Mathematics (attitude).
 - The group's examples mentioned specific mathematical skills in more detail, such as measurement.



Learning activity 3: Group work and presentation (10 minutes)

1. Student teachers discuss the question in groups: 'What role should a teacher play in helping students achieve these goals?'
2. Representatives of the groups make presentation.



Assessment

Listen to each group's presentation. They should demonstrate an understanding of the four goals of primary Mathematics education, and they should clearly outline some attributes, skills and activities that teachers should have to support students to achieve these goals.



Possible student teacher responses

As there are no correct/incorrect answers, student teachers should discuss this freely. Some sample opinions for each of the four goals are listed below.

Table 1.2. Goals of primary Mathematics education

Goal 1	<p>Teachers must learn and teach students to help them understand clearly about Numbers, Measurement, Geometry and Mathematical relations.</p> <p>Teachers must have knowledge of and skills in all aspects of primary school Mathematics as should understanding the connections between areas of the curriculum.</p> <p>Teachers must have the knowledge and skills to enable students to learn the contents in effective ways.</p> <p>Teachers must have knowledge of the content of the Mathematics curriculum for higher level and so must be able to help students to prepare for further learning.</p>
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Goal 2	<p>Teachers must train primary students so that they can think logically.</p> <p>Teachers have to present the learning contents and activities for primary students so that they can learn to think by themselves.</p> <p>Teachers have to provide opportunities to primary students to share the opinion and to listen to others.</p> <p>Teachers have to teach primary students to develop mastery of basic operations in Mathematics.</p> <p>Teachers have to train primary students to give answers with reasons.</p> <p>Teachers must have knowledge and skills to conduct lessons on problem-solving.</p>
Goal 3	<p>Teachers have to give examples with which students are familiar in their daily lives.</p> <p>Teachers have to ask primary students how to apply what they learned to their daily lives. Teachers must have the knowledge and skills to identify mathematical problems from their daily lives to relate to their learning contents.</p>
Goal 4	<p>Teachers have to provide learning activities that are interesting and which motivate students to learn and see the usefulness of Mathematics.</p> <p>Teachers have to provide lessons so that primary students enjoy Mathematics.</p>



Learning activity 4: Student teachers capture personal goals as a teacher (15 minutes)

1. Student teachers write down their own goals to be a teacher who can support primary students to achieve the goals of primary Mathematics. They should write their goals on a piece of paper with their name on it.
2. The Teacher Educator collects the goals from student teachers and closes the lesson.



Assessment

You will be able to understand student teachers' current image of Mathematics by checking the individual worksheet. Assess to what extent student teachers understand the goals of primary Mathematics. You will also understand the goals which student teachers set to be a good teacher who is able to support primary students by checking their individual work.



Possible student teacher responses

There are no correct/incorrect answers. Some sample student teacher goals for each of the four goals of primary Mathematics education are listed below.

Table 1.3. Brainstorming student teachers' personal goals

Goal 1	<p>I will learn all of the primary Mathematics curriculum well so that I can explain it clearly to students.</p> <p>I will identify effective teaching strategies for the topics in the curriculum.</p>
Goal 2	<p>I will design Mathematics activities that encourage students to think logically and work through their own problem-solving processes.</p> <p>I will create a welcoming environment and encourage students to share their thoughts and logical processes with the class.</p>
Goal 3	<p>I will prepare examples that students are familiar with in their daily lives.</p> <p>I will encourage students to apply the Mathematics skills they learn to their daily lives.</p>
Goal 4	<p>I will have a positive attitude around Mathematics, and will encourage students to have the same.</p> <p>I will provide learning activities that are interesting and which motivate student teachers to learn and see the usefulness of Mathematics.</p>



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand and be able to explain why Mathematics is taught to primary students.
2. Ask student teachers to turn to their partner and to explain why it is important to teach Mathematics to primary students.
3. Remind student teachers that they explored the four goals of primary Mathematics education, and set personal goals for helping students meet these when they are teachers.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty, ask one or two volunteers to share their ideas with the class before working in groups.

Learning activity 2: After reflecting on the similarities and differences between the goals and student teachers' own ideas, you may ask if there are any changes they would make to the four goals of primary Mathematics education.

Learning activity 3: If student teachers have difficulty, ask one or two volunteers to share their ideas with the class before working in groups.

Learning activity 4: Encourage student teachers to be specific in their goals. If they have difficulty, they may use the goals their group wrote from learning activity 3 as guidance.

1.2.2.

Mathematics in the primary curriculum

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain the linkage between the goals of primary Mathematics, principles of basic education and the features of grade-wise content of primary Mathematics.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum.



Time: One period of 50 minutes



Learning activity 1: Individual presentations (10 minutes)

1. Ask student teachers to recall the four goals of primary Mathematics education and share them with the class.
2. Nominate some student teachers to share their goals to be a teacher who is able to support primary school students to achieve the goals in primary Mathematics education, which they completed in the previous lesson.



Assessment

Choose four student teachers and ask them each to give one of the four goals of primary Mathematics education to assess how well they recall the goals.

Listen to student teachers' presentations of their personal goals to assess their understanding of the four goals and whether their personal goals support the goals of primary Mathematics education.



Possible student teacher responses

The four goals of primary Mathematics education:

- Have basic mathematical knowledge and skills regarding numbers, quantities, geometrical figures and data representation. (Knowledge and understanding, skills)
- Be able to reason and explain logically in problem-solving. (Thinking)
- Be able to apply mathematical knowledge and skills to the problems in daily life as well as in their learning. (Knowledge and understanding, skills)
- Appreciate the usefulness of mathematical ideas and approaches. (Attitude)

Examples of personal goals:

Table 1.4. Examples of personal goals

Goal 1	<p>I will learn all of the primary Mathematics curriculum well so that I can explain it clearly to students.</p> <p>I will identify effective teaching strategies for the topics in the curriculum.</p>
Goal 2	<p>I will design Mathematics activities that encourage students to think logically and work through their own problem-solving processes.</p> <p>I will create a welcoming environment and encourage students to share their thoughts and logical processes with the class.</p>
Goal 3	<p>I will prepare examples that students are familiar with in their daily lives.</p> <p>I will encourage students to apply the Mathematics skills they learn to their daily lives.</p>
Goal 4	<p>I will have a positive attitude around Mathematics, and will encourage students to have the same.</p> <p>I will provide learning activities that are interesting and which motivate student teachers to learn and see the usefulness of Mathematics.</p>



Learning activity 2: Group work on the guiding principles (20 minutes)

1. Ask student teachers to read the 'Guiding principles for basic education' in the Student Teacher Textbook.
2. Ask the student teachers to find the connections between the contents of the guiding principles and the four goals of primary school Mathematics. They should discuss in groups and record their ideas clearly.
3. Ask a representative of each group to share their ideas with the class.



Assessment

Listen to student teachers' presentations to assess how well they understand the guiding principles and whether they can identify the linkages between the goals of primary Mathematics and the guiding principles of basic education.



Possible student teacher responses

As there are no correct/incorrect answers, student teachers should discuss this freely. Some sample opinions are listed below.

Table 1.5. Goals and guiding principles of basic education

Goal	Guiding principle with linkages
Have basic mathematical knowledge and skills regarding numbers, quantities, geometrical figures and data representation. (Knowledge and understanding, skills)	e. Preparation for higher learning g. Balance in academic literacy
Be able to reason and explain logically in problem-solving. (Thinking)	c. 21 st century skills
Be able to apply mathematical knowledge and skills to the problems in daily life as well as in their learning. (Knowledge and understanding, skills)	c. 21 st century skills g. Preparation for one's life in Myanmar society and modern economy
Appreciate the usefulness of mathematical ideas and approaches. (Attitude)	a. All-round, balanced development



Learning activity 3: Interpreting the grade-wise Mathematics curriculum (15 minutes)

1. Ask student teachers to look at the 'Grade-wise Mathematics curriculum' in the textbook with their groups.
2. Briefly describe how to read the table:
 - This diagram shows how the contents of primary Mathematics are organised and how the topics are ordered and also connected to one another.
 - The first row shows the Grade (1-5). The left column shows the 'strand'. The second and third columns show the sub-strand.
 - There are four strands in primary Mathematics. These are Number, Geometry, Measurement and Mathematical Relations.
 - In each strand, the relationship between the topics is indicated with arrows.
3. Student teachers should work with their groups to identify how the diagram is organised. They should be able to understand its structure. Note that they do not need to read it carefully. Student teachers will analyse each strand in later lessons, so all they are expected to grasp at this stage are the overall features and arrangement of the contents.



Assessment

Walk around and ask the groups questions about the table to assess to what extent student teachers are able to interpret the information. These are example questions you may ask:

- What are the four strands in primary Mathematics?
- Where are the four strands found in the table?
- Where are the sub-strands found in the table?
- Where are the grade levels found in the table?
- What are some topics covered in the strand 'number' during Grade 2?
- What are some topics covered in the strand 'Geometry' during Grade 4?



Possible student teacher responses

Student teachers should identify the answers using the table. Answers for the example questions are provided:

- What are the four strands in primary Mathematics?
 - Answer: Number, Geometry, Measurement and Mathematical Relations
- Where are the four strands found in the table?
 - Answer: In the first column.
- Where are the sub-strands found in the table?
 - Answer: In the second and third columns.
- Where are the grade levels found in the table?
 - Answer: In the top row.
- What are some topics covered in the strand ‘number’ during Grade 2?
 - Example answer: Counting, reading and writing of numbers up to 1,000.
- What are some topics covered in the strand ‘Geometry’ during Grade 4?
 - Example answer: Meaning, property and drawing of parallel and perpendicular lines.



Check student teachers’ understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand how the goals of primary Mathematics education are linked to the guiding principles of basic education; and
 - Explain the features of grade-wise content of the primary Mathematics curriculum.
2. Ask student teachers to turn to their partner and explain how to interpret the table that gives the grade-wise content of the primary Mathematics curriculum.
3. Remind student teachers that they explored two new documents: the guiding principles of basic education and the grade-wise content of the primary Mathematics curriculum. These are both important documents that should guide them as teachers.



Extension and differentiation activities

Learning activity 1: As student teachers, explain their personal goals, you may ask them to describe some specific steps that they could take to achieve the goals.

Learning activity 2: Encourage student teachers to give the logic and reasons for their identified linkages between the goals of primary Mathematics education and the guiding principles of basic education.

Learning activity 3: As you walk around and ask groups questions about the table, start with simple questions. Ask more challenging questions based on student teachers' level of understanding. If needed, explain the table further to individual groups.



Review questions: Possible student teacher responses

Question 1: Why is it important to teach Mathematics to primary school students?

Answer: It is important for children's daily lives and development of skills such as logical thinking and problem-solving. It supports the goals of primary education and the guiding principles for basic education.

Question 2: What are the four goals of primary Mathematics education?

Answer: 1. Have basic mathematical knowledge and skills. 2. Be able to reason and explain logically in problem-solving. 3. Be able to apply mathematical knowledge and skills to the problems in daily life and learning. 4. Appreciate the usefulness of mathematical ideas and approaches.

Question 3: What is the grade-wise Mathematics curriculum?

Answer: It describes the topics that should be taught in each of the four strands of Mathematics (number, Geometry, measurement and mathematical relations) at each level of primary education.

1.3. Course overview of Education College Mathematics curriculum

This short sub-unit is designed to provide student teachers with an overview of the curriculum they will follow for their first year in Mathematics. Apart from providing this overview, the lesson is designed to demonstrate how the course they will follow will equip them to teach Mathematics to primary school students.

1.3.1. The Mathematics curriculum in Year 1

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Identify the EC Mathematics curriculum in Year 1 and identify the component parts of the syllabus; and
- Recognise the connections between the strands in the EC Mathematics syllabus and the primary Mathematics syllabus.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum.



Time: One period of 50 minutes



Learning strategies: Reading; discussion; group work



Preparation needed: Prepare a PowerPoint presentation summarising the key points around the student teacher syllabus (use the notes in the Student Teacher Textbook)



Resources needed: Flipcharts; pens; copies of primary Mathematics syllabus



Learning activity 1: Introducing the lesson and the Year 1 syllabus (15 minutes)

1. After providing an overview of the objectives of this lesson, either request the student teachers to read the text for this lesson or give a PowerPoint presentation.



Assessment

Select a few student teachers to answer questions about this lesson. You may allow them to reference their textbook for the answers. These are example questions:

- How many units are there in this course?
- What are the units in this course?
- What was covered in the first unit of the course?
- What is the next unit in the course?



Possible student teacher responses

Assess students' understanding of the course and its content based on their answers:

- How many units are there in this course?
 - Answer: 5
- What are the units in this course?
 - Answer: Introduction to Mathematics, Problem-Solving and Misconceptions, Understanding Mathematics, How we learn and How we teach Mathematics, Mathematical Modelling and Representation.
- What was covered in the first unit of the course?
 - Answer: Introduction to Mathematics, including the nature and objectives of Mathematics, and Mathematics in the Basic Education Curriculum.
- What is the next unit in the course?
 - Answer: Problem-Solving and Misconceptions.



Learning activity 2: Reviewing the Year 1 Mathematics syllabus (15 minutes)

1. Request the student teachers to review the syllabus in their groups. Suggest that this should be done in two ways:
 - Student teachers should identify questions they have where they do not understand concepts or where they require clarification about the course.
 - Where the syllabus is covering specific mathematical topics (numbers, measurement, algebra, Geometry and so on.) they should try to identify where they think they may need revision to refresh their knowledge of the topics.
2. Request that groups capture their ideas on flipcharts for discussion in the class



Assessment

Walk around to observe groups and check for understanding of the activity. You may listen to their conversations or read what they are writing on the flipchart. There are no correct/incorrect answers.



Possible student teacher responses

Possible questions on the content or course:

- Why do we need a unit on ‘understanding Mathematics’ when we have already completed Mathematics courses?
- Why do primary students need to learn statistics and probability?

Possible gaps in knowledge:

- Not familiar with the meaning of some phrases in the unit descriptions, such as ‘inquiry-based learning’ (IBL) and ‘mathematical modelling’.
- Not familiar with how scatterplots can be used to represent real world situations.
- Need revision of algebra.



Learning activity 3: Discussing issues and identifying gaps in knowledge (10 minutes)

1. Facilitate a discussion on questions on the content or course. Invite groups to present their questions and allow discussion. Offer clarification to their questions as much as possible.
2. Facilitate a discussion on ‘gaps in knowledge’. Invite groups to present their gaps and allow discussion. Suggest ways in which student teachers can revise the topics (using primary school texts, working through grade level assessment papers, searching the internet, and so on.)



Assessment

Listen as students present their questions and gaps in knowledge. There are no correct/incorrect answers. Also listen as they share ideas during the discussion.



Possible student teacher responses

Examples of student teacher responses are given above for Learning activity 2. During Learning activity 3, student teachers may respond and reflect on other groups' ideas, or give them suggestions. These are examples of possible responses to the questions and gaps identified above:

Questions on the content or course:

- Why do we need a unit on 'understanding Mathematics' when we have already completed Mathematics courses?
 - Example response: Although we may understand mathematical processes, this unit will look at methods of teaching them to primary students.
- Why do primary students need to learn statistics and probability?
 - Example response: Statistics and probability help us to process and understand information in our daily lives.

Gaps in knowledge:

- Not familiar with the meaning of some phrases in the unit descriptions, such as 'inquiry-based learning' and 'mathematical modelling'.
 - Example response: These phrases will be addressed in the relevant unit. If student teachers have further questions that are not addressed, they may speak to their peers or educator, or search the internet.
- Not familiar with how scatterplots can be used to represent real world situations.
 - Example response: Review primary Mathematics textbooks.
- Need revision of algebra.
 - Example response: Review primary Mathematics textbooks.



Learning activity 4: Review the primary Mathematics syllabus (5 minutes)

Provide copies of the primary Mathematics syllabus (or reference to electronic access). Student teachers should read this carefully for homework and identify all the areas across the syllabus where they think they need revision. They should keep it as a reference throughout the course.



Assessment

You may assess students after they have a chance to review the syllabus by asking them to share which topics they identified as ones they need revision on.



Possible student teacher responses

There are no correct/incorrect answers. Student teachers may identify that they need revision on any topic in the syllabus.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand the EC Mathematics curriculum in Year 1 and identify the component parts of the syllabus; and
 - Recognise the connections between the strands in the EC Mathematics syllabus and the primary Mathematics syllabus.
2. Ask student teachers to turn to their partner and explain the content of this course.
3. Remind student teachers that they were introduced to the content of this course. For their homework, they will review the content of the primary Mathematics syllabus.



Extension and differentiation activities

Learning activity 1: If student teachers have read the textbook before the lesson, you may ask them to explain the content in their own words before your presentation.

Learning activity 2: If they have time after listing their questions and gaps, you may ask student teachers to work with their groups to try to answer their own questions.

Learning activity 3: Depending on their level of understanding, you may encourage student teachers to answer other groups' questions. If they do not understand the content well, you may answer the groups' questions yourself. It is also okay to leave some questions unanswered – they will likely be answered during this course.

Learning activity 4: You may ask student teachers to write down the topics in the primary Mathematics syllabus they need revision on, and come up with a plan for revising these topics.



Review questions: Possible student teacher responses

Question 1: What are the main parts of the syllabus for this course?

Answer: There are five units: Introduction to Mathematics; Problem-solving and Misconceptions; Understanding Mathematics; How we Learn and How we Teach Mathematics; Mathematical Modelling and Representation.

Question 2: How is the syllabus for this course related to the syllabus for primary Mathematics?

Answer: The two syllabi cover the same strands of the subject. This course requires an understanding of the topics in the primary Mathematics syllabus.

Unit Summary



Key messages

During this introductory unit, we have covered topics (referred to in the syllabus as sub-strands) including the nature and objectives of Mathematics, Mathematics in the Basic Education Curriculum and teaching Mathematics to inspire young students to be fascinated by the subject.

The objective of the unit is to provide student teachers with an essential understanding of Mathematics, why it is an important subject in primary education, what is contained in the syllabus in the first year and how teachers can inspire students.



Unit reflection

What were the key lessons you learnt in this unit?

- It is important to motivate children to have a positive attitude toward Mathematics.
- Mathematics is applicable to our daily lives and many different fields.
- The logical thinking and problem-solving processes are important in Mathematics.
- It is important to teach Mathematics to primary students.
- There are primary Mathematics goals and a grade-wise Mathematics curriculum to guide teachers.

What are some ideas you have for motivating young students to be enthusiastic about learning Mathematics? Introducing problem-solving activities relevant to their daily lives is a particularly useful technique, but often require some work on your part, as you may find you have to create such tasks yourself.

Have your feelings about Mathematics changed as a result of this unit? If so, in what ways have they changed? Do you agree that Mathematics is important for everyone and we are all mathematicians in our daily lives?

What gaps have you identified in your knowledge of Mathematics for primary school? How are you planning to address these gaps?

What are some tools that you have learned about in this unit? How will these be useful to you as a teacher? Make sure you understand and know the grade-wise Mathematics curriculum, and understand which strands to teach during each grade level as a teacher.



Further reading

1.1.

Mensa Education and Research Foundation. (2019). Fabulous Fibonacci. Retrieved from Mensa for Kids website: <https://www.mensaforkids.org/teach/lesson-plans/fabulous-fibonacci/>

Vaidyanathan, S. (2019). Teaching coding to kids: What programming language Should we use? Retrieved from EdSurge website: <https://www.edsurge.com/news/2019-03-11-teaching-coding-to-kids-what-programming-language-should-we-use>

1.2.

Grade-Wise Mathematics Curriculum. (n.d.).

Veldhuis, M., & Zhao, X. (2018). The effectiveness of Mathematics teaching in primary schools. *Research in Mathematics Education.*

1.3.

International Bureau of Education, & UNESCO. (2013). *Glossary of curriculum terminology.* Retrieved from http://www.ibe.unesco.org/fileadmin/user_upload/Publications/IBE_GlossaryCurriculumTerminology2013_eng.pdf

Unit 2

Problem-Solving and Misconceptions

This unit explores strategies for problem-solving that can be used to build primary students' Mathematics skills. It examines some research in Mathematics, including some common errors and misconceptions in Mathematics. It also describes the importance of creating an inclusive learning environment, and gives ideas for how this can be accomplished.

Expected learning outcomes



By the end of this unit, student teachers will be able to:

- Describe the current problems of teaching Mathematics in our schools in Myanmar;
- Reflect on the issues and propose some solutions to them;
- Explain problems of the conventional teaching of Mathematics;
- Explain the new teaching strategy which improves children's mathematical thinking skills;
- Explain Polya's four phases for solving problems;
- Identify how teachers can promote the mathematical thinking of primary students by understanding Polya's four phases;
- Understand the concept of heuristics;
- Explain how to use 'Back to Definition', 'Visualise by Drawing', 'Finding a Pattern', and 'Using a Simpler Analogous Problem' heuristics in problem-solving;
- Describe some of the common errors and misconceptions that students have in adding and subtracting in columns;
- Recognise examples of how to avoid the development of these errors and misconceptions is addressed in the Grade 2 Mathematics textbooks;
- Identify the errors or misconceptions that a student has made and help him/her correct them;
- Explain how all children are different and have potential to be mathematician;
- Demonstrate the advantages of developing an inclusive approach to teaching and learning in schools;
- Demonstrate how to promote and develop inclusive approaches in schools and in the classroom; and
- Prepare as Mathematics teachers for the challenges in promoting inclusive education in Myanmar.

2.1. Enquiry-based Learning

This sub-unit explores new approaches to Mathematics that are based on applying specific phases and strategies to solve Mathematics problems. Student teachers will examine techniques that they can encourage their own students to use to solve Mathematics problems, and they will apply these strategies to solve problems themselves. They will compare these new ideas with the current way of teaching and learning Mathematics in Myanmar, and identify some benefits and challenges to implementing these ideas in their own classrooms.

2.1.1. Developing thinking skills for primary school students

Expected learning outcomes

By the end of the lesson, the student teachers will be able to:

- Describe the current problems of teaching Mathematics in our schools in Myanmar
- Reflect on the issues and propose some solutions to them





Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources; and B1.3. Demonstrate good lesson planning and preparation in line with student teachers' learning ability and experience.



Time: One period of 50 minutes



Learning strategies: Individual work, group discussion



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; pens



Learning activity 1: Group work (5 minutes)

1. Form the class of student teachers into groups of four which will work together throughout the lessons in this unit.
2. Supply each group with flipchart paper and ask them to capture their ideas.
3. Ask them to reflect on the Mathematics classrooms that they have been in. Suggest that they firstly make a personal list and then discuss these areas of difficulty in their group. They should answer the questions: 'What do Mathematics teachers do well in Myanmar? What are some ways that Mathematics teachers could improve their teaching?'



Assessment

Walk around to observe groups. Make sure they understand the questions and are participating.



Possible student teacher responses

There are no correct/incorrect answers. Encourage student teachers to share their opinions. Examples follow.

Things that teachers do well:

- They have good technical knowledge of Mathematics.
- They explain Mathematics concepts and steps clearly.

Things teachers could improve:

- They could use a more student-centred approach.



Learning activity 2: Individual and group work (15 minutes)

1. Student teachers should solve the area problem in the textbook individually in as many ways as they can.
2. After student teachers finish solving, they should share their ideas in their groups.
3. A representative of each group should make a presentation of the group's discussion to the whole class.



Assessment

Listen to the groups' presentations to assess if they have a good understanding of the problem. Each group should have solved the problem in different ways. They should understand that there are multiple paths to the same answer. You may give the following hints if student teachers are struggling and can't find solutions, but not give the answers. The Teacher Educator should model what the teacher should be doing for primary school students.

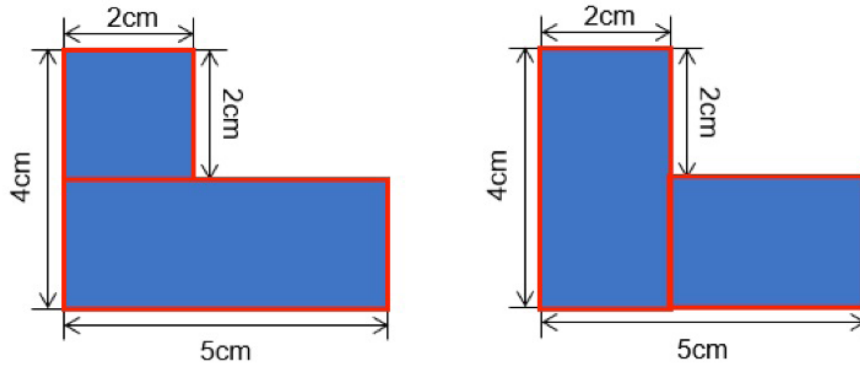
1. Perhaps you could cut or divide the shape?
2. If you add something to the shape will this help?
3. What happens if you double the shape to make another shape? Does that help?



Possible student teacher responses

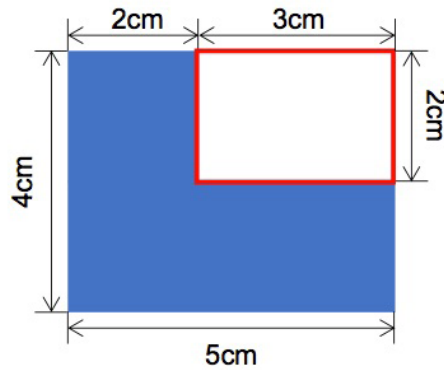
The problem can be solved in several ways:

Solution 1: Cut/divide the shape to make 2 rectangles

Figure 2.1. Finding the area of a shape (1)

$$(2\text{cm} \times 2\text{cm}) + (5\text{cm} \times 2\text{cm}) = 14\text{cm}^2 \quad (4\text{cm} \times 2\text{cm}) + (3\text{cm} \times 2\text{cm}) = 14\text{cm}^2$$

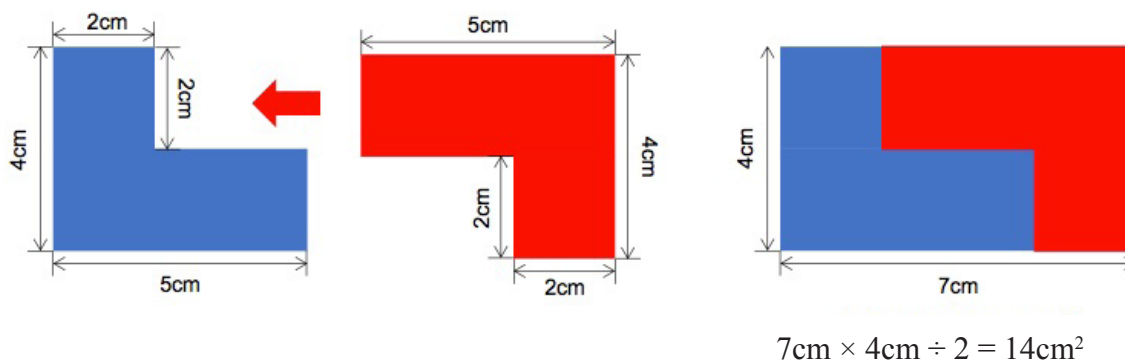
Solution 2: Add a small rectangle to make a big rectangle and subtract the area of the small rectangle from the big rectangle

Figure 2.2. Finding the area of a shape (2)

$$(5\text{cm} \times 4\text{cm}) - (3\text{cm} \times 2\text{cm}) = 14\text{cm}^2$$

Solution 3: Duplicate the shape to make a big rectangle and divide it into 2

Figure 2.3. Finding the area of a shape (3)



Learning activity 3: Discussion of problem-solving (20 minutes)

1. Student teachers will discuss the questions in their groups:
 - Are there common ideas in these solutions? What are these?
 - In our primary schools, many of primary students cannot think through these solutions even if they already know how to calculate the area of rectangles, addition, subtraction, multiplication, and division. What are the disadvantages if primary students cannot think through solutions?
 - Why do you think many primary students cannot think of these solutions?
 - Are there possible problems in the way Mathematics is taught at primary level?
2. Request student teachers in their groups to reflect on the questions posed in the discussion sheet and to make a record on flipchart paper.
3. Facilitate a class discussion on matters discussed in their groups.



Assessment

Listen to group presentations to assess to what degree student teachers reflect on the questions and understand the purpose of the problem-solving activity. Student teachers should notice the importance of thinking ideas and solutions by themselves, rather than just memorising formula. Student teachers should be aware of the problems of current teaching methodology, which do not focus on students' mathematical thinking skills, and realise the constraints of the conventional ways of teaching Mathematics.



Possible student teacher response

Student teachers may notice the following points:

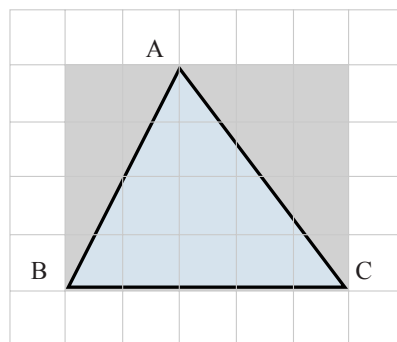
- We can solve the question by using prior knowledge, such as the formula of rectangle area, addition, subtraction, division and multiplication. It requires thinking of various ideas and solutions, not only the knowledge.
- Primary student teachers are weak in mathematical thinking. They are weak in applying their prior knowledge to the new question, which means that they always face difficulties when they encounter new types of questions. At the same time, it would be difficult for primary student teachers to apply their mathematical knowledge into real situation.
- In current teaching, the teacher always teaches how to solve problems/questions first, and student teachers just follow the way they are told. Student teachers are not given enough opportunities to think solutions by themselves. Additionally, student teachers are not given enough chances to share their ideas with others. This limits the student's ability to think and learn in different ways.



Learning activity 4: Assignment on conceptualising a lesson (5 minutes)

1. Student teachers should work with their groups to complete this assignment.
2. Explain that the task for the student teachers is to conceptualise a lesson on area using the shape below (also in their textbooks), and to explain each step of the lesson.

Figure 2.4. Building a lesson activity on finding the area of a shape





Assessment

You will assess student teachers' lesson plans when they submit them. Student teachers should think of the best ways of facilitation, focusing on how to give primary students opportunities to think by themselves.



Possible student teacher responses

An example of a group's work for this homework activity is provided in Learning Activity 2 in the following lesson, in which student teachers present their work.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Be aware of the current problems of teaching Mathematics in our schools in Myanmar; and
 - Think of some solutions.
2. Ask student teachers to turn to their partner and explain some of the problems with the current way of teaching Mathematics in Myanmar.
3. Remind student teachers that they practiced problem-solving in their groups. For their homework they will create a teaching and learning activity that would encourage their own students to apply problem-solving skills.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty reflecting and responding to the questions, provide an example from your own experience to guide them and make them feel more comfortable answering the question.

Learning activity 2: If they have difficulty thinking of solutions to the problem, you may ask questions to guide them. For example, 'Perhaps you could cut or divide the shape?'

Learning activity 3: You may ask additional questions to prompt student teachers to reflect further. For example: 'How can you address this issue in your own classroom?'

Learning activity 4: If student teachers have difficulty understanding the assignment, spend a few extra minutes explaining. You may guide them to resources in the library or on the internet that can support their lesson planning.

2.1.2. Teaching thinking through enquiry

Expected learning outcomes



By the end of the lesson, the student teachers will be able to:

- Explain problems of the conventional teaching of Mathematics; and
- Explain the new teaching strategy which improves children's mathematical thinking skills.



Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources; and
B1.3. Demonstrate good lesson planning and preparation in line with students' learning ability and experience.



Time: One period of 50 minutes



Learning strategies: Presentation; explanation; group discussion



Preparation needed: Review the presentation materials for the new strategy of teaching Mathematics



Resources needed: Flipchart paper; pens



Learning activity 1: Introducing the lesson (5 minutes)

1. Ask a few student teachers to explain briefly what they learned in the previous lesson.



Assessment

Listen to student teachers' responses to assess whether they can understand and recall the previous lesson.



Possible student teacher responses

Student teachers should pick up some problems in teaching Mathematics referring to the activities in the previous lesson. Example responses are:

- There can be many different ways to solve a single problem.
- Students may have difficulty solving a problem even if they have the mathematical skills needed to solve it.
- The old way of teaching does not encourage creativity in problem-solving.



Learning activity 2: Presentation (20 minutes)

A representative of each group should give a presentation on the ideas that the group developed in response to the homework assignment.



Assessment

Listen to each group present their homework and assess to what extent their activity encourages students to think for themselves and apply problem-solving skills.

You will be able to assess to what extent student teachers can think of Mathematics lesson plans which give primary student teachers enough opportunities to think by themselves. The table below may be used to guide your assessment.

Table 2.1. Thinking about Mathematics lesson plans

	Advanced	Fair	Need to improve
Teaching method (Activity 1)	Can make a lesson plan which can give primary student teachers opportunities of thinking by themselves, share ideas with others, and find the formula of triangle area by themselves.	Can make a lesson plan in which teacher does not teach the solution/formula before student teachers' thinking (at least it is important)	The teaching method applied in the lesson plan is similar to the current teaching, which is teacher-centred (teacher teaches the solution/formula before student teachers' thinking).



Possible student teacher responses

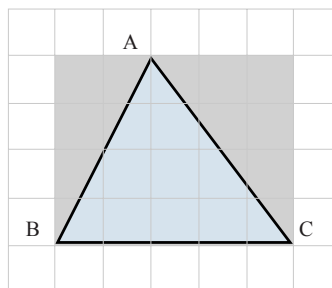
The following is a sample lesson activity. Keep in mind that there is no single answer for the homework. The given sample is just one of the ways. If student teachers come up with better ways, please appreciate these.

- The teacher asks students to recall their understanding on area of rectangle and area of parallelogram, and confirm their prior knowledge.
- The teacher gives the question and asks students to think by drawing pictures.
- Students think individually.
- The teacher asks students to share their ideas in small groups.
- Students make presentation on their idea. Students draw pictures and mathematical sentences on black/ white board, while explaining to the whole class.

Student A

First, I divided the given triangle into 2 small right triangles by drawing a line from A to make a right angle to the base (B-C). Then, I doubled the both right triangles to make 2 rectangles respectively. Area of the given triangle would be half of the both rectangles.

Figure 2.5. Finding the area of a triangle



Therefore, the mathematical sentence would be;

$$((4\text{cm} \times 2\text{cm}) \div 2) + ((4\text{ cm} \times 3\text{ cm}) \div 2) = 10\text{ cm}^2$$

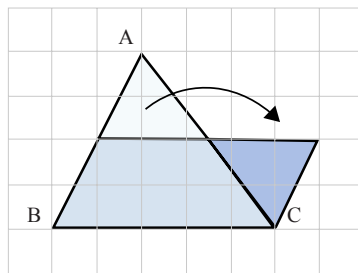
Other students may present that the area of the given triangle would be half of the both rectangles, which means the half of the big rectangle. So the mathematical sentence can be $((5\text{ cm} \times 4\text{ cm}) \div 2) = 10\text{ cm}^2$.

Student B

I cut the top triangle and moved to the right side to make a parallelogram. The area of parallelogram can be found by base x height.

Therefore, the mathematical sentence would be:

Figure 2.6. Finding the area of parallelogram (1)



$$5\text{cm} \times 2\text{cm} = 10\text{cm}^2$$

Student C

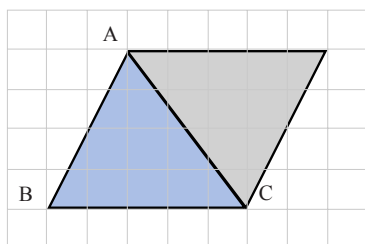
I doubled the given triangle to make a parallelogram.

The area of parallelogram can be found by base x height.

Therefore, the mathematical sentence would be:

$$(5\text{cm} \times 4\text{cm}) \div 2 = 10\text{cm}^2$$

Figure 2.7. Finding the area of parallelogram (2)



The teacher asks students to check whether all answers are the same.

The teacher asks students to analyse the mathematical sentences presented by students by asking some questions. The teacher asks student B how he/she found 2 cm and what 2 cm means. student teachers can find out that $2 \text{ cm} = 4 \text{ cm} \div 2$. Therefore, $5 \text{ cm} \times 2 \text{ cm} = 5 \text{ cm} \times (4 \text{ cm} \div 2) = (5 \text{ cm} \times 4 \text{ cm}) \div 2$. Then, teacher confirms with students that the mathematical sentences from all of solutions can be the same; $(5 \text{ cm} \times 4 \text{ cm}) \div 2$.

The teacher gives students other triangles to check whether those three ways can be applied. The teacher writes down the mathematical sentences which students found from those triangles and confirms that '(Base x Height) ÷ 2' is derived from all of the mathematical sentences found by students. Therefore, the formula of triangle area is Base x Height ÷ 2.

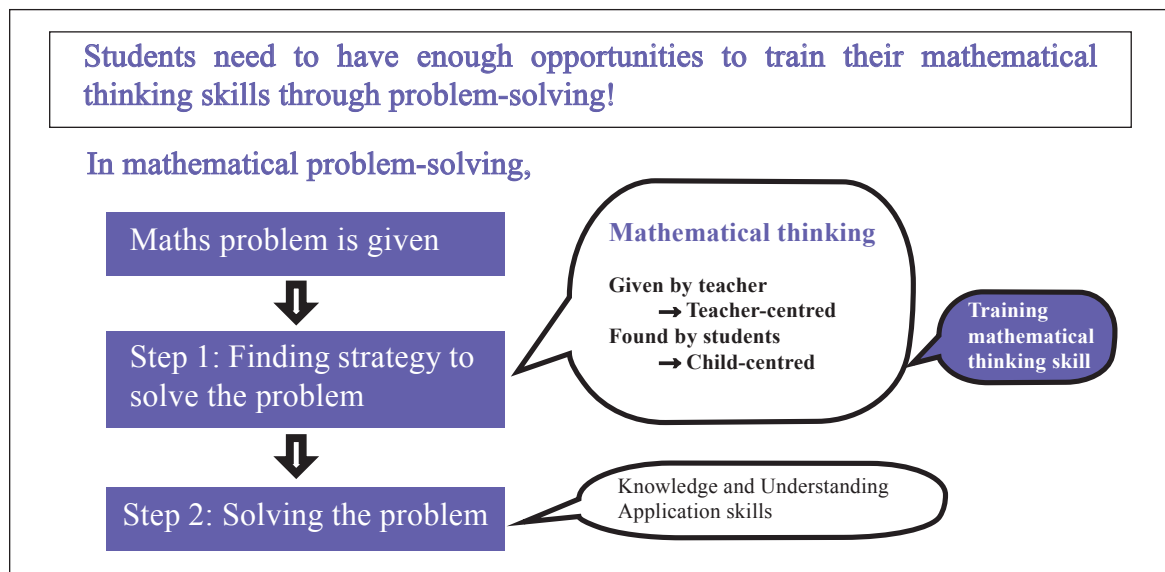
The teacher confirms the understanding of the students and closes the lesson.



Learning activity 3: Explanation of the new approach (10 minutes)

1. Explain the new approach to teaching Mathematics in primary schools based on the presentation slides in the student text. The presentation slides are below.

Figure 2.8. Presentation slides



Example(1): addition with carrying (TCA)

32 Addition with number rearrangement

Practical Calculation

Ten	One
2	3
+	9
3	2

Steps taken in calculation
Write 23 + 9 vertically.

Ten	One
2	3
+	9
	2

Add the one digits.
3 + 9 = 12
12 = 1 Ten and 2

Ten	One
2	3
+	9
	2

Add the ten digits.
1 + 2 = 3

Ten	One
2	3
+	9
3	2

Exercise 2

1. Compute them.

A)

Ten	One
5	7
+5	9

B)

Ten	One
7	5
+1	4

C)

Ten	One
7	5
+	7

D)

Ten	One
4	2
+	8

E)

Ten	One
7	5
+	6

F)

Ten	One
4	6
+2	6

G)

Ten	One
8	7
+3	7

H)

Ten	One
6	9
+	9

Exercise 3

1. Compute them.

A)

Ten	One
3	9
+4	6

B)

Ten	One
2	9
+4	1

C)

Ten	One
5	6
+3	4

D)

Ten	One
6	2
+1	9

E)

Ten	One
1	7
+7	6

F)

Ten	One
2	3
+2	8

G)

Ten	One
5	7
+1	9

H)

Ten	One
4	4
+4	8

Students learn new topic.

Teacher explains how to do this addition.

Students do not really have chance to train their mathematical thinking skills.

Students practise addition with carrying here.

Students acquire understanding and skills on this topic.

Students should have enough chances to nurture their **mathematical thinking skills** in mathematics lessons.



Problem-solving oriented lesson

Figure 2.9. Presentation slides continued

The common format of a problem-solving oriented lesson has 4 stages as below.

1. **Posing key question (or main problem)**
2. **Solving individually or in groups**
3. **Sharing ideas in whole class (by presentation)**
4. **Confirmation problems and Summarizing.**

- At the beginning of lesson before posing a key question the stage of recalling, students prior knowledge is added in necessary cSES.

Example (2): addition with carrying

Step 1: Posing key question
This key question is directly related to the lesson objective of Lesson 1 (To do long hand addition without carrying within 100).

There are 37 girls and 28 boys in a class room. How many children are there in all?

Write a math sentence. $\square + \square = \square$

(E) Think about how to calculate using blocks.

Tens	Ones
3	7
2	8
5 15	

When we add all blocks in ones place, we have 15 ones. How many tens and ones does this represent?

There are 65 blocks in the ones place. What should we do?

(F) Think about how to calculate in vertical form.

3	7	+	2	8	=	□

Step 3: Sharing ideas in whole class
Students share and examine their ideas in whole class. Then, they confirm how to do this addition.

Step 2: Solving Individually
At this stage, each student think about how to do this addition individually.

Step 4: Confirmation question and summarizing
Students work out similar questions to confirm their understanding. Then, at the last part of the lesson, they summarise what they learned in this lesson.



Assessment

Ask a few student teachers to briefly summarise the new approach to teaching Mathematics in their own words.



Possible student teacher responses

Example responses include:

- Students are encouraged to think through problems on their own and find their own strategies to solve them.
- In the old way of teaching, students would solve the same type of problem again and again. The new way of teaching is problem-solving oriented.
- In the new way of teaching, students work individually or in groups to solve a problem, then present their ideas to the class.



Learning activity 4: Group discussion (10 minutes)

1. Student teachers will discuss this topic in their groups. They will focus on the changes of method, the advantages and possible difficulties of the primary Mathematics. Each group will take notes.
2. Student teachers will share the results of their discussion results with the class.



Assessment

Walk around and listen to the examples of the advantages and difficulties that student teachers identify in their groups. You will be able to assess whether student teachers understood the advantages of new teaching strategy. The table below may be used to guide your assessment.

Table 2.2. Advantages and difficulties of teaching the curriculum

	Advanced	Fair	Need to improve
Advantages of new teaching strategy (Activity 3)	Can identify that the new teaching strategy can improve student teachers' mathematical thinking skills and other skills such as communication skills.	Can identify at least that the new teaching strategy can improve student teachers' mathematical thinking skills.	Can't identify the advantages of new teaching strategy.



Possible student teacher responses

Examples of advantages:

- Students will build many skills through the problem-solving process, including creativity, communication and teamwork.
- Students will be more active and engaged because the lesson is student-centred.

Examples of difficulties:

- Teachers are used to the old way of teaching and learning.

- Students are not used to discussing and solving problems in groups.
- Students' prior knowledge may not be correct or sufficient.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcomes of this lesson:
 - Be aware of problems of the conventional teaching of Mathematics; and
 - Understand the new teaching strategy which improves children's mathematical thinking skills.
2. Ask student teachers to turn to their partner and explain why the approach to teaching Mathematics has changed.
3. Remind student teachers that they were introduced to the new approach to teaching Mathematics which they will practice and apply to their own teaching.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty recalling the previous lesson, you may remind them of the learning outcomes or ask guiding questions.

Learning activity 2: You may ask student teachers questions after their presentations to get them to reflect, and explain their thought process behind designing the activity in the way that they did.

Learning activity 3: If student teachers cannot easily identify and summarise the new approach, you may spend a few more minutes explaining the approach and then check for understanding again.

Learning activity 4: Student teachers may have difficulty identifying advantages of the new approach because they are not used to it. You may ask guiding questions, such as: 'What type of skills do you think students will gain through this approach?'

2.1.3. Polya's four phases for solving problems

Expected learning outcomes



By the end of the lesson, the student teacher will be able to:

- Explain Polya's four phases for solving problems; and
- Identify how teachers can promote the mathematical thinking of primary students by understanding Polya's four phases.



Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources.



Time: One period of 50 minutes



Learning strategies: Individual work, group discussions and presentation



Preparation needed: Ensure the resources below are available



Resources needed: Question sheet (included); flipchart paper; pens



Learning activity 1: Introduction (5 minutes)

The teacher educator asks the following question to student teachers and requests that they recall what they learned in the previous lessons: 'How can we improve children's mathematical thinking skills in the primary level?'



Assessment

Listen to student teachers' responses to assess to what extent they recall and understand the new approach to mathematical problem-solving.



Possible student teacher responses

Student teachers may vary, and should show an understanding of the new approach. Example responses are:

- Students should be encouraged to apply their previous knowledge and use creativity to solve problems.
- There can be many different ways to solve a single problem, and students should be encouraged to find their own solution by applying problem-solving skills.



Learning activity 2: Individual work, group discussion & presentation (20 minutes)

1. Student teachers should read ‘Polya’s four phases’ individually. Then they should discuss the following questions in groups:
 - What are ‘Polya’s four phases’ for problem-solving? What does each phase entail?
 - How can teachers help students in each phase? (What are the possible questions which teachers can ask students to help their thinking/learning in each phase?)



Assessment

Request that a representative of each of the groups make a presentation of their discussion.



Possible student teacher responses

Polya’s four phases and example descriptions of each:

1. Understanding the problem: Understand the meaning of the problem and the conditions.
2. Devising a plan: Students use their knowledge and experiences they have had in the past to develop a strategy to solve the problem.
3. Carrying out the plan: Carry out the problem-solving process, checking to make sure each step is correct.

4. Looking back: Students should re-examine their solution and the steps they took.

In Phase 1, the following questions can be asked to help students:

- What are the given conditions?
- What is to be found as the answer?

In Phase 2, the following questions can be asked to help students:

- Have we solved similar problems before?
- Can we use the way which was used for the similar problems to solve this one?
- Can we use the result of the similar problems for this problem?
- Can we draw figures to explore the given problem?
- Can we rephrase the given problem?
- Can we use any definitions?
- Have we considered all the given conditions?

In Phase 3, teachers should check if students are carrying out their plan correctly. If not, teachers should help students come back on the right track. For example, students make mistakes in calculation even their plan is fine.

In Phase 4, the following questions can be asked to help students:

- Do you get a result which we are asked to find?
- Can we check the result by applying it to the given case?
- Are there any other ways to find the answer?
- Can we apply the way we used for this problem to other problems?



Learning activity 3: Group discussion and presentation (20 minutes)

1. Refer student teachers to the homework problem they did after the first lesson in this unit. Student teachers will discuss:
 - How can Polya's four phases be applied to helping students think through solutions for finding the area of the triangle?

- What questions and instructions can be asked of students to help their thinking and learning in each phase without providing answers?
2. Ask group representatives to make presentation on the group's discussion.



Assessment

Listen to the group representatives' presentations to assess how well they understand and apply Polya's four phases to the activity they created previously. Although this assessment is not for marking purposes, you may assess whether student teachers can come up with relevant questions to ask students. The table below provides some guidance.

Table 2.3. Rubric for assessment

	Advanced	Fair	Need to improve
Questioning and instruction skill	Can identify several questions/instructions for all of four phases which are effective to support student teachers' thinking/ learning.	Can identify at least one question/ instruction for all of four phases which are effective to support student teachers' thinking/ learning.	Cannot identify possible questions/instructions well.



Possible student teacher responses

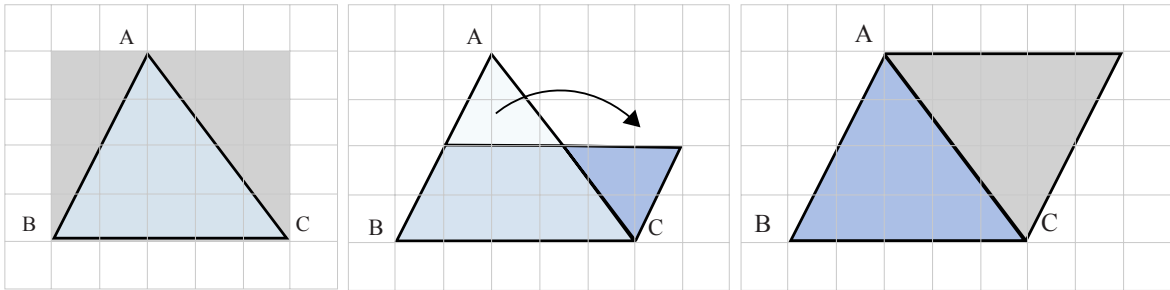
Student teacher responses may vary. They should give specific ideas and instructions for applying Polya's four phases to the activity. Example responses are given below.

In Phase 1, the following points should be checked:

- What are given conditions? → base = 5 cm, height = 4 cm
- What is to be found as the answer? → area of given triangle

In Phase 2, the following points should be checked:

- Did we solve similar problems before? → area of rectangle, area of parallelogram
- Can we use the way which was used for the similar problems? → area of rectangle = base × height, area of parallelogram = base × height can be used
- Can we draw figures to explore the given problem? → several figures can be drawn

Figure 2.10. Finding the area of a shape

In Phase 3, teachers should check if student teachers are carrying out their plan correctly. If not, teachers should help student teachers come back on the right track.

In Phase 4, the following points should be checked:

- Is the result what we are asked to find? → the result have to show the area of triangle
- Are there any other ways to find the answer? → let student teachers try to find other ways
- Can we apply the way we used for this problem to find the areas of other triangles? → Let student teachers think about the areas of other triangles. Then, let them find the formula of the area of a triangle.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Identify Polya's four phases for solving problems; and
 - Identify how teachers can promote the mathematical thinking of primary students by understanding Polya's four phases.
2. Ask student teachers to turn to their partner and explain Polya's four phases.
3. Remind student teachers that they identified Polya's four phases and applied them to a teaching and learning activity. They should consider the four phases throughout this course as they design Mathematics activities.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty describing the new approach to Mathematics, you may ask guiding questions. Briefly review the new approach if needed.

Learning activity 2: If student teachers have difficulty identifying and describing the four phases, you may explain or invite a few volunteers to identify and describe the four phases for the class.

Learning activity 3: You may ask additional questions to prompt student teachers to reflect further. For example: 'What challenges do you think you'll encounter in applying Polya's four stages to this activity? How would you overcome these?'

2.1.4. Introduction to heuristics

Expected learning outcomes



By the end of the lesson, the student teachers will be able to:

- Understand the concept of heuristics



Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources.



Time: One period of 50 minutes



Learning strategies: Individual work; class plenary discussion; explanation



Preparation needed: Ensure the resources below are available



Resources needed: A piece of paper without right-angles for each student teacher; flipchart paper; pens



Learning activity 1: Introducing the lesson (5 minutes)

To introduce this next lesson ask student teachers to recall what they learned in the previous lesson: 'How can understanding Polya's four phases help teachers to support primary students to promote their mathematical thinking skill?'



Assessment

Select a few student teachers and listen as they share their ideas to assess whether they understand the application of Polya's four phases.



Possible student teacher responses

Student teachers may vary, and should show an understanding of Polya's four phases.

Example responses are:

- The four phases provide a clear approach to problem-solving that can be understood and used by primary student teachers.
- The four phases help students to identify and apply their thinking skills to the problem-solving process, instead of solving problems in a rote or repetitive way.



Learning activity 2: Explanation of heuristics (10 minutes)

Explain how heuristics can help primary students improve their mathematical thinking skills. Outline the main ideas in the textbook.



Assessment

Ask a few student teachers to briefly summarise heuristics in their own words.



Possible student teacher responses

Example responses include:

- Heuristics are the strategies used to solve a problem.
- There are many different strategies that can be considered heuristics, including:
 - a. Back to definition;
 - b. Drawing a figure;
 - c. Finding a pattern; and
 - d. Solving a simpler analogous problem.
- Heuristics are mainly used in the second phase of Polya's four phases (devising a plan).



Learning activity 3: Solving problems (25 minutes)

1. Ask student teachers to solve the four example problems in their texts. Student teachers should think about and solve these four problems individually.
2. Explain that student teachers should not simply find the answers but need to think of heuristics to solve each question. Student teachers need to write down the ways they solved the questions.
3. Distribute one piece of paper per student. The paper should not have any right angle on its surface and should not have any lines on it.
4. Appoint student teachers to make presentations one by one.
5. Explain the heuristics after each student's presentation if necessary.



Assessment

Assess each student teacher's presentation by using the following rubric.

Table 2.4. Rubric for assessment

	Advanced	Fair	Need to improve
Explanation skill	Can explain heuristics step by step by drawing and writing on board clearly	Can explain heuristics verbally but can not draw or write on board clearly	Cannot explain or draw/write heuristics well.
Use of heuristics	Can use heuristics appropriately and also showed different ways of solving questions by using other heuristics.	Can use heuristics appropriately.	Cannot use heuristics appropriately.



Possible student teacher responses

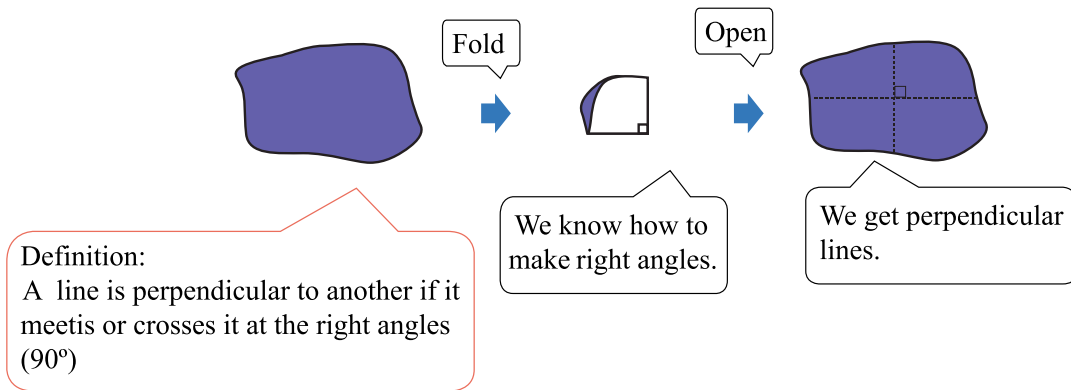
Example 1:

In the student teacher's presentation, ask the student teacher 'why do you fold the paper like this to make perpendicular lines?' and should ask student teachers the definition of a perpendicular line. Student teachers need to have the definition as this is crucial to the heuristic used here: *A line is perpendicular to another if it meets or crosses it at right angles (90 degrees).*

Figure 2.11. Perpendicular lines

Example(1): Back to definition

Q. Make perpendicular Lines by folding paper



Example 2:

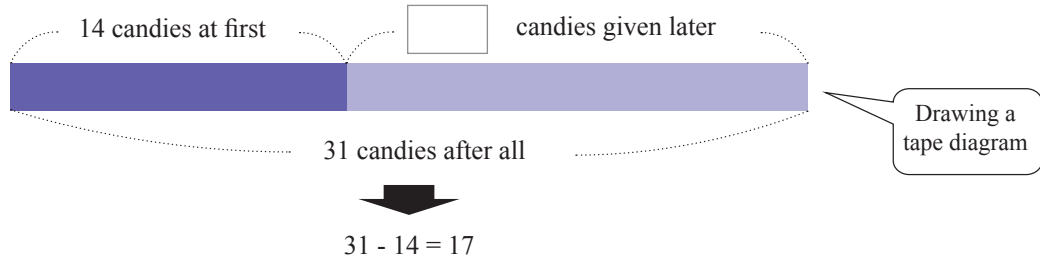
In this example, student teachers can draw various types of figures to solve the problem. Some student teachers may draw pictures of candies, or draw semi-concrete objects, or draw tape diagram, and so on. Ask student teachers who have different ideas to make presentation. If there are some student teachers who drew a tape diagram, ask the reasons why they drew a tape diagram. Student teachers need to notice that tape diagram can be applied even if the number used in the mathematical problem is large. It can help student teachers interpret the word problem into a mathematical sentence.

If no-one drew a tape diagram, you should demonstrate. First, draw a blue line which stands for 14 candies, then draw a red line which stands for additional candies mother gave her, and write down the total number of candies under the whole tape. Then, it is easy to come up with the mathematical sentence $31 - 14 = 17$.

Figure 2.12. Drawing a figure

Example (2): Visualize by drawing

Q. Aye Aye had 14 candies at first. Then, her mother gave her some more candies later on and she has 31 candies now. How many candies did her mother give her?



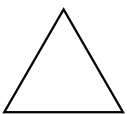
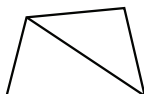
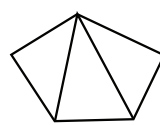
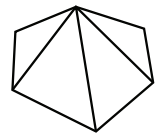
Example 3:

In middle school, students need to identify and know the generalised formula ($180^\circ \times \text{Number of sides} - 2$). But it is not necessary to generalise here as this example is for primary level. In the student teachers' presentation, ask: 'Why does the sum of the interior angles increase by 180 degrees one by one according to the number of side?' This question is to help student teachers to find the pattern. Student teachers need to find out that it is because the number of triangles increases one by one according to the number of sides.

Figure 2.13. The sum of interior angles

Example (3): Finding a pattern

Q. Find the sum of interior angles of hexagon.

Triangle	Quadrilateral	Pentagon	Hexagon
			
Sum of interior angles = 180°	Quadrilateral = 2 triangles Sum of interior angles of a quadrilateral is $180^\circ \times 2 = 360^\circ$	Pentagon = 3 triangles The sum of interior angles of a penlagon is $180^\circ \times 3 = 540^\circ$	Hexagon = 4 triangles The sum of interior angles of a hexagon is $180^\circ \times 4 = 720^\circ$

Example 4: Using a simpler analogous problem

The heuristic used in this example may be new for most of EC student teachers as it is not used in the old primary curriculum. If student teachers use calculation by vertical form, teacher can ask student teachers how to solve this without using the calculation by vertical form. Student teachers need to understand that 0.1 is considered as one unit in this mathematical problem. Therefore, it is a key that 1.2 is regarded as 12 of 0.1. In other mathematical problem, one unit can be another value, for example, 0.01 is the one unit of 1.23 (123 of 0.01).



Learning activity 2: Heuristics assignment (5 minutes)

1. Show student teachers that there are another four example problems in their texts. They should complete these individually for homework.
2. Remind student teachers that they should not only solve the problems, but identify their problem-solving approaches (heuristics) as well.



Assessment

You will assess this assignment when student teachers share their answers and heuristics during the following class.



Possible student teacher responses

Possible student teacher responses are included in the following lesson plan where student teachers share their answers and the heuristics they used to solve the problems.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand the concept of heuristics.
2. Ask student teachers to turn to their partner and explain the concept of heuristics.
3. Remind student teachers that they applied the concept of heuristics to their problem-solving approaches. They will do a similar activity for homework.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty describing Polya's four phases, you may ask guiding questions. Briefly review the four phases if needed.

Learning activity 2: If student teachers have difficulty identifying and describing heuristics, you may explain or invite a few volunteers to identify and describe heuristics for the class.

Learning activity 3: You may allow student teachers to discuss in pairs if they are struggling. If they cannot explain the heuristics of their approach, you may help them do so.

Learning activity 4: You may allow student teachers to work on the assignment in pairs if they do not understand heuristics well yet.

2.1.5. Heuristics for problem-solving

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain how to use ‘Back to Definition’, ‘Visualise by Drawing’, ‘Finding a Pattern’, and ‘Using a Simpler Analogous Problem’ heuristics in problem-solving.



Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources.



Time: One period of 50 minutes



Learning strategies: Pair work; presentations



Preparation needed: Check answers from the homework assigned in the previous lesson



Resources needed: Homework and answers assigned in the previous lesson



Learning activity 1: Introducing the lesson (5 minutes)

Introduce the lesson by asking the questions: 'What is a heuristic?' and 'Why is it important for primary school students to learn heuristics?'



Assessment

Listen to student teachers' responses to assess to what extent they recall and understand the new approach to mathematical problem-solving.



Possible student teacher responses

Student teachers may vary, and should show an understanding of the new approach. Example responses are:

- Heuristics is all the strategies we use in problem-solving.
- By learning heuristics, primary student teachers will be able to improve their mathematical thinking skill.



Learning activity 2: Pair work (10 minutes)

Student teachers share the heuristics they used in the homework in pairs.



Assessment

Walk around and listen to pairs discuss in order to assess their understanding of heuristics. Observe how well they describe their problem-solving processes.



Possible student teacher responses

There are no correct/incorrect answers, and students may describe their problem-solving processes in their own language. They may identify that they used the following specific heuristics: Back to Definition, Visualise by Drawing and Finding a Pattern.



Learning activity 3: Presentation of heuristics (30 minutes)

1. Appoint a sample of student teachers to make presentation by using the board to explain the heuristics they used for homework.
2. Ask if there are other heuristics used for each question. If there are, then allow the student teachers who used other heuristics to also make a presentation.
3. Summarise the possible heuristics for each problem.



Assessment

Listen to presentations. You can assess student teachers who made presentations by using the following rubric.

Table 2.5. Rubric for assessment

	Advanced	Fair	Need to improve
Explanation skill	Can explain heuristics step by step by drawing and writing on board clearly	Can explain heuristic verbally but can not draw or write on board clearly	Cannot explain or draw/write heuristics well.
Use of heuristics	Can use heuristics appropriately and also showed different ways of solving questions by using other heuristics.	Can use heuristics appropriately.	Cannot use heuristics appropriately.



Possible student teacher responses

Exercise 1

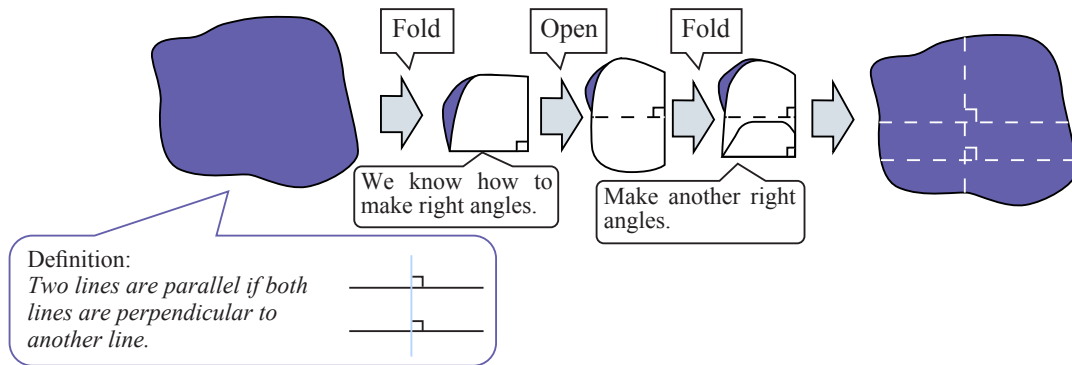
In student teachers' presentations, ask: 'Why do you fold a paper like this to make parallel lines?' and ask student teachers the definition of the parallel line. Additionally, provide the following explanation:

In the old primary curriculum, 'parallel line' was defined as that 'two lines are parallel, if the distance between them is always the same.' But in the new primary curriculum, it is defined as 'two lines are parallel, if both lines are perpendicular to another line'. This is because we need to use a perpendicular line to find the distance between parallel lines.

If no one can find the answer, explain how to make parallel lines by demonstrating how to hold a paper. First, fold a paper to make a right angle. Then, hold the paper again by making a right angle on the same line as the first right angle.

Figure 2.14. Parallel lines

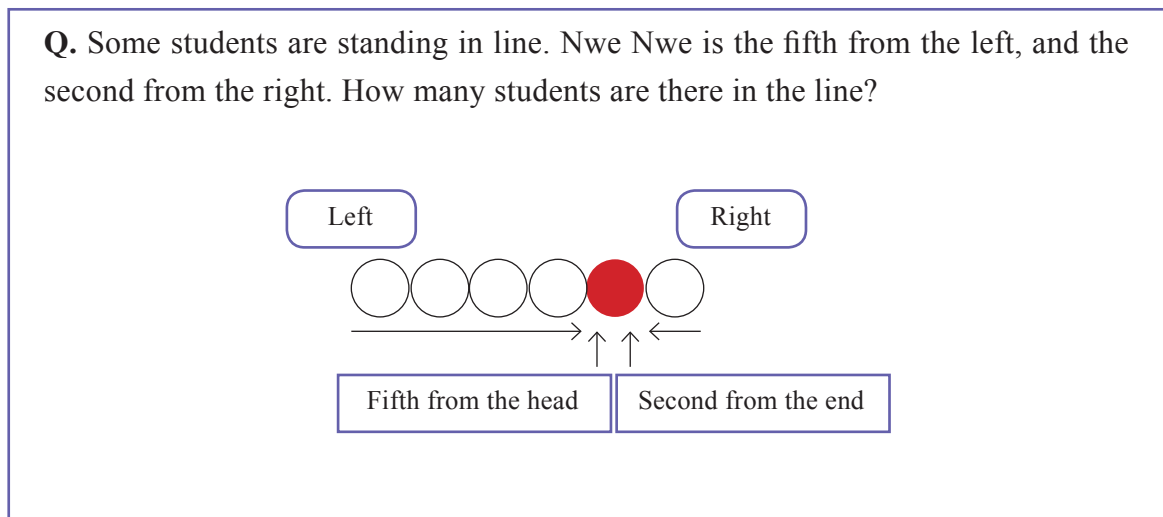
Q. Make parallel lines by folding paper. Recall the definition first.



Exercise 2

Student teachers need to explain by drawing a figure on the board. First, draw a coloured circle that indicates Nwe Nwe. Then, draw four circles in the left of her so that she comes to the fifth from the left, and one more circle in her right so that she comes to the second from the right. Then, count the number of circles, which is 6.

Figure 2.15. Drawing a figure




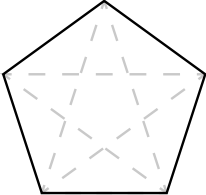
Exercise 3

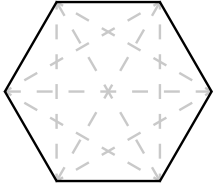
Student teachers need to draw pictures of a rectangular, a pentagon and a hexagon. Then, you should draw diagonals from each angle. It will be clear to understand that ‘when all angles make diagonals, there are double the number of diagonals’ if they use different colours when drawing same diagonals.

Figure 2.16. Find a pattern

Q. How many diagonals are there in a hexagon?







Each angle can make 1 diagonal
 Rectangular has 4 angles.
 When all angles make diagonals,
 there are double number of
 diagonals.
 $1 \times 4 \div 2 = 2$

Each angle can make 2 diagonal
 Rectangular has 5 angles.
 When all angles make diagonals,
 there are double number of
 diagonals.
 $2 \times 5 \div 2 = 5$

Each angle can make 3 diagonal
 Rectangular has 6 angles.
 When all angles make diagonals,
 there are double number of
 diagonals.
 $3 \times 6 \div 2 = 9$

Exercise 4

In this exercise, student teachers need to understand each step. (1) Firstly, we know $2 \times 3 = 6$. (2) 20 is 10 times 2. So, the answer would be 10 times $6 = 60$. (3) 30 is 10 times 3. So, the answer would be 10 times $60 = 600$. (4) 200 is 10 times 20. So, the answer would be 10 times $600 = 6,000$. (5) 300 is 10 times 30. So, the answer would be 10 times of $6,000 = 60,000$.

Figure 2.17. A simpler analogous problem

Q. Calculate 200×300 (Think how to teach a student who knows multiplication table up to 10 and knows how to make a group of 10.

$2 \times 3 = 6$	
10 times	
$20 \times 3 = 60$	
	10 times
$20 \times 30 = 600$	
10 times	
$200 \times 30 = 6000$	
	10 times
$200 \times 300 = 60000$	

1) First, we know $2 \times 3 = 6$

2) 20 is 10 times of 2. So, the answer would be 10 times of $6 = 60$

3) 30 is 10 times of 3. So, the answer would be 10 times of $60 = 600$

4) 200 is 10 times of 20. So, the answer would be 10 times of $600 = 6,000$.

5) 300 is 10 times of 30. So, the answer would be 10 times of $6,000 = 60,000$.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Explain how to use 'Back to Definition', 'Visualise by Drawing', 'Finding a Pattern', and 'Using a Simpler Analogous Problem' heuristics in problem-solving
2. Ask student teachers to turn to their partner and share an example of an interesting heuristic they saw during class today.
3. Remind student teachers that they identified different heuristics that can be used in problem-solving processes. Remind them that there are no single heuristics for each mathematical problem but there are some heuristics can be used for solving mathematical problems.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty describing heuristics, you may ask guiding questions. Ask a few volunteers to give specific examples of heuristics that they considered in the previous lesson.

Learning activity 2: If student teachers have difficulty explaining the heuristics they used, encourage them to simply explain their problem-solving process in their own words.

Learning activity 3: You may ask additional questions to prompt student teachers to reflect further. For example: 'What challenges do you think primary students would have in following this process? How would you address these?'



Review questions: Possible student teacher responses

Question 1: How is the new approach to teaching Mathematics different from the current approach being used in our schools in Myanmar?

Answer: The current approach encourages rote learning, and students simply memorise processes. The new approach encourages creativity in problem-solving.

Question 2: What are Polya's four phases for solving problems and why are they important to Mathematics?

Answer: The four phases are understanding, devising a plan, carrying out the plan, and looking back. They are important because they can guide students through problem-solving processes and improve their approach to solving Mathematics problems.

Question 3: Explain the meaning of 'heuristics' in your own words.

Answer: Heuristics describes strategies that can be used to solve Mathematics problems.

2.2. Research in Mathematics

Two important areas of research that have an impact on Mathematics education are covered in this sub-unit. Firstly, we focus on the common errors that primary students often make (in this case addition and subtraction) and misconceptions they develop usually because they have misunderstood concepts or processes. We look at the Grade 2 textbook to see how these are addressed.

Secondly, we investigate research into the cognitive development of children and how these stages have an impact on the development of their understanding of mathematical concepts and operations. We also investigate some theory in this area of cognitive development and mathematical thinking and its implications for teaching practice.

2.2.1.

Common errors and misconceptions in addition and subtraction

Expected learning outcomes



By the end of the lesson, the student teachers will be able to:

- Describe some of the common errors and misconceptions that students have in adding and subtracting in columns;
- Recognise examples of how to avoid the development of these errors and misconceptions is addressed in the Grade 2 Mathematics textbooks; and
- Identify the errors or misconceptions that a student has made and help him/her correct them.



Competencies gained: A1.2. Demonstrate understanding of how different teaching methods can meet students' individual learning needs; and A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources.



Time: One period of 50 minutes



Learning strategies: Discussion and reflection



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; pens; Grade 2 primary Mathematics textbooks (at least one for each small group)



Learning activity 1: Introducing the lesson (5 minutes)

1. Provide an overview of the objectives and what will be covered in this lesson.
2. Form the class of student teachers into groups of four.
3. Supply each group with flipchart paper and ask them to capture their ideas.
4. Ask them to reflect on what aspects of Mathematics at school they found difficult to understand. Suggest that they firstly make a personal list and then discuss these areas of difficulty in their group. They should answer the questions: 'Did you finally understand? How did this happen? Could you teach this now?'



Assessment

Walk around to observe groups. Make sure they understand the questions and are participating.



Possible student teacher responses

Responses could be a range of areas of Mathematics that students could find difficult to understand. Student teachers should answer the second set of questions as well.



Learning activity 2: Introduction to misconceptions (10 minutes)

1. Introduce the next part of the lesson where they will consider common errors made by primary students in adding as a result of misconceptions. Introduce the definitions of ‘error’, ‘mistake’ and ‘misconception’, or ask student teachers to explain the terms in their own words based on their reading.
2. Introduce the concepts of ‘place value’ and ‘number bonds’. Ask student teachers if they can remember how they learnt number bonds at school.



Assessment

Ask a few student teachers to define the following terms using their own words: errors, misconceptions, mistakes.



Possible student teacher responses

These are the definitions from the textbook. Student teachers may rephrase these in their own words:

- ‘*Errors*’ are pervasive (found everywhere) and are often repeated in different contexts.
- ‘*Misconceptions*’ is a term used to describe the alternative understandings that people may have about concepts. These understandings may be either incorrect or incomplete.
- ‘*Mistakes*’ are incorrect answers that have been obtained as a result of ‘slips’.



Learning activity 3: Reviewing the Mathematics textbooks (15 minutes)

1. Request student teachers in their groups to examine the lessons Addition 2 and Subtraction 2 in the Grade 2 Mathematics textbooks and to reflect on what methods and visual tools were used to teach these topics.
2. They should consider how the textbook is attempting to help students to understand place value and adding in columns of 1s and 10s and make points.

3. They should think about how they learnt these topics at school. Was it useful to help you understand? Do you find the methods indicated in the textbooks useful? Could you use these to teach a lesson on the topics?



Assessment

Walk around and listen to students' discussion of the Grade 2 textbook as they work in groups. Assess whether the groups answer the questions and reflect on the way addition and subtraction are taught in the textbook.



Possible student teacher responses

There are no incorrect/correct answers. Student teachers may make any reasonable observations about the Grade 2 text.



Learning activity 4: Common mistakes (15 minutes)

1. Briefly go through the examples given in the lecture notes of mistakes students make with subtraction.
2. Request student teachers in their groups, using the examples in subtraction and addition as a base, make a short worksheet on a flipchart of new examples that could be used to test common errors and misconceptions in addition and subtraction.



Assessment

Walk around and observe groups as they work. You may ask them to explain how the examples on their flipchart can be used to test common errors.



Possible student teacher responses

The groups may use any addition and subtraction problems to demonstrate common mistakes. For example, student teachers may include the problem $62 - 28$ on their worksheet.

This problem can be miscalculated:

$$\begin{array}{r}
 62 \\
 - 28 \\
 \hline
 46
 \end{array}$$

This problem can be used to test whether students can apply their understanding of place value to borrowing in subtraction. The problem demonstrates the importance of understanding place value and borrowing from a larger place value in subtraction.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Describe some of the common errors and misconceptions that students have in adding and subtracting in columns;
 - Understand how avoiding the development of these errors and misconceptions is addressed in the Grade 2 Mathematics textbooks; and
 - Identify the errors or misconceptions that a student has made and help them correct them.
2. Ask student teachers to turn to their partner and explain a common error that students make when adding or subtracting.
3. Remind student teachers that they identified some common errors and discussed the importance of developing a strong Mathematics background during primary grades, including topics such as place value and number bonds.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty reflecting and responding to the questions, provide an example from your own experience to guide them and make them feel more comfortable answering the question.

Learning activity 2: You may spend more time reviewing and discussing the content of the text if needed. You may ask questions that prompt further discussion of the terms, for example, What is the difference between an error and a mistake?

Learning activity 3: As you walk around the room, you may ask questions that encourage student teachers to consider the textbook content further. For example, If you were the textbook writer, is there anything you would have done differently? What could you add to this information to help your students understand and avoid misconceptions?

Learning activity 4: If student teachers do not understand, you may spend a few minutes reviewing the common mistakes in the textbook. You may ask a group with a well-developed worksheet to explain their worksheet to the class.



Review questions: Possible student teacher responses

Question 1: Why is it important that teachers can distinguish between the mathematical errors and mistakes?

Answer: Because, then, the teacher will know when a student's answer is wrong due to error or mistake. If error, the teacher can go back to Math concepts and if mistake, search for miscalculations in the student's problem-solving.

Question 2: How can students' misconceptions be used to develop their logical thinking skill?

Example answer: By comparing their misconceptions with correct Math conceptions, guide the students to realise their shortcomings and deviations.

2.3. Inclusive approach to teaching Mathematics

The topic of inclusive education will be covered in Educational Studies. However, in the one lesson for this sub-unit, we look at the broad ideas of inclusive education and then examine specifically what Mathematics teachers could do to ensure inclusive Mathematics classes and the challenges these teachers are likely to face.

2.3.1. The challenges for Mathematics teachers in developing an inclusive classroom

Expected learning outcomes



By the end of the lesson, the student teachers will be able to:

- Explain how all children are different and have potential to be mathematician;
- Describe the advantages of developing an inclusive approach to teaching and learning in schools;
- Demonstrate how to promote and develop inclusive approaches in schools and in the classroom; and
- Prepare as Mathematics teachers for the challenges in promoting inclusive education in Myanmar.



Competencies gained: A1.2. Demonstrate understanding of how different teaching methods can meet students' individual learning needs; and A5.2. Demonstrate understanding of how to vary delivery of subject content to meet students' learning needs and the learning context.



Time: One period of 50 minutes



Learning strategies: Individual reading; group discussion; presentation



Preparation needed: Ensure copies of the report 'Towards inclusive education for children with disabilities: a guideline' are available: <http://unesdoc.unesco.org/images/0019/001924/192480e.pdf>



Resources needed: Flipchart paper; pens, Grade 2 Mathematics textbooks; report referenced above



Learning activity 1: Introducing the lesson (5 minutes)

1. Explain that over the next lesson, you will explore the potential of all primary school students, why schools need to seek to include all students, and how teachers can develop approaches to include all students of different abilities in their classes.
2. Student teachers should have read this section of the book for homework. Ask a few students to summarise what they read.
3. Explain that the lesson will take the format of a seminar discussion. Student teachers will have to read and study the notes provided and the UNESCO report from which the notes are extracted.
4. After short group discussions about the key points highlighted and of the possibilities for inclusive education in Myanmar, groups will discuss the particular challenges that face Mathematics teachers.



Assessment

Ask a few student teachers to summarise what they read for homework to assess their understanding of the content.



Possible student teacher responses

Students may describe any of the content in Lesson 2.3.1 of the textbook. These are examples:

- All children are different. Some children have disabilities or difficulties at home that affect their participation in and understanding of learning activities.
- All children can learn, but some children learn in different ways and need special considerations.
- It is challenging to address the learning needs of diverse students, but there are strategies that teachers can use to manage this, including using group work and communicating clearly.
- Teachers should collaborate, support and learn from each other.



Learning activity 2: Group work (20 minutes)

1. Request that student teachers study the notes included in their text on 2. Challenges for Teachers and then work in groups to: Discuss how best to promote inclusive Mathematics education in Myanmar and what the challenges and constraints are that will make this difficult. Return to the Grade 2 Mathematics textbook and the lessons on addition and subtraction we examined in Lesson 2.2.1. What special approaches should teachers take to ensure that all students are able to understand addition and subtraction when teaching these?
2. They should develop a short presentation on the key points they discussed to the class group after their discussion.



Assessment

Walk around and listen to students' discussion as they work in groups. Assess whether the groups answer understand inclusive education and can apply this concept to the addition and subtraction content of the textbook.



Possible student teacher responses

These are example ideas for promoting inclusive education in Myanmar:

- Use simple language in the classroom and communicate clearly;
- Learn local sign language for deaf students;
- Invite students with special needs to sit near the chalkboard; and
- Use group work in classrooms.

These are example ideas of the challenges that will make this difficult:

- Classrooms are crowded and there is limited space for group work; and
- There are limited resources for learning sign language.

These are examples of approaches that can be taken to make addition and subtraction activities inclusive:

- Have students solve problems in groups so that they learn from each other;
- Show how to solve problems in different ways, such as using diagrams or a place value chart; and
- Do a variety of activities, such as worksheets, story problems, and games.



Learning activity 3: Group presentations (20 minutes)

1. Invite presentations from groups.
2. Facilitate a discussion on the advantages of an inclusive approach to teaching Mathematics and the constraints and challenges schools and teachers are likely to face.



Assessment

Listen to the presentations to assess the learning outcomes of this lesson.



Possible student teacher responses

Student teachers should present the ideas they discussed in Learning activity 2. Their ideas and responses will be similar to those examples outlined above.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand that all children are different and have potential;
 - See the advantages of developing an inclusive approach to teaching and learning in schools;
 - Understand how to promote and develop inclusive approaches in schools and in the classroom; and
 - Prepare as Mathematics teachers for the challenges in promoting inclusive education in Myanmar.
2. Ask student teachers to turn to their partner and explain why it is important to use an inclusive approach in a Mathematics classroom.
3. Remind student teachers that they explored why it is important to create an inclusive classroom environment, identified the challenges to doing this in Myanmar, and brainstormed some ways to address these challenges in lessons on addition and subtraction.
4. Request that student teachers read the full report, 'Towards inclusive education for children with disabilities: a guideline' online for homework:
<http://unesdoc.unesco.org/images/0019/001924/192480e.pdf>



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty describing the content of the lesson they read for a homework, spend a few minutes reviewing and explaining it.

Learning activity 2: If student teachers have difficulty applying these concepts to addition and subtraction activities, you may refer them to specific guidelines in the textbook, and ask them how they would apply those to lesson activities on addition and subtraction.

Learning activity 3: If student teachers have difficulty presenting their ideas, you may ask guiding questions to facilitate the discussion.



Review questions: Possible student teacher responses

Question 1: What are some features of an inclusive classroom environment?

Answer: The layout of the classroom and seating assignments support any students with special needs. The activities used in the classroom are varied, and the teacher uses clear explanations.

Question 2: What are some concrete steps you can take to create an inclusive classroom environment?

Answer: Learn about the special needs of my students by communicating with their families or other teachers. Spend time supporting students with special needs one-on-one.

Unit Summary



Key messages

2.1. of this unit explored strategies for problem-solving that can be used to build primary students' Mathematics skills, such as Polya's four phases for problem-solving and heuristics. Student teachers applied these problem-solving processes in the classroom, and discussed the importance of problem-based learning in Mathematics.

2.2. explored research in Mathematics, including some common errors and misconceptions and how these can be addressed.

2.3. examined the characteristics of an inclusive learning environment, and discussed the challenges and benefits of creating such an environment in Myanmar.



Unit reflection

What were the key lessons you learnt in this unit?

What are some ideas you have for shifting to the new, problem-based approach to teaching Mathematics?

What are some ideas you have for creating an inclusive learning environment?

What gaps have you identified in your knowledge of Mathematics for primary school?
How are you planning to address these gaps?

What are some tools that you have learned about in this unit? How will these be useful to you as a teacher?



Further reading

2.1.

Larson, L. C. (1983). *Problem-Solving through Problems*. New York: Springer-Verlag Publishers.

Polya, G. (1957). *“How to Solve it”*. Garden City, New York: Doubleday.

Tsukahara, S. (1994). *Heuristics in Secondary Mathematics: Introduction to Heuristics. (In Japanese)*. Japan: Toyokan publishing.

Posamentier, A. S., & Krulik, S. (1998). *Problem-solving strategies for efficient and elegant solutions: A resource for the Mathematics teacher*. Thousand Oaks, Calif.: Corwin Press.

2.2.

Sapire, I. Shalem, Y. and Reed, Y. (2015). *Assessment for Learning: Using learners’ test data for professional development*. Johannesburg: University of Witwatersrand and Saide.

2.3.

UNESCO. (2001). *Understanding and Responding to Children’s Needs in Inclusive Classrooms- A Guide for Teachers*.

UNESCO. (2009). *Towards inclusive education for children with disabilities: a guideline*. Retrieved from <https://unesdoc.unesco.org/ark:/48223/pf0000192480>

Unit 3

Understanding Mathematics

This unit comprises of four sub-units. In the first sub-unit on numbers, we explore number systems and the history of number systems. We learn to work in a base 5 system in order to understand some of the difficulties young students might have in understanding operations in our base 10 system. Then there four lessons on teaching fractions, during which student teachers will identify some common misconceptions, and identify ways to address them. In the second sub-unit, we will explore measurement, including how to introduce concepts to students at different levels. The third sub-unit examines problem-solving methods and problem types appropriate for students in different grade levels. During the fourth sub-unit, student teachers will explore some scenarios and identify the importance of using real-life scenarios in a Mathematics classroom to give students practice applying their Mathematics skills to real-world problems.

Expected learning outcomes



By the end of this unit, student teachers will be able to:

- Identify different aspects when primary students learn about numbers;
- Explain the central aspects of a positional number system, in other words, a number system like ours which is based on place value;
- Explain the basic properties of addition, namely:
 - commutative property;
 - associative property;
 - distributive property;
 - Zero as the additive identity;
- Identify the notion of grouping and regrouping through working with a base 5 positional number system;
- Explain how addition and subtraction are inverse operations;
- Explain the basic properties of multiplication, namely:
 - commutative property;
 - associative property;
 - distributive property;
 - 1 as the multiplicative identity;
 - multiplication and division are inverse operations;
- Compare different meaning of fractions;
- Work practically with different models for representing fractions;
- Use these models to help students to both identify fractions and to understand the concept of fractions;
- Identify common student misconceptions around part-whole relationships;
- Develop students' proportional thinking skills whilst teaching fraction concept;
- Use various models to find equivalent fractions;
- Identify different kinds of errors, in particular the differences between mistakes and misconceptions;
- Explain the benefits to both teaching and learning of working with students' misconceptions;
- Identify student misconceptions around fractions on a number line;
- Explain that there are an infinite number of fractions between any two fractions;



- Recognise the common student misconception that they can add numerators and denominators;
- Recognise that in order to add or subtract fractions, we make equivalent fractions with common denominators;
- Use paper folding to visually demonstrate the logic of making common denominators;
- Design follow up parts of lessons that work with pictorial and abstract ways to add and subtract fractions with different denominators;
- Multiply fractions and whole numbers;
- Multiply fractions with fractions;
- Simplify fractions by factorising and re-arranging the factors (using the commutative or associative properties) to form fractions equivalent to 1;
- Use 1 as the multiplicative identity to simplify fractions;
- Generalise towards a procedure for multiplying fractions from examining patterns in several calculations;
- Explain that when we make common denominators, we are really making equivalent fractions by multiplying by a fraction that is equivalent to 1;
- Describe informal and formal measurement systems that can be used to teach measurement concepts;
- Demonstrate practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students have are addressed;
- Explain how to teach standard units in the Myanmar context using metric, imperial and Myanmar units;
- Demonstrate practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students experience are addressed;
- Explain how to teach standard units in the Myanmar context using metric units;
- Explain what are the different measurement systems for weight;
- Demonstrate practical ideas to teach measurement of weight;
- Develop practical tools for primary students to establish standard weight, to measure and to compare;
- Propose ideas about how to convey the concept of density of materials;
- Explain the measurement systems that are used in Myanmar;



- Explain the approaches to teach the measurement of volume of liquids;
- Describe ideas of practical tools to make and mark out for measurement;
- Explain how best to teach volume of three-dimensional spaces;
- Identify in what contexts these Myanmar units of length and weight are used;
- Adapt a number of the work station lessons from the previous lessons and developed some others to be used to teach measurement with these units;
- Explain how to derive the area of parallelograms, rhombuses and trapezia from the area of rectangles;
- Teach Grade 5 students to find the area of parallelograms, rhombuses and trapezia from the area of rectangles;
- Explain the importance of linking topics and formulae when teaching Mathematics;
- Explain the value of visual images when teaching Mathematics;
- Teach the dynamic concept of angle as an amount of turning;
- Plan lessons on angle measurement that minimise common student misconceptions about angles;
- Name angles;
- Link dynamic and static notions of angles;
- Teach Grade 4 students how to use a protractor;
- Describe the importance of considering the language of Mathematics particularly when teaching early grade students;
- Explain the levels through which mathematical learning develops for young students through the early grades;
- Explain the problem types that are useful to help early grade students develop problem-solving skills;
- Describe the kinds of problem-solving questions that are useful to extend mathematical thinking for students to Grade 3;
- Categorise problem types and develop similar examples Grade 3 students;
- Explain the level Grade 3 students should be able to attain;
- Discuss the differences between problem-solving and routine calculations;
- Examine different processes for solving problems;
- Describe the kinds of problem-solving questions that are useful to extend mathematical thinking for students from Grade 4 to Grade 6;



- Categorise problem-types and develop similar examples for Grade 4 to Grade 6 students;
- Understand the level Grade 6 students should be able to attain;
- Use the skills with measurement of length, weight, volume, time and area and graphs to design a lesson around an agricultural or food project;
- Develop similar practical lessons to demonstrate how numeracy skills and mathematical thinking are used in real life situations;
- Use the skills with measurement of length, weight, volume, time and area to plan a class excursion;
- Develop similar lessons to demonstrate how numeracy skills and mathematical thinking is used in real life situations;
- Use skills with measurement, data and graphs to design a lesson around climate in Myanmar;
- Develop graphic posters as a teaching and learning resources for classroom display; and
- Develop similar practical lessons to demonstrate how numeracy skills and mathematical thinking are used in real life situations.

3.1. Numbers

This sub-unit explores numbers, including common misconceptions that students may have when they start learning Mathematics and how teachers can address these. The first three sections of this sub-unit introduce base 5, and apply operations to numbers in base 5. Student teachers will likely have misconceptions as they learn to work in this number system which will help them to understand the misconceptions that primary students have when they learn arithmetic in the base 10 system. The last five lessons in this sub-unit are on fractions. Student teachers will identify teaching aids and activities that can be used to demonstrate the concept of fractions, equivalent fractions, and operations on fractions. At the end of this sub-unit, they should be prepared to build primary students' understanding of numbers and arithmetic.

3.1.1.

Number systems: The history of numbers and learning about a different place value number systems

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Identify different aspects when primary students learn about numbers;
- Explain the central aspects of a positional number system, in other words, a number system like ours which is based on place value; and
- Explain the basic properties of addition, namely:
 - commutative property;
 - associative property;
 - distributive property; and
 - Zero as the additive identity.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Group discussion; calculating; reading



Preparation needed: Read the article 'A brief history of the development of the number system' in the Student Textbook. Read through the lesson, especially the notes in Learning Activity 1, and do all of the calculations. Prepare copies of

- 5-frame for each student; and
- Base 5 images of Dienes blocks for each student.



Learning activities

The purpose of the next three lessons is for student teachers

- To realise that when students start to calculate with numbers there is a lot to learn;
- To consolidate the central aspects of a positional number system, in other words, a number system like ours which is based on place value; and
- To consolidate the basic properties of operations:
 - commutative property;
 - associative property;
 - distributive property;
 - Zero as the additive identity;
 - 1 as the multiplicative identity;
 - addition and subtraction are inverse operations; and
 - multiplication and division are inverse operations.



Learning activity 1: Introducing the lesson (10 minutes)

1. Ask student teachers some questions about our Base 10 place value number system. For example:
 - Ask student teachers to draw a place value table for four-digit numbers. Compare students' headings. Did all the student teachers write the headings in words (Thousands, Hundreds, Tens, Ones)? Did some student teachers write the headings in numbers? Show student teachers that they can write the headings in three ways.
 - Check that student teachers remember what exponents are? Some student teachers may remember that 10^2 is 10×10 and that 10^3 is $10 \times 10 \times 10$, but they may well have forgotten that 10^1 is 10 and that 10^0 is 1.

2. Ask student teachers how many digits we have in our number system.
Explain that because we use 10 digits, it is called a base 10 system.
Explain that we group numbers according to powers of 10. Explain by using numerical examples that this means that each unit can be decomposed or broken up into 10 units of the next lower power and that 10 units of any power can be composed into the next higher unit.
3. Discuss the difference between value of a digit in a number and the place value of that digit.
4. Give student teachers a few examples of numbers to write in expanded notation.



Assessment

Ask student teachers to solve a simple problem using the base 10 system. Walk around to assess their understanding as they write.



Possible student teacher responses

Examples of numbers in a place value table:

Table 3.1. Place value table

	Thousands	Hundreds	Tens	Ones
3,250 =	3	2	5	0
6,096 =	6	0	9	6

Examples of exponents:

3. $10^0 = 1$
4. $10^1 = 10$
5. $10^2 = 10 \times 10 = 100$
6. $10^3 = 10 \times 10 \times 10 = 1,000$

Examples of expanded notation:

7. $145 = 100 + 40 + 5$
8. $608 = 600 + 8$
9. $4,590 = 4,000 + 500 + 90$



Learning activity 2: Experiencing and working with a base 5 number system (15 minutes)

1. Explain that there is a lot about our number system that we have internalised and take for granted. It is often difficult for primary school teachers to understand what it is that students have to learn, and why they may find it difficult.

For this reason we are going to work with an unfamiliar base 5 system, so that we experience both the frustration of learning about a number system and how to calculate but also so that we begin to appreciate the underlying logic. Student teachers will see how particular tools help them to understand a number system better.

2. Introduce the number system by explaining that it has 5 digits, you can show these using the fingers of one hand and using the table below. This table uses both 5-frames and loose blocks, but shows that once there are 5 blocks they are grouped into one group of 5. Similarly, once there are 5 groups of 5 can be shown by cubes arranged in a 5×5 block. When there are 5 groups of 5 groups of 5 this can be shown in a cube of $5 \times 5 \times 5$ cubes.

Figure 3.1. Base 5 number system

Symbol	Number name	Image on 5 -frame	Number shown in blocks
0	Nul		
I	Een		
Γ	twee		
Π	Dree		
□	Feer		
10	Fayf		

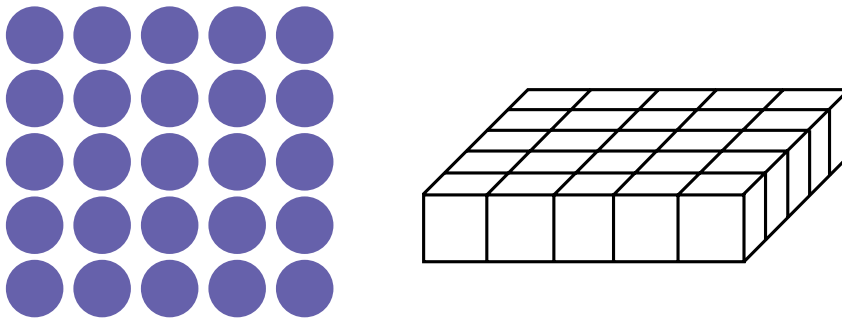
10 indicates one group of 5.

Figure 3.2. Composing into one unit (1)



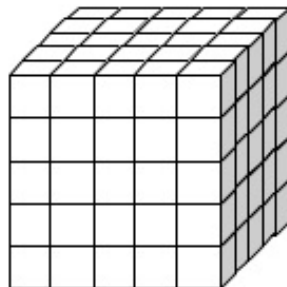
100 indicates 5 groups of 5, or 5 times 5, or 5^2 .

Figure 3.3. Composing into one unit (2)



1000 indicates 5 groups of 5 groups of 5 or $5 \times 5 \times 5$ or 5^3 .

Figure 3.4. Composing into one unit (3)



2. Ask student teachers to work through questions 1-3 in the Student Teacher Textbook in pairs.



Assessment

Walk around and listen to student teachers as they discuss the base 5 system in pairs. As you assess their understanding, clarify any misconceptions.

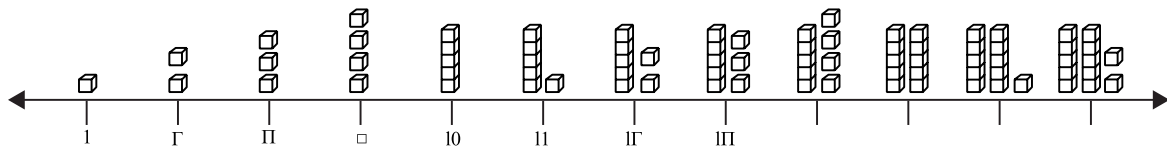


Possible student teacher responses

These correspond to questions 1-3 in the Student Teacher Textbook.

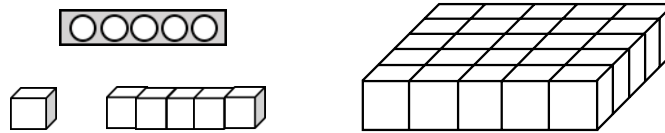
- Talking about the symbols 0, 1, Γ , Π , \square :
 - Student teachers should notice that the number of straight lines in each symbol represents the value of the number. 0 – 0 straight lines, 1 – 1 straight line, Γ – 2 straight lines, Π – 3 straight lines, \square - 4 straight lines.
- Student should complete the number line by adding in the numbers below.

Figure 3.5. Representing numbers in symbols



- Student teachers should look at the base 10 number grid. They should talk about how the numbers are arranged in rows of 10 and that all the numbers in a row have the same tens-value and all the numbers in a column have the same ones-value.

Student teachers should complete the number grid for base 5. The aim is for them to see how to construct the next number in a sequence by counting on in ones, but also to see the patterns of the symbols. Student teachers should not convert each of the numbers to base 10 and then back to base 5. If student teachers struggle to fill in the numbers let them either work with the image of a 5-frame or visual equivalents of Base 5 Dienes blocks. For example

Figure 3.6. A base 5 system**Figure 3.7. Base 5 multiplication table using special digits**

0	1	Γ	Π	□
10	11	1Γ	1Π	1□
Γ0	Γ1	ΓΓ	ΓΠ	Γ□
Π0	Π1	ΠΓ	ΠΠ	Π□
□0	□1	□Γ	□Π	□□
100	101	10Γ	10Π	10□

**Learning activity 3: Adding in a Base 5 number system (15 minutes)**

1. Explain that student teachers will now try addition in the base 5 system.
2. Show student teachers the addition table for base 10 in problem 4. Ask them to work out how it works or explain to them how it works. Ask student teachers to complete the addition table for the base 5 number system.

Solution**Figure 3.8. Base 5 addition table using special digit**

+	0	1	Γ	Π	□	10
0	0	1	Γ	Π	□	10
1	1	Γ	Π	□	10	11
Γ	Γ	Π	□	10	11	1Γ
Π	Π	□	10	11	1Γ	1Π
□	□	10	11	1Γ	1Π	1□
10	10	11	1Γ	1Π	1□	Γ□

3. Ask student teachers to discuss in pairs the questions in question 5 of the textbook and then report back on their answers.
4. Ask student teachers to use the addition table to add the numbers in part a of question 6 in the Student Teacher Textbook.
5. Ask student teachers to answer the questions in part b and c of question 6.



Assessment

Listen as pairs share their answers with the class. Walk around as student teachers solve the addition problems in problem 6.



Possible student teacher responses

Answers to question 5:

10. What happens to the numbers when you add zero to them? (The number does not change.)
11. What happens to the numbers when you add 1 to them? (You get the next consecutive number.)
12. What stays the same when you add 10 to a number? (The ones digit does not change, only the 10s digit, that is, 10's digit)

Answers to question 6:

a.

- i. $\Gamma + I$ (=II)
- ii. $I + \Gamma$ (=II)
- iii. $\Pi + \Pi$ (=II)
- iv. $\square + \square$ (=I II)
- v. $\Pi + \square$ (=I Γ)
- vi. $\square + \Pi$ (=I Γ)
- vii. $\square + \Pi + \Gamma$ (=I \square)
- viii. $\Pi + \Gamma + \square$ (=I \square)
- ix. Which was easier to add (g) or (h)?

- b. Can you use the addition table to add three numbers? (Yes)

Student teachers might find it easier to add $\Pi + \Gamma + \square$ because they might add the first two numbers $\Pi + \Gamma$ which give 10 and then add \square to get 1 \square .

However, in the previous example, they could also start by adding the same two numbers. They do not have to add the numbers in the order given.

Did student teachers use the addition table to add 3 numbers in questions (vii) and (viii)? There are two possible ways to get the answers here.

If they want to add $I + \Gamma + \square$, they can first add $I + \Gamma$ and get Π and then add $\Pi + \square$ to get $II\Gamma$. The second option in an example like $\square + \Pi + \Gamma$ is to add $\square + \Pi = II\Gamma$, then to ignore for a moment the 5s digit, that is, 105's digit and add $\Gamma + \square = I\Gamma$ and then to add $II\Gamma + I\Gamma = \Gamma\Gamma$

- c. Does the answer change if you change the order in which you add numbers? (The answer does not change)



Learning activity 4: Place value table for base 5 (5 minutes)

1. Ask student teachers to complete question 7 in pairs. They should look back to the place value table for base 10, and then write a place value table for base 5.
2. Walk around to check for understanding and help them. Use the base 5 versions of illustrations of Dienes blocks to assist them if necessary.



Assessment

Walk around to watch and listen to pairs as they work.



Possible student teacher responses

In one row, they should use the normal base 10 word name. In another row, they should use our normal number digits. In yet another row, they use whichever of our base 5 digits you need 0, 1, Γ , Π , \square .

Table 3.2. Problem solutions

One hundred and twenty fives	Twenty-fives	Fives	Ones
125s	25s	5s	1s
1 000 ₅ s	100 ₅ s	10 ₅ s	1 ₅ s
10 ³ 5	10 ² 5	10 ¹ 5	10 ⁰ 5

**Check student teachers' understanding (5 minutes)**

1. Remind student teachers of the learning outcome of this lesson:
 - To realise that when students start to calculate with numbers there is a lot to learn;
 - To consolidate the central aspects of a positional number system, that is, a number system like ours which is based on place value;
 - To consolidate the basic properties of addition, namely:
 - commutative property;
 - associative property;
 - distributive property;
 - Zero as the additive identity;
2. Ask student teachers to turn to their partner and explain what they understood through today's lesson.
3. Remind student teachers that they worked in a base 5 number system with a new set of digits. They should understand that there is a lot for primary students to learn when they begin working in our usual base 10 number system.



Extension and differentiation activities

Learning activity 1: The purpose of this activity is to check for a basic understanding of the base 10 system. If student teachers need revision, briefly explain and discuss the system.

Learning activity 2: The base 5 system can be difficult to grasp at first. If student teachers have difficulty, explain and discuss it for a few minutes before they discuss in pairs.

Learning activity 3: Make sure student teachers understand the addition table in base 10, and that the addition table in base 5 follows the same pattern. Solve a few examples as a class if needed before asking them to work in pairs.

Learning activity 4: If student teachers do not understand, you may start writing the place value chart on the board and have them complete it on paper in their pairs.

3.1.2. Adding and subtracting in base 5

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Identify the notion of grouping and regrouping through working with a base 5 positional number system; and
- Explain how addition and subtraction are inverse operations.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Calculating



Preparation needed: Make copies of the diagrams of the base 5 Dienes blocks and 5-frame



Resources needed: As above



Learning activities

The focus here is on representations which help student teachers to make sense of grouping and regrouping in 5s and powers of 5. This is so that they better understand how these representations will assist young students.



Learning activity 1: Introducing the lesson and addition in base 5 (20 minutes)

1. Explain to student teachers that they will do more work on adding in base 5. Instead of using the addition table they will work with regrouping into 5 and powers of 5. Demonstrated using a 5-frame how to add:

$$+ \quad \rightarrow \quad + \quad \rightarrow \quad +$$

2. Ask student teachers to repeat this calculation using images of base 5 Dienes blocks.
3. Demonstrate this with an arrow and circle diagram

□

$$1 + \text{II}$$

$$\text{So } \square + \text{II} \rightarrow \square + 1 + \text{II} = 10 + \text{II} = \text{III}$$



Assessment

Ask student teachers to complete the calculations in their textbooks.

You can demonstrate the ‘hidden’ aspects of calculations that we have shown in the arrow diagrams by using 5-frame, arrow and circle diagrams or diagrams of base 5 Dienes blocks.



Possible student teacher responses

$$\begin{array}{r}
 \text{i.} \\
 \begin{array}{r}
 100_5s \quad 10_5s \quad 1_5s \\
 1 \quad \Gamma \quad \text{II} \\
 + \quad 1 \quad \Gamma \quad 1 \\
 \hline
 \Gamma \quad \square \quad \square
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 100s \quad 10s \quad 1s \\
 1 \quad \text{II} \quad \square \\
 + \quad \Gamma \quad \Gamma \quad \Gamma \\
 \hline
 \square \quad 1 \quad 1
 \end{array}$$

	<i>100s</i>	<i>10s</i>	<i>1s</i>
	1	11	□
+	11	□	11
	1	0	11
			1

- ii. $1111 + 11□1 = 1110$
- iii. $11□1 + 10□ = 1000$
- iv. $□□□ + 1 = 1001$

Learning activity 2: Subtraction in base 5 (20 minutes)

1. Ask student teachers to work on the subtraction problems in the textbook in pairs. Provide explanation as needed.
2. Ask a few student teachers to share their answers with the class and explain.



Assessment

Ask student teachers to complete the calculations in their textbooks. Walk around to check for understanding.



Possible student teacher responses

1. $□ + 11 = 11$, so $11 - □ = 11$. Because addition and subtraction are inverse operations.
2. Student teachers should use the number line provided to work out the answers to question 2.
 - $1 - 1 = 1$
 - $10 - 1 = □$
 - $□ - 11 = 1$
 - $10 - 11 = 1$
 - $11 - 1 = 11$
 - $1□ - 11 = 11$
3. Student teachers should do the multi-digit subtraction. They should check their answers. Many of the calculations require regrouping using addition.

a. $\text{III} - \text{II} = 10$

b.

	100_5	10_5	1_5
	1	II	I
-	1	1	I
		I	0

c.

	100_5	10_5	1_5
	□	II	I
-	1	1	I
	II	I	0

d.

		10_5	1_5
		I	II
-		1	II
		1	□

e. $\text{II} - \square = \text{II}$

f. $\text{III} - \square = \text{II}$

g. $\text{III} - \text{II} = \square$



Check student teachers' understanding (10 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Better understand the notion of grouping and regrouping through working with a base 5 positional number system;
 - Group and regroup numbers into groups of 5 and groups of powers of 5; and
 - Understand that addition and subtraction are inverse operations.
2. Ask student teachers to turn to their partner and explain what they learned during today's lesson.

3. Remind student teachers that they added and subtracted 2- and 3-digit numbers in base 5. This is similar to adding and subtracting in base 10, but seems difficult because it is a system that we are not used to. Suggest that this is how primary students feel when they begin to learn Mathematics.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty solving addition problems with grouping, solve a few examples on the board before asking them to complete the problems in the textbook.

Learning activity 2: If student teachers have difficulty solving subtraction problems with grouping, solve a few examples on the board before asking them to complete the problems in the textbook.

3.1.3.

Understanding multiplication in a positional number system by working in a base 5 system

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain the basic properties of multiplication, namely:
 - commutative property;
 - associative property;
 - distributive property;
 - 1 as the multiplicative identity; and
 - multiplication and division are inverse operations.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Discussion; calculations



Preparation needed: Read the textbook and this lesson plan carefully, and be prepared to explain multiplication and division in base 5.



Resources needed: None



Learning activity 1: Introducing the lesson (5 minutes)

1. Briefly review addition in base 5. Write a few problems on the board and ask student teachers to solve them alone or in pairs.



Assessment

Walk around student teachers as they work. Remind them how to use the addition table to add in base 5 if needed.



Possible student teacher responses

Examples of addition problems and responses:

1. $\Gamma + \Gamma = \square$
2. $I + \Pi = \square$
3. $\Pi + \square = I\Gamma$



Learning activity 2: Multiplication in base 5 (20 minutes)

1. Ask student teachers to work in pairs to complete the multiplication table for base 5 in questions 1 of the textbook.
2. After they complete the table, they should try to complete questions 2 and 3 in pairs.
3. Ask volunteers to write the solutions to questions 1-3 on the board.



Assessment

Ask student teachers to complete the calculations in their textbooks. Walk around to check for understanding.



Possible student teacher responses

- To build the multiplication table, student teachers should understand that:
 - Student teachers should know that anything multiplied by zero is zero. So completing the 1st row and 1st column will be easy.
 - Student teachers should know that anything multiplied by 1 remains unchanged. So completing the 2nd row and 2nd column will be easy.
 - Help student teachers to use repeated addition to build up a multiplication table for base 5 using our special base 5 digits.
 - Multiplying by 2 or Γ is doubling. This should help student teachers to complete the 3rd row and 3rd column. When student teachers double Π , that is, $\Gamma \times \Pi$ or $\Pi + \Pi$ they will need to regroup into one group of 5 and 1, that is, Π . A similar approach can be used for $\Gamma \times \square$.
 - Multiplying by Π is just adding the number 3 times. Student teachers will need to regroup.
 - Multiplying by \square is just adding the number 4 times. Student teachers will need to regroup.

The completed multiplication table:

Table 3.3. Complete multiplication table

x	0	1	Γ	Π	\square	10
0	0	0	0	0	0	0
1	0	1	Γ	Π	\square	10
Γ	0	Γ	\square	Π	$\Pi\Pi$	$\Gamma 0$
Π	0	Π	Π	$\Gamma 0$	$\Gamma\Gamma$	$\Pi 0$
\square	0	\square	$\Pi\Pi$	$\Gamma\Gamma$	$\Pi\Pi$	$\square 0$
10	0	10	$\Gamma 0$	$\Pi 0$	$\square 0$	100

2. Answers to the multiplication problems:

a. $\Gamma \times \Gamma = \square$

b. $\Pi \times \square = \Gamma\Gamma$

c. $\Gamma \times \Pi \times \square = \Gamma\Gamma \times \Gamma = \square\square$

(explanation $\Pi \times \square = \Gamma\Gamma$ from (b) above. $\Gamma \times \Gamma = \square$ from (a) above, so $\Gamma\Gamma \times \Gamma = \square\square$)

d. $\Pi \times 10 = \Pi 0$

e. $\Pi \times 100 = \Pi 00$

3. Answers to the multi-digit column multiplication problems:

$$\begin{array}{r} 10_5s \quad 1_5s \\ \Pi \quad \Gamma \\ \times 1 \quad \Gamma \\ \hline 1 \quad 1 \quad \square \end{array}$$

$$\begin{array}{r} \Pi \quad \Gamma \quad 0 \\ \hline \square \quad \Pi \quad \square \end{array}$$

$$\begin{array}{r} 10_5s \quad 1_5s \\ \Pi \quad \Gamma \\ \times \Gamma \quad \square \\ \hline \Gamma \quad \Pi \\ 1^1 \quad 1^1 \Pi \quad \square \\ \hline 1^1 \square \quad 1^1 0 \quad \Gamma \end{array}$$



Learning activity 3: Thinking about the properties of multiplication (10 minutes)

1. Ask student teachers to work in pairs to discuss questions in problem 4 of the textbook.
2. After they have a few minutes to discuss, invite a few student teachers to share their responses with the class.



Assessment

Walk around and listen to student teachers' discussions to check for understanding.



Possible student teacher responses

- a. Can you use the table to multiply three numbers? Explain your answer using examples.

Yes, for example $\Gamma \times \Gamma \times \square = (\Gamma \times \Gamma) \times \square = \square \times \square =$ (from the table, or by repeated addition) $\text{III}\Gamma$

- b. Does the order in which you multiply the numbers matter? Explain your answer using examples.

No, you can change the order of numbers when you multiply. Show student teachers several examples from the multiplication table, for example, $\Gamma \times \square = \text{III}$ (from the table) and $\square \times \Gamma = \text{III}$

- c. Do you see the symmetry in the multiplication table? Why does this symmetry arise?

The answers (products) are symmetrical about the diagonal that runs from the top left to the bottom right of the table. This is because of the commutative property of multiplication: the order in which we multiply two numbers does not matter. This in turn means that students only need to learn half of the multiplication facts on a base 10 multiplication grid. Let student teachers look at the base 10 multiplication grid to confirm that this is true.

Table 3.4. Multiplication grid

x	0	1	Γ	Π	□	10
0	0	0	0	0	0	0
1	0	1	Γ	Π	□	10
Γ	0	Γ	□	11	111	Γ0
Π	0	Π	11	Γ0	ΓΓ	Π0
□	0	□	111	ΓΓ	111	□0
10	0	10	Γ0	Π0	□0	100



Learning activity 4: Thinking about the properties of division (10 minutes)

1. Ask student teachers to find the set of questions for ‘Division and the properties of arithmetic’ in the textbook.
2. Let student teachers discuss the questions in groups. Each group can get one question to answer. They can write this up and then do a gallery walk where one person from each group presents to the other groups as they move around the room.



Assessment

Walk around and listen to student teachers’ discussions to check for understanding.



Possible student teacher responses

1. Does order matter when doing division?
Try to think of some examples of simple division.
Remember if one example does not work, then the statement has been disproved.

$$10 \div 5 = 2$$

$$5 \div 10 = 0.5$$

So, division is not commutative.

2. Is division associative?

What happens when we divide more than two numbers? Does it matter which of the pairs of numbers we associate when we perform division?

Student teachers may take three numbers 8, 4 and 2. What are the different expressions of division consisting of different arrangements of the numbers that we can make?

$$(8 \div 4) \div 2 = 2 \div 2 = 1$$

$$(4 \div 2) \div 8 = 2 \div 8 = 1/4$$

So division is not associative.

3. Are there any special numbers for division?

What about one?

Any number divided by 1 gives that number as a result. For example, $2 \div 1 = 2$, $3 \div 1 = 3$, $4 \div 1 = 4$ and so on.

What about zero?

When we divide 0 by any non-zero number, the answer is 0. For example, $0 \div 1 = 0$,

This is because $0 \times 1 = 0$.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand the basic properties of multiplication, namely;
 - commutative property;
 - associative property;
 - distributive property;
 - 1 as the multiplicative identity;
 - Multiplication and division are inverse operations.
2. Ask student teachers to turn to their partner and explain a few facts they know about multiplication and division.

3. Remind student teachers that they applied multiplication and division to numbers in base 5. This is similar to multiplying and dividing in base 10, but seems difficult because it is a system that we are not used to. Suggest that this is how primary students feel when they begin to learn Mathematics.



Extension and differentiation activities

Learning activity 1: If student teachers do not recall or understand addition in base 5, spend a few times reviewing it. They will need to understand addition to complete the multiplication problems in this lesson.

Learning activity 2: If student teachers do not understand multiplication in base 5, work out the multiplication table on the board together and solve a few example problems before student teachers work in pairs.

Learning activity 3: Student teachers should identify the properties of Mathematics. If they have difficulty, you may write some example problems in base 10 to remind them of the properties they should already be familiar with. Then ask them how these properties apply to base 5.

Learning activity 4: If student teachers have difficulty understanding division in base 5, spend a few minutes discussing it as a class before they work in groups.

3.1.4.

The concept of fractions (1)

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Compare different meaning of fractions;
- Work practically with different models for representing fractions;
- Use these models to help students to both identify fractions and to understand the concept of fractions;
- Identify common student misconceptions around part-whole relationships; and
- Develop students' proportional thinking skills whilst teaching fraction concept.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Paper folding; drawing; discussing; addressing common student misconceptions; writing lesson plan



Preparation needed: Prepare a tangram puzzle to demonstrate, and one for each student teacher if possible. Photocopy copies of tangram sets for each pair of student teachers. Alternatively, you could ask student teachers to fold their own set of tangrams: see the following two websites for how to do this:

<https://www.uen.org/lessonplan/view/11079>

<https://www.youtube.com/watch?v=96nqx2eia0s>



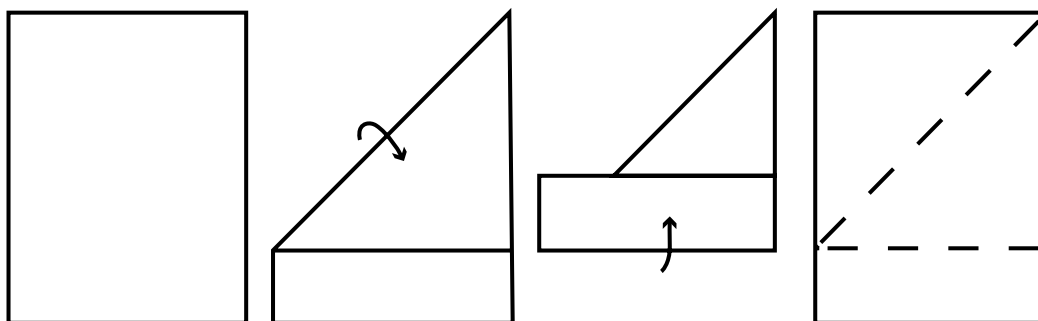
Resources needed: A sheet of paper for each student; pairs of scissors; loose counters



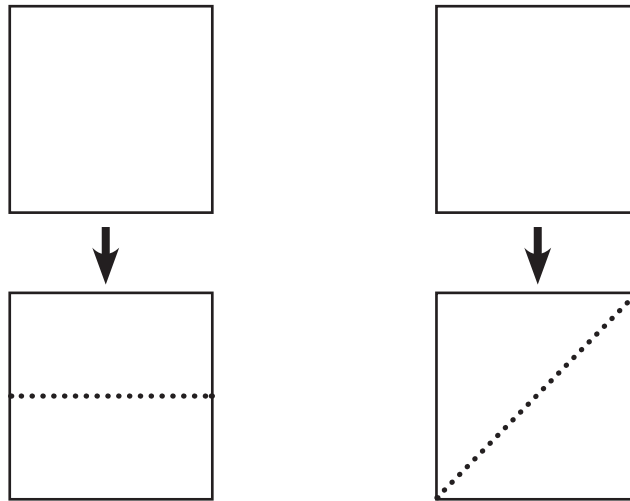
Learning activity 1: Introducing the lesson (15 minutes)

1. Give each student teacher a piece of paper. Ask them to follow the following steps to make a square and fold it. Through this exercise they should realise that equal parts can look different.
 - **Step 1:** Show student teachers how to fold the paper it to make a square.

Figure 3.9. Folding paper (1)



- **Step 2:** Tear or cut off the extra flap: throw it away. Use only the square piece of paper.
- **Step 3:** Ask every second student to fold his/her square in half vertically or horizontally. The other student teachers should fold their squares in half diagonally.

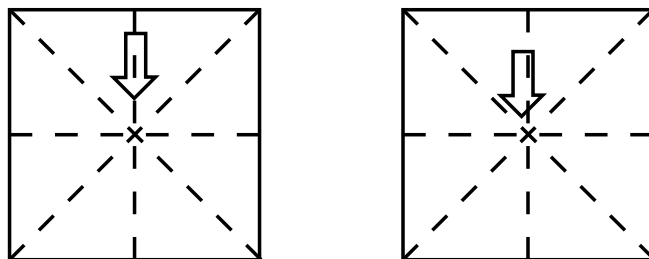
Figure 3.10. Folding paper (2)

- **Step 4:** Each student teachers should then fold their sheets half in half again. The first group should fold to make squares. The second group should fold to make triangles again.
- **Step 5:** Let student teachers compare their quarters to see how different they look.
- **Step 6:** Ask them whether the triangles and squares (that represent of the sheets of paper) are the same size or different sizes.

Figure 3.11. Folding paper (3)

- **Step 7:** Ask student teachers to fold their pieces of paper in half again. The student teachers who folded their pages into squares should now fold into triangles. The student teachers who originally folded into triangles, should continue to fold into triangles.
- **Step 8:** Ask them what fraction parts they have made now.
- **Step 9:** Ask them to compare their pages, to see that they now look the same

Figure 3.12. Folding paper (4)



2. Ask student teachers to work in pairs to complete questions 2 and 3. These are linked and focus on the relationship between division and fractions



Assessment

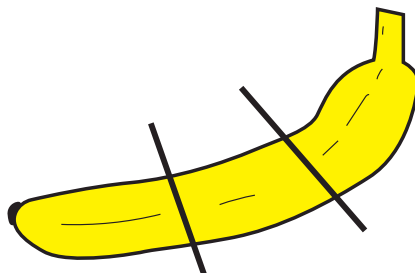
For question 1, ask a few student teachers what they learned from this exercise. For questions 2 and 3, walk around to listen to student teachers as they work and check their written work.



Possible student teacher responses

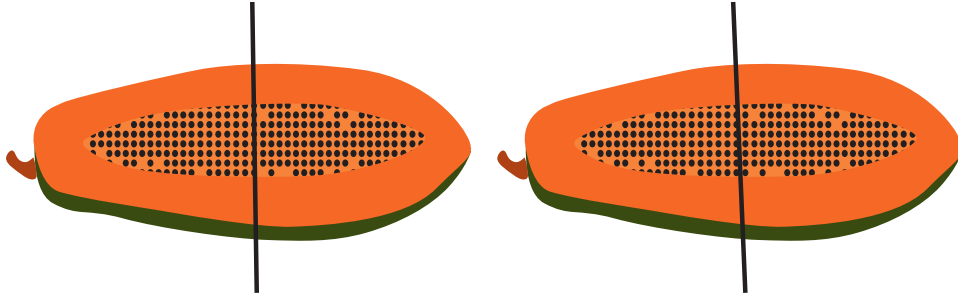
1. Student teachers should identify that the size of the parts in the folded papers are the same although the papers were folded differently.
2. Student teachers should draw the shapes divided approximately equally into the number of parts given. These are example drawings:
 - a. 1 banana is divided among 3 children.

Figure 3.13. Dividing a banana



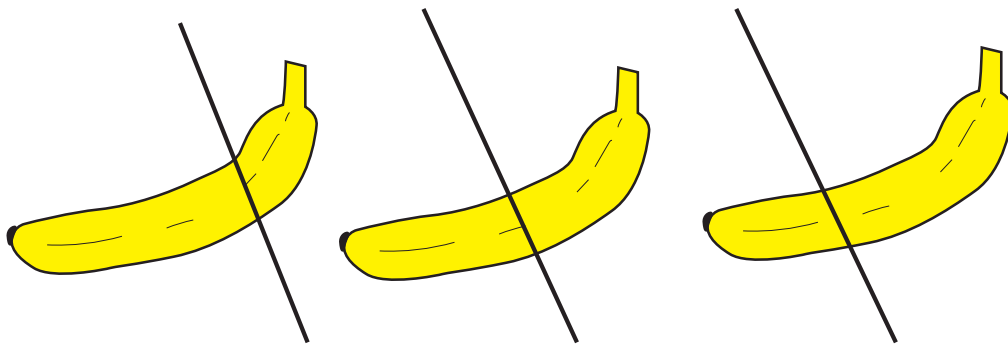
- b. 2 papayas are divided among 4 children.

Figure 3.14. Dividing papayas



- c. 3 bananas are divided among 4 children.

Figure 3.15. Dividing bananas



3. Completed table:

Table 3.5. Fractions

Sharing problem	Expression	Fraction that each child got
1 banana is divided among 3 children	$1 \div 3$	$\frac{1}{3}$
3 bananas are divided among 4 children	$3 \div 4$	$\frac{3}{4}$
2 oranges are divided among 5 children	$2 \div 5$	$\frac{2}{5}$



Learning activity 2: Making wholes by repeating equal parts (15 minutes)

1. Ask student teachers to find the section ‘Making wholes by repeating equal parts’ in the textbook.
2. Ask them to work with a partner to complete the two problems in this section.
3. Ask a few student teachers to share what they learned through this exercise.



Assessment

Walk around to listen to pairs as they work and check their written work.



Possible student teacher responses

1. There is sometimes more than one correct drawing.
 - a. The diagram represents $\frac{3}{8}$, and there are 3 small squares. Student teachers should draw a figure with 8 small squares.
 - b. Student teachers should draw a figure with 7 small triangles.
 - c. Student teachers should draw a figure with 6 trapezoids.
 - d. Student teachers should draw a figure with 5 trapezoids.
2.
 - a. 9 square metres
 - b. 16 counters
 - c. 60 counters
 - d. 100 students
 - e. 40 chickens



Learning activity 3: Same shape? Same size? (10 minutes)

1. Ask student teachers to find the section 'Same shape? Same size?' in the textbook.
2. Ask them to work with a partner to complete the two problems in this section.
3. Ask a few student teachers to share what they learned through this exercise. Guide them to identify two common fraction misconceptions:

- a. Firstly, students assuming that because fraction pieces have different shapes, they cannot be equal.
 - b. Secondly, students not paying attention to the size of the parts.
- In both of the examples provided students have to think about proportionality.



Assessment

Walk around to listen to pairs as they work and check their written work.



Possible student teacher responses

1. Student teachers should answer in their own words. These are examples:
 - a. $\frac{1}{4}$
 - b. Students identify that the shape is divided into 4 equal parts although the parts look different. It is first divided into 2 halves, and each half is divided differently. Thus the shaded triangle is $\frac{1}{4}$.
 - c. Show them that the parts are equal using a square paper and folding it into 4 equal parts in two different ways.
2. Student teachers should answer in their own words. These are examples:
 - a. $\frac{1}{8}$
 - b. The shaded part is one of 5 parts shown in the diagram, but the shape is not divided equally.



Learning activity 4: Assignment (5 minutes)

1. Ask student teachers to find the sections 'Parts and whole: unequal parts and proportionality' and 'Write a lesson plan' in the Student Teacher Textbook.
2. Explain that these two parts are an assignment that they should complete before the next class.
3. Briefly explain the tangram puzzle if needed. Make sure student teachers understand the assignment. They will first explore tangrams, then write a lesson plan about them. It is best if student teachers have physical examples of the tangram puzzle.



Assessment

You will assess student teachers' written work when they submit the assignment.



Possible student teacher responses

For 'Parts and whole: unequal parts and proportionality':

Student teachers need to consider the size of each part in relation to the whole. This lays a visual basis for the notion of common denominator which they will work with later. They can then see that two large triangles make up half the tangram area. So four of the large triangles make up the whole tangram area. From there they should superimpose pieces. For example, if they superimpose the medium triangle on the large triangle, they will see that it is half of the large triangle, and so, eight of these pieces will fill the area of the whole tangram. They can superimpose the small triangles on the medium triangle and also on the square and the parallelogram.

The completed table:

Table 3.6. Parts and whole

Shape	Number of pieces that will cover the area of the whole tangram	Fraction of the tangram
Large triangle	4	$\frac{1}{4}$
Medium triangle	8	$\frac{1}{8}$
Square	16	$\frac{1}{16}$
Parallelogram	16	$\frac{1}{16}$
Small triangle	16	$\frac{1}{16}$

For 'Write a lesson plan':

Lesson plans should show an understanding of tangrams and effective methods to teach primary students about fractions using them.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand different meaning of fractions;
 - Work practically with different models for representing fractions;
 - Use these models to help students to both identify fractions and to understand the concept of fractions;
 - Identify common student misconceptions around part-whole relationships; and
 - Develop students' proportional thinking skills whilst teaching fraction concept.
2. Ask student teachers to turn to their partner and explain some common misconceptions about fractions and how they plan to address these in their classrooms.
3. Remind student teachers that they identified some common misconceptions about fractions. It is important to recognise misconceptions that students may have about Mathematics concepts, and be prepared to address them.



Extension and differentiation activities

Learning activity 1: Make sure student teachers have a good understanding of the first definition of fractions during this activity. You may ask them to describe other ways they could demonstrate to students that fractions are equal parts of a whole.

Learning activity 2: If student teachers have difficulty understanding these questions, work through 1-2 as a class before releasing them to complete the activity in pairs.

Learning activity 3: You may ask a few student teachers to demonstrate for the class how they would address the common misconceptions highlighted here.

Learning activity 4: Make sure student teachers understand the assignment before leaving the classroom. If they are unfamiliar with tangram puzzles, you may spend a few minutes explaining and demonstrating.

3.1.5. The concept of fractions (2): Equivalence

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Use various models to find equivalent fractions;
- Identify different kinds of errors, in particular the differences between mistakes and misconceptions;
- Explain the benefits to both teaching and learning of working with students' misconceptions;
- Identify student misconceptions around fractions on a number line; and
- Explain that there are an infinite number of fractions between any two fractions.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Paper folding; working with pictorial representations; reading about equivalent fractions; reading about misconceptions; working with student misconceptions about fractions on a number line; role play remediation



Preparation needed: Prepare a poster size fraction wall, or prepare a slide so that you can project it onto the wall. Ask student teachers to read the article about equivalent fractions before the lesson.



Resources needed: Sheets of paper for each student.



Learning activity 1: Working with fraction strips (10 minutes)

1. In this activity, student teachers begin by folding fraction strips. This is a precursor to using a fraction wall to find equivalent fractions.
2. Ask student teachers to:
 - a. Make 4 equal sized strips of paper.
 - b. Fold one strip to make halves. Write $\frac{1}{2}$ on each half.
 - c. Fold another strip to make quarters. Write $\frac{1}{4}$ on each quarter.
 - d. Fold another strip to make eighths. Write $\frac{1}{8}$ on each eighth.
3. Then ask student teachers questions about which fractions are equal to which other fractions, as indicated in the Student Teacher Textbook. Show student teachers that they can superimpose the different strips on top of each other to 'prove' the result.
4. Explain that making equivalent fractions forms the foundations of several mathematical procedures, for example, adding and subtracting fractions with different denominators; factorising fractions to simplify them.



Assessment

Assess students understanding by checking that they folded fraction strips correctly and whether they are able to show equivalent fractions using strips.



Possible student teacher responses

Student teachers should identify some equivalent fractions by lining up their strips. For example, they may identify that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$.



Learning activity 2: Equivalent fractions on a fraction wall (15 minutes)

1. Ask student teachers to find the section 'Using a fraction wall to understand equivalent fractions' in their textbook.
2. Explain that fractions that have different names and look different to each other in their numerical form can have the same value. We call these equivalent fractions.
3. Show student teachers how use a ruler or any straight edge, for example, side of a book, to make a fraction wall.

4. Ask student teachers to look at the fraction wall and name fractions that are equal to 1.
5. They can first answer from the examples on the fraction wall and then generalise. (Answer: $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, ...).
6. Ask student teachers to list fractions that are equal to. They can first answer from the examples on the fraction wall and then generalise. (Answer: $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, ...)
7. Let student teachers answer the other questions from their book.
8. Discuss with student the value of students working with fractions walls as a way of informally laying the basis for calculating equivalent fractions numerically.



Assessment

Walk around to check for understanding as students complete the problems from their textbooks.



Possible student teacher responses

The answers to the problems in the textbook are:

1. a. 3 thirds equal 1
 b. two sixths equal $\frac{1}{3}$
 c. three ninths are equal to $\frac{2}{6}$
 d. two thirds are equal to $\frac{4}{6}$
2. a. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$
 b. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$
 c. $\frac{3}{4} = \frac{6}{8}$
 d. $\frac{2}{5} = \frac{4}{10}$
 e. $\frac{3}{6}$, $\frac{2}{6}$, $\frac{4}{6}$
 f. $\frac{4}{8}$, $\frac{4}{8}$, $\frac{6}{8}$, $\frac{8}{8}$



Learning activity 3: Using a number line to understand the density of fractions (10 minutes)

1. Explain that when we add 1 to any whole number, we generate the next whole number. This is called the successor function of whole numbers. Fractions do not work in this way. Fractions with the same denominators are evenly spaced on a number line. For example, 1 third is the same distance from 2 thirds as 2 thirds is from 3 thirds and so on.
2. Hand out a sheet of paper to each student. Read aloud the instructions from the student textbook. Student teachers should make all of their folds in the same direction, in other word, to cut their number line. Ask them to predict how many fold lines they will make each time. Ask student teachers to write the fractions that should be placed at each fold line. Write up the fractions on a number line on the board.
3. After student teachers have made eighths, ask them state any fraction between zero and $\frac{1}{8}$.
4. Repeatedly ask for any fraction that is halfway between the last answer provided and zero.
5. Stress that there is no smallest fraction, and that there are an infinite number of fractions between any two fractions.



Assessment

To check for understanding, ask questions and invite a few student teachers to respond. For example: 'How many fractions are there between $\frac{1}{2}$ and $\frac{1}{4}$?'



Possible student teacher responses

Student teachers should realise that between any two fractions are an infinite number of other fractions. This activity aims to consolidate student teachers' knowledge of this.



Learning activity 4: Misconceptions around fractions on a number line (10 minutes)

1. Ask student teachers to find the section 'Misconceptions around fractions on a number line' in their textbook.
2. Let student teachers answer Question 1 independently.
3. Then let them examine the student responses. Student teachers should try to understand and express the students' logic when choosing the incorrect answer C.
4. Student teachers should then work in pairs to prepare a role play of how they would assist a student who chose C as their answer (question 4 in the textbook).



Assessment

Use the remediation role play (question 4) as an assessment.



Possible student teacher responses

Student teachers could identify the following points.

4. Why a student chose arrow C: The fraction from the problem contains $\frac{3}{4}$, and C is the third arrow on the number line.
5. Develop feedback to the student to explain what is incorrect: Ask the student to find 1 on the number line, and tell them that $1\frac{3}{4}$ means $\frac{3}{4}$ of a unit past 1 on the number line.
6. Suggest how the student should proceed so that he / she arrives at the correct answer. Include a focus on scale on a number line: Ask the student to label all of the fourths on the number line and use the labels to find the given fraction.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Use various models to find equivalent fractions;
 - Read about different kinds of errors, in particular the differences between mistakes and misconceptions;
 - Consider the benefits to both teaching and learning of working with student's misconceptions;
 - Consider student misconceptions around fractions on a number line; and
 - Understand that there are an infinite number of fractions between any two fractions.
2. Ask student teachers to turn to their partner and explain some common misconceptions about equivalent fractions and how they plan to address these in their classrooms.
3. Remind student teachers that they identified equivalent fractions, and practice representing fractions using paper, a poster and number line. All of these tools can be used to explain fractions in their classrooms and clarify any misconceptions their students have.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty understanding how fraction strips are used to identify equivalent fractions, you may have volunteers come to the front of the class and demonstrate equivalent fractions using their strips.

Learning activity 2: If student teachers have difficulty understanding, you may review the solutions to the problems as a class. Ask a few volunteers to explain how they found the answers.

Learning activity 3: If student teachers have difficulty understanding that there are an infinite number of fractions between any two fractions, you may give an example to help them understand. For example, have volunteers find fractions between 0 and $\frac{1}{2}$, and write as many as they can on the board. They should begin to see that there are infinitely many: $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \dots$

Learning activity 4: You may give additional examples of similar misconceptions of fractions on a number line.

3.1.6.

Adding and subtracting fractions with the same denominator

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Recognise the common student misconception that they can add numerators and denominators.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Self-study; micro-teaching; remediating misconceptions; using pictorial representations (semi-concrete apparatus) in the form of fraction strips and number lines to teach adding and subtracting of fractions with the same denominators.



Preparation needed: Ask student teachers to read the lesson the day before. They should work through all the calculations and the question on misconceptions. Break the student teachers into six groups. Group 1 prepares to teach question 1, group 2 prepares to teach question 2 and so on. Groups 5 & 6 prepare to remediate the misconception.



Resources needed: None



Learning activity 1: Introducing the lesson (5 minutes)

1. Revisit the term misconception: ask student teachers why it is useful for both teachers and students if teachers analyse and remediate student misconceptions.
2. Explain that in this lesson student teachers will explore some common misconceptions about adding fractions.



Assessment

Ask a few student teachers to describe ‘misconception’ in their own words.



Possible student teacher responses

This is the definition from the textbook. Student teachers may rephrase this in their own words:

- ‘*Misconceptions*’ is a term used to describe the alternative understandings that people may have about concepts. These understandings may be either incorrect or incomplete.



Learning activity 2: Micro-teaching addition of fractions (25 minutes)

1. Give student teachers 10 minutes to prepare for the activity. They should have worked through the problems for homework and prepared to teach their assigned problem.
2. Ask Groups 1 to 4 to take turns micro-teaching the calculation they prepared. Ask other groups for feedback.



Assessment

Assess student teachers on their micro teaching.



Possible student teacher responses

The answers to the four problems are below. Assess whether groups explain these clearly. They should use explanation and/or teaching aids that would clarify any misconceptions, such as fraction strips or the fraction poster.

1. a. $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$ of the candies
b. $\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$ of the candies
2. a. $\frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$ of the fruit salad
b. $\frac{8}{8} - \frac{7}{8} = \frac{1}{8}$ of the fruit salad
3. a. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ cup of water
b. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{4} = 1 \frac{1}{4}$ cup of water
4. a. $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ mile



Learning activity 3: Working with misconceptions about adding fractions (15 minutes)

1. Ask student teachers to find this section in their textbook.
2. Ask group 5 to explain the incorrect logic used by the 25% of students who chose as their answer.
3. Ask group 6 to role play the remediation, and different members of the group to present different examples to help the ‘students’ understand correctly how to add and subtract fractions with the same denominator.



Assessment

Assess student teachers on their presentations.



Possible student teacher responses

Group 5 should note that students who incorrectly chose $\frac{2}{8}$ added both the numerators and the denominators of the fractions: $\frac{1}{4} + \frac{1}{4} = \frac{(1+1)}{(4+4)} = \frac{2}{8}$. This is a common misconception.

Group 6 may use any reasonable explanation to help students understand. For example, they may use fraction strips to show that $\frac{1}{4} + \frac{1}{4}$ is the same as $\frac{1}{2}$. They should then explain to students that only the numerators of fractions are added. They should also make sure their students understand equivalent fractions and simplifying.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Add and subtract fractions with the same denominator;
 - Teach adding or subtracting fractions with the same denominator use contexts, number lines and fractions strips to add or subtract fractions with the same denominator; and
 - Work with students' misconceptions about adding fractions.
2. Ask student teachers to turn to their partner and explain a common misconception that primary students have about adding fractions.
3. Remind student teachers that they explored how to teach addition of fractions, and common misconceptions that students have around this topic. They should be prepared to address these misconceptions in their own classrooms.



Extension and differentiation activities

Learning activity 1: You may ask some volunteers to describe common misconceptions about adding fractions to make sure other student teachers understand what these are (for example, adding the denominators of fractions, or adding unlike fractions).

Learning activity 2: If student teachers do not use teaching aids, you may ask them to explain the problems using specific teaching aids, such as fraction strips or the fraction poster.

Learning activity 3: Other groups should act as students with common misconceptions about adding fractions. Encourage the presenting groups to use different methods to explain the concepts to student teachers and make sure they understand.

3.1.7. Adding and subtracting fractions with different denominators

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Recognise that in order to add or subtract fractions we make equivalent fractions with common denominators;
- Use paper folding to visually demonstrate the logic of making common denominators; and
- Design follow up parts of lessons that work with pictorial and abstract ways to add and subtract fractions with different denominators.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and 1.1. Demonstrate capacity to teach subject-related subject concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Watch video; discussion; micro teaching



Preparation needed: Watch the video clips A third plus a quarter, and A third minus a fifth from <https://vitalmaths.com/videos/English>. These are silent movies but they do have some written English, which you may have to translate.



Resources needed: Six sheets of paper for each pair of student teachers; scissors for each pair of student teachers



Learning activities

In the videos, they show how to find a common denominator by paper folding. This is a version of the area model of multiplication.

The paper folding method used produces a common denominator if one folds the one fraction along the length of the sheet of paper and another along the width of the sheet of paper. You will not necessarily get the same result if you fold both fractions in the same direction. This is important to point out to student teachers.

This method works best for fractions with small denominators for example it will be difficult to demonstrate if you add $\frac{1}{24} + \frac{3}{15}$. Student and students need to identify a more algebraic method to be able to add fractions of these magnitudes.



Learning activity 1: Introducing the lesson (45 minutes)

1. Show the video clip '*A third plus a quarter*' to student teachers.
2. Ask student teachers for responses to the video.
3. Explain to student teachers that the paper folding method used produces a common denominator if one folds the one fraction along the length of the sheet of paper and another along the width of the sheet of paper.
4. Also that although it produces a common denominator, it is not necessarily the lowest common denominator.
5. Ask pairs of student teachers to choose two other fractions (with denominators less than 10) to add. Hand out the sheets of paper and pairs of scissors. Ask them to work out how to model the addition of their chosen fractions using the sheets of paper.
6. Ask student to write down the numbers that show the steps of the calculations that they demonstrate physical. It is important for student teachers to see this connection. For example, they need to be able to see that what they did was

$$\begin{aligned}
 & \frac{1}{3} + \frac{1}{4} \\
 = & \frac{4}{12} + \frac{3}{12} \\
 = & \frac{4+3}{12} \\
 = & \frac{7}{12}
 \end{aligned}$$

7. Ask student teachers to explain the relationship between the initial two denominators and the final denominator. Similarly, ask them about the relationships between each numerator and the final denominator.
8. Show the video *A third minus a fifth* to student teachers. Repeat the process outlined above in steps 2 to 7.
9. Ask some pairs of student teachers to teach either adding fractions or subtracting fractions whose denominators are not multiples of each other by paper folding.
10. Ask other pairs of student teachers to teach a follow up lesson-snippet using diagrams and yet other pairs of student teachers to teach a more algebraic method with fractions that have larger denominators.



Assessment

Assess student teachers on their responses to questions and micro-teaching.



Possible student teacher responses

Relationship between the initial two denominators and the final denominator:

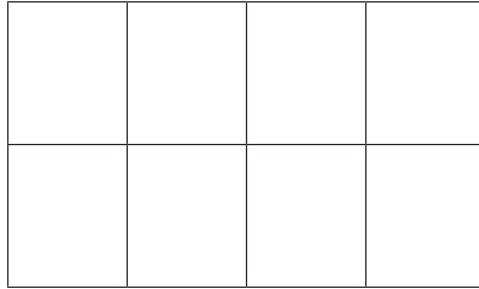
- The final denominator is the product of the initial two denominators.
- The initial denominators are factors of the final denominator.

Relationships between each numerator and the final denominator:

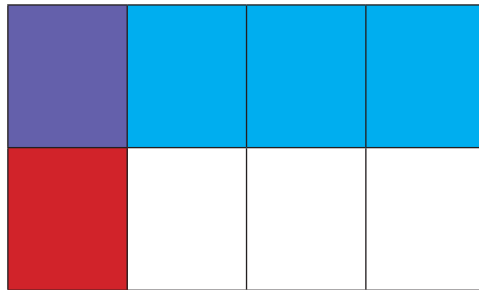
- Each numerator is a factor of the final denominator.

Teachers who demonstrate addition or subtraction of fractions using paper should fold the paper with the appropriate number of horizontal and vertical pieces to demonstrate. For example, consider $\frac{1}{2} + \frac{1}{4}$:

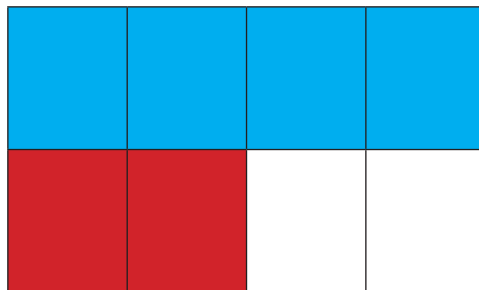
- The paper is folded using the steps in the video:

Figure 3.16. Fraction grid (1)

- $\frac{1}{2}$ is one row, and $\frac{1}{4}$ is one column:

Figure 3.17. Fraction grid (2)

- Change the rectangles under consideration so there is no overlap:

Figure 3.18. Fraction grid (3)

- Count the number of segments and find that 6 out of 8 are selected. The answer is $\frac{6}{8}$.

Pairs who show a method using diagrams may draw diagrams similar to those above that are meant to show folded paper.

Pairs who show an algebraic method should show their process of making the denominators the same. For example, this is an example for $\frac{3}{8} + \frac{1}{10}$:

- Make the denominators the same by multiplying the numerators and denominators:
 $\frac{30}{80}$ and $\frac{8}{80}$
- Add the numerators: $\frac{(30+8)}{80} = \frac{38}{80}$
- Student teachers should identify that the answer needs to be simplified: $\frac{19}{40}$



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand that in order to add or subtract fractions we make equivalent fractions with common denominators;
 - Use paper folding to visually demonstrate the logic of making common denominators; and
 - Design follow up parts of lessons that work with pictorial and abstract ways to add and subtract fractions with different denominators.
2. Ask student teachers to turn to their partner and explain a method they would use to teach addition of fractions in their own classroom.
3. Remind student teachers that they explored how to teach addition of fractions in the previous lesson. In this lesson, they used concrete materials to visually motivate the notion of a common denominator that students need to transform one or both of the fractions so that they both have the same denominator.



Extension and differentiation activities

Learning activity 1: Relate the content of this lesson to previous lessons by asking questions such as: How are equivalent fractions related to this lesson? Which of Piaget's stages of cognitive development do you think the paper folding exercise would be most appropriate for?

3.1.8. Multiplying fractions

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Multiply fractions and whole numbers;
- Multiply fractions with fractions;
- Simplify fractions by factorising and re-arranging the factors (using the commutative or associative properties) to form fractions equivalent to 1;
- Use 1 as the multiplicative identity to simplify fractions;
- Generalise towards a procedure for multiplying fractions from examining patterns in several calculations; and
- Explain that when we make common denominators, we are really making equivalent fractions by multiplying by a fraction that is equivalent to 1.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Discussion, independent and group work



Preparation needed: Ensure the resources below are available



Resources needed: Sufficient sheets of paper for each student to have one. Tape to stick folded sheets to the wall. Sufficient counters for each student to have 36: use counters that teachers would find easy to access in schools, for example, beans, bottle tops



Learning activity 1: Taking a fraction of a whole number or (multiplying fractions and whole numbers)

1. The aim of this activity is to develop a conceptual understanding of multiplying fractions by whole numbers through practical work and visual thinking.
 - During these activities, reinforce the conceptual relationship between division and fractions. For example, $\frac{1}{3}$ of 27 is equivalent to $\frac{27}{3}$ which is equivalent to $27 \div 3$.
 - It is important that student teachers also understand that we if we repeatedly add a unit fraction, it makes other fractions, for example, $= \frac{3}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$, in other word, $3 \times \frac{1}{4} = \frac{3}{4} = 3$ quarters.
 - After doing the activity student teachers can work towards a general rule that when we take a fraction of a whole number, we divide by the denominator (because of the relationship between division and fractions) and multiply by the numerator (because a non-unitary fraction is just $n \times$ a unitary fraction).
2. Ask student teachers to pack out the arrangements for questions 1, 2 and 3 in turn and follow the instructions in their books. They could pack the counters out on their desks and draw circles in chalk or pack them out on their notebooks and circle with pen. If using bottle tops as counters, one option is to turn over the tops of the required number of fractions instead of circling them.
3. Student teachers could also pack out counters for the first question and then draw circles instead of packing out counters for questions 2 and 3.
4. Ask a few student teachers to share the answers to the questions with the class.



Assessment

Walk around to check for understanding of the activity as student teachers work. Also assess a few student teachers by asking them to share the answers with the class.



Learning activity 2: Taking a fraction of a fraction or (multiplying a fraction by a fraction)

1. Divide the class into groups. Give each student a sheet of paper.
2. Each group folds their sheet into a different number of panels. The fold lines must run in the same direction. It is easier if you use halves, quarters, eighths; thirds are also relatively easy to fold.
3. Student teachers should shade 1 part, in other word, $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{3}$
4. Then ask student teachers to fold the sheets in the other direction. Let some groups halve the halves, others fold the halves into quarters, others fold the quarters in half, the eighths in half, the thirds into quarters and so on.

Ask student to mark the outline on their unfolded sheet of a fraction of a fraction for example, $\frac{1}{2}$ of $\frac{1}{2}$; $\frac{1}{4}$ of $\frac{1}{2}$; $\frac{2}{3}$ of $\frac{1}{2}$ depending on how they folded their paper. They should stick these on the wall. Do a gallery walk – one person from the group stays behind and explains what they did and shows the fraction portion arrived at for example, $\frac{2}{3}$ of $\frac{1}{2}$ equals $\frac{2}{6}$

5. Ask student teachers to pack out 36 counters (these can be beans or bottle tops or anything easily available) in 4 rows of 9. They should first circle $\frac{1}{4}$ of the counters (this is easiest if you see that each row is 1 out of 4 rows. Then they should circle $\frac{1}{3}$ of the row of 9 counters circled. This will give them 3 counters.
6. In question 10, students are asked to generalise a rule by look back at all the preceding multiplication calculations. The calculations are presented both in number sentence form and calculated. By examining the similarities in the calculations, student teachers are asked to make or rule or procedure for multiplying a fraction by a fraction. Student teachers are expected to deduce that when one multiplies fractions you multiply their numerators and you multiply their denominators. Generalising from patterns is at the heart of Mathematics. Many mathematicians and Mathematics educators say that ‘Mathematics is the Science of pattern’



Assessment

Walk around to assess student teachers as they work in groups. Assess them based on their responses and ideas for question 10.



Possible student teacher responses

Question 10 possible responses:

- What patterns do you notice in the calculation strategies?
'The numerators and denominators are multiplied. The denominator in the answer is the product of the denominators in the problem'.
- Explain how to multiply a fraction by a fraction.
'Multiply the numerators and denominators. Simplify the result if needed'.



Learning activity 3: Link multiplying by a fraction equivalent to 1 with making common denominators.

- Now that student teachers know how to multiply fractions with fractions, demonstrate to them how when they make common denominators they are really multiplying each fraction by a fraction that is equivalent to 1.
- First ask student to tell you any fraction that is equivalent to 1. Write this up in a long string on the board:

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{x}{x}$$

Then show student teachers how one uses fractions equivalent to 1 to make a common denominator. For example,

Figure 3.20. Fractions equivalent to 1

$$\frac{1}{3} + \frac{2}{8} = \left(\frac{1}{3} \times \frac{8}{8}\right) + \left(\frac{2}{8} \times \frac{3}{3}\right) = \frac{8+6}{24} = \frac{14}{24} = \frac{7 \times 2}{12 \times 2} = \frac{7}{12} \times \frac{2}{2} = \frac{7}{12}$$

Fractions equivalent to 1



Assessment

Ask a few student teachers to explain how fractions equivalent to 1 are used in adding fractions.



Possible student teacher responses

Fractions equivalent to 1 are used to change the denominators of two fractions so that they are the same and can be added. Each fraction in the addition problem can be multiplied by a different fraction equivalent to 1.



Check student teachers' understanding (5 minutes)

1. Remind student teachers that this lesson focused on:
 - taking a fraction of a whole number or (multiplying a whole number by a fraction); and
 - taking a fraction of a fraction (multiplying fractions by fractions).
2. Remind student teachers that we demonstrated that starting with practical apparatus (concrete) and moving on to diagrams (semi-concrete or pictorial) helps students to first understand the concept before learning the procedure. The standard procedure for multiplying fractions is 'multiply the numerators and multiply the denominators'.



Extension and differentiation activities

Learning activity 1: Walk around and support student teachers if they have difficulty working with the counters and understanding how they are used to solve the problems.

Learning activity 2: If student teachers have difficulty understanding how the demonstrations with paper and counters show multiplication of fractions, do an example in front of the class and explain.

Learning activity 3: If there is time, you may ask student teachers to solve addition problems of their own by multiplying the fractions by another fraction equivalent to 1.



Review questions: Possible student teacher responses

Question 1: What are some Mathematics concepts you identified by working in base 5?

Answer: Addition and subtraction are inverse operations. Sometimes numbers need to be regrouped in order to add or subtract. Multiplication and division are inverse operations.

Question 2: What are some methods you identified for introducing fractions to students?

Answer: Folding paper; using arrays of counters, using diagrams.

Question 3: What are some ways of demonstrating equivalent fractions?

Answer: Using fraction strips or a fraction poster.

3.2. Measurement

This sub-unit explores different systems of measurement, and how to introduce these to primary students in line with the grade-wise curriculum. It first explores different stages of teaching measurement, then individual lessons are dedicated to specific types of measurement: length, weight, and volume. After solving problems using metric and imperial units, there is a lesson on Myanmar units such as ‘*taung*’ and ‘*htwa*’. There is then a lesson on teaching area, and two lessons on teaching angles. Throughout the unit, there is a focus on designing problems that are appropriate for the level being taught.

3.2.1. Teaching Measurement

Expected learning outcomes



By the end of the lesson, the student teacher will be able to:

- Describe informal and formal measurement systems that can be used to teach measurement concepts;
- Demonstrate practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students have are addressed; and
- Explain how to teach standard units in the Myanmar context using metric, imperial and Myanmar units.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related subject and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Individual work, reading and listening, group discussion and application



Preparation needed: If possible, create presentation slides from the diagrams in this lesson.



Resources needed: Flipchart paper; presentation slides



Learning activity 1: Introducing the lesson (10 minutes)

1. Introduce the topic of Measurement. Form the class into new groups of four to work together throughout the lessons in this unit.
2. Supply each group with flipchart paper and ask them to capture their ideas. Ask them to reflect on: Why do many students find measurement difficult? What knowledge and skills do students need to be able to measure successfully?



Assessment

Facilitate a class discussion on the responses



Possible student teacher responses

A range of areas of measurement primary students generally find difficult to understand.



Learning activity 2: How do we measure? (15 minutes)

1. Introduce the topic 'How we measure' Present the key ideas from the first diagram, below.
2. Request that student teachers read the first diagram in their textbooks and answer the questions in the second diagram in pairs.
3. Facilitate a class review of the answers.



Assessment

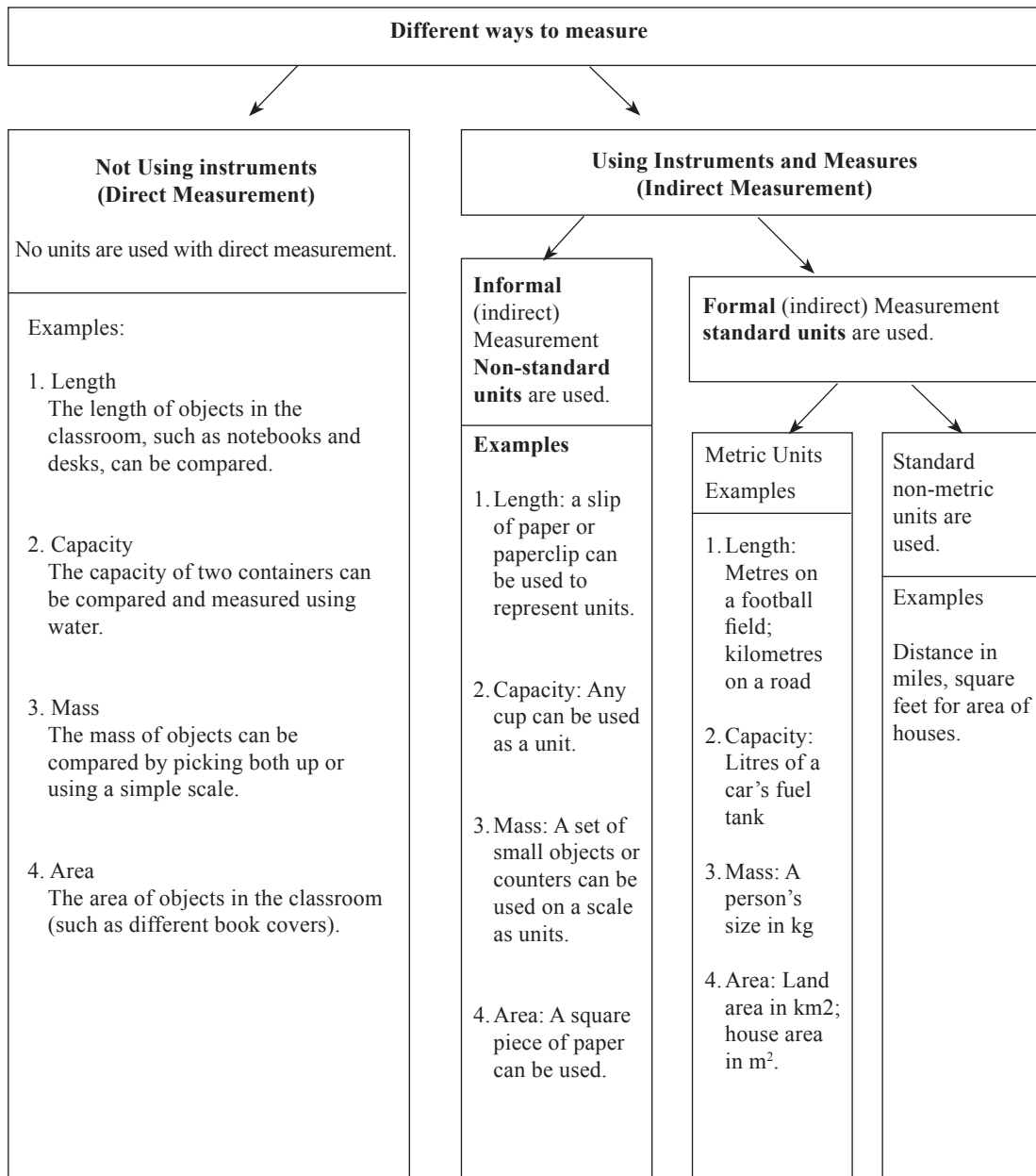
Assess student teachers based on their contributions during the class discussion.



Possible student teacher responses

Student teachers may fill the diagram with any examples of measurement used in Myanmar. Some examples are given below. Ensure that student teachers understand the key categorisations here; the difference between direct and indirect measuring and the difference between formal and informal units of measurement. Highlight also that standards are national and international.

Figure 3.21. Different ways to measure





Learning activity 3: Principles of measuring (10 minutes)

1. Provide a short presentation on ‘Principles of measuring’ using the information in the textbook.
2. Facilitate a discussion on the points during the presentation.



Assessment

Ask questions and assess student teachers based on their responses.



Possible student teacher responses

These are example questions and answers. You may ask any other questions relevant to the text.

1. What knowledge do students need to have before they measure with standard units and calibrated instruments?
 - a. a sense of the size of each unit used and of the sub-units
 - b. skills to position the instruments and themselves in relation to the instruments
 - c. knowledge on how to read the instruments, the numbered and unnumbered intervals
2. What are the principles of indirect measurement, according to Booker, Bond, Briggs and Davies (1998)?
 - a. Always state the unit of measurement
 - b. Never change the unit of measure halfway through
 - c. Comparison of measurements is possible when using the same unit
 - d. Units are chosen for convenience



Learning activity 4: Stages of learning to measure (10 minutes)

1. Now provide a presentation/facilitation on the key points on ‘Stages of learning to measure’ using the information in the textbook.
2. Now outline the groups assignment that is an application of the knowledge developed in this lesson.

3. Request that student teachers in their groups try to conceptualise a lesson 'How tall are we' for early grade students introducing measurement. Explain that the task for the student teachers is to make use of the principles and stages in composing the lesson.
4. Allow student teachers to start in class and complete the lesson as homework.



Assessment

The teacher educator can check student teachers' understanding of the concepts covered the principles and stages by assessing whether these are covered in the sequencing of the lesson design that has been given as a homework assignment.



Possible student teacher responses

The lesson plans should use non-standard units. Height may be measured using any object, such as a pencil or a piece of paper. The lesson should apply concepts from the textbook. For example, a standard unit can be used to measure multiple students in a class and then the heights of those students compared.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Be aware of informal and formal measurement systems that can be used to teach measurement concepts;
 - Have practical ideas to teach measurement of length, height and distance to ensure that the difficulties that students have are addressed; and
 - Have some perspective of how to teach standard units in the Myanmar context using metric, imperial and Myanmar units.
2. Ask student teachers to turn to their partner and explain a method they would use to teach indirect measurement in their classrooms.
3. Remind student teachers that they explored how to teach measurement to primary students. Remind them that students should be taught through indirect measurement first, and then they can be introduced to direct measurement using standard units.



Extension and differentiation activities

Learning activity 1: If certain groups struggle to respond to the questions, you may give them an example answer and ask them to brainstorm other responses.

Learning activity 2: If student teachers have difficulty with this exercise, complete one example of each type as a class (for example, identify examples for ‘length’ in all boxes of the diagram), then ask them to complete the diagram in their pairs.

Learning activity 3: You may take the discussion a step further by asking student teachers to give examples of how they can apply the principles of indirect measurement during lesson activities.

Learning activity 4: Make sure student teachers understand the assignment before leaving class.

3.2.2. Length, height and distance

Expected learning outcomes



By the end of the lesson, the student teachers will be able to:

- Demonstrate practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students experience are addressed; and
- Explain how to teach standard units in the Myanmar context using metric units.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentations of lesson ideas; class discussion; teacher educator presentation and facilitation; individual work



Preparation needed: Presentation slides from lecture notes on the key points of the metric system



Resources needed: None



Learning activity 1: Introducing the lesson and group presentations (15 minutes)

1. Provide an overview that the lesson will entail detailed presentations and discussion on their measuring lesson for early grade students.

2. Request each group present, provide comment and guidance and facilitate questions and discussion.
3. Hopefully some of the groups will have included in their lesson the development of measuring instruments. (Metre sticks of bamboo made from a standard could be an example.) Facilitate discussion on what stage to introduce standard measures. Get student teachers to refer to the grade-wise curriculum.



Assessment

Check to what extent student teachers have grasped ideas of teaching measurement in the early grades through assessing the thinking in each of the lesson ideas that are presented from the homework assignment.

Assess student teachers' ability to interpret the grade-wise curriculum based on whether they can identify the stage to introduce standard measures.



Possible student teacher responses

Lessons should be creative and interesting for young students. Hopefully, some of the groups will have included in their lesson the development of measuring instruments. (Metre sticks of bamboo made from a standard could be an example.)

According to the grade-wise curriculum, standard measures should be introduced in Grade 2. Student teachers should be able to identify this in the grade-wise curriculum in their textbook.

Figure 3.22. Grade-wise objectives

		Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Measurement	Basic SI units	Length	Direct comparison of lengths (U9)	Measuring lengths with cm & mm(U7)	Distance with km	Finding perimeter of polygons
			Indirect comparison of lengths (U9)	Relation between cm & mm(U7)	Relation between km & m	
			Measuring lengths with arbitrary units (U9)	Simple calculation with cm & mm (U7)	Simple calculation with km & m	
			Measuring lengths with m & cm (U16)			
			Relation between m & cm (U16)			
			Simple calculation with m & cm (U16)			
	Weight		Comparison of weights of objects with balance (U19)	Measuring weight with kg & g		
				Relation between kg & g		
				Simple calculation with kg & g		
				Relation between \square , $k \square$, $c \square$ & $m \square$		
	Time	Time with o'clock & half past (U13)	Time with o'clock, minutes AM & PM (e.g. 5:48 AM)	Relation between minute & second		
			Time duration with hour, minute (e.g. 2 hours and 27 minutes)(U2)	Time & time duration with minute & second		
			Relations between day & hour, hour & minute (U10)	Finding times & time during by calculation		



Learning activity 2: Introduction to the metric system (10 minutes)

1. Now provide an interactive presentation on the basic metric units, the usefulness of the metric system and on its origins, based on the information in the textbook.



Assessment

Assess student teachers’ understanding of the key points by asking questions after each section of the presentation.



Possible student teacher responses

These are example questions and answers. You may ask any other questions relevant to the text.

1. What are some units for measuring length or distance in the metric system?
 - a. Metres, kilometres, centimetres, millimetres
2. What are some units for measuring mass in the metric system?
 - a. Kilograms, grams
3. What are some units for measuring volume or capacity in the metric system?
 - a. Litres, kilolitres, millilitres
4. How was 1 metre defined?
 - a. The metre was defined as one ten millionth ($\frac{1}{10000000}$) of the distance from the equator to the North Pole.
5. How was 1 litre defined?
 - a. A litre was defined as the volume of a cube with each edge 10 cm long.
6. How was 1 kilogram defined?
 - a. A kilogram was defined as the mass of one litre of pure water (at the melting point of ice).



Learning activity 3: Connections between units (10 minutes)

1. Now provide a presentation on the connection between derived units in the metric system and the connection between these units and the base 10 place value system that we use. Demonstrate with presentation slides.
2. Continue the presentation on how the other metric units are derived from the base units. Present the table of metric prefixes for review.
3. Ask questions to check for understanding.



Assessment

Assess student teachers' understanding of the key points by asking questions after each section of the presentation.



Possible student teacher responses

These are example questions and answers. You may ask any appropriate questions about the content.

1. How other units are derived from base units?
 - b. The units that are bigger or smaller than metres, grams and litres have standard prefixes, for example, kilometres, kilolitres, centimetres, millimetres, millilitres, milligrams and are related to the base unit by being a positive or negative power of 10 of the base unit.
2. What are examples of some units derived from metres?
 - c. Millimetres, centimetres, and so on.
3. What are examples of some units derived from litres?
 - d. Millilitres, decalitres, and so on.



Learning activity 4: Create a worksheet (10 minutes)

1. Provide explanation to the unit conversion system.
2. Request student teachers review the example conversion problems in the textbook and then use these as a template. Each student should create a worksheet with five units to convert.
3. Collect the worksheets and redistribute them to different student teachers. Provide time for them to complete and check answers.



Assessment

Assess student teachers' understanding of the use and meaning of prefixes and of conversions of units based on the worksheet they create and the worksheet they complete.



Possible student teacher responses

Student teachers' worksheets should appear similar to the example problems in the textbook. Some example problems are provided below, with the completed tables and answers.

km	hm	Dm	m	dm	cm	mm

Use the table to convert the following:

1. Convert 4 m to cm.

km	hm	Dm	m	dm	cm	mm
			4	0	0	

$$4 \text{ m} = 400 \text{ cm}$$

2. Convert 3 km to m.

km	hm	Dm	m	dm	cm	mm
3	0	0	0			

$$3 \text{ km} = 3000 \text{ m}$$

3. Convert 8 km to cm.

km	hm	Dm	m	dm	cm	mm
8	0	0	0	0	0	

$$8 \text{ km} = 800,000 \text{ cm}$$



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Have practical ideas about how to teach measurement of length, height and distance to ensure that the difficulties that students experience are addressed; and
 - Have some perspective of how to teach standard units in the Myanmar context using metric units.
2. Ask student teachers to turn to their partner and explain a method they would use to teach the metric system in their classrooms.
3. Remind student teachers that they explored how to teach measurement with the metric system in this lesson. An understanding of how the units relate to each other help students to understand how to convert between units.



Extension and differentiation activities

Learning activity 1: Review the grade-wise curriculum if needed. It is important that student teachers build their familiarity with this document.

Learning activity 2: Make sure student teachers understand the metric system well and spend a few more minutes reviewing it if needed.

Learning activity 3: Make sure student teachers understand the connection between different units in the metric system well and spend a few more minutes reviewing it if needed.

Learning activity 4: If student teachers have difficulty writing a worksheet, you may allow them to work in pairs to create the worksheet and trade with another pair of student teachers to solve their problems.

3.2.3. Weight

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain what are the different measurement systems for weight;
- Demonstrate practical ideas to teach measurement of weight;
- Develop practical tools for primary students to establish standard weights, to measure and to compare; and
- Propose ideas about how to convey the concept of density of materials.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation and discussion; ‘work stations’; individual work; group discussion



Preparation needed: Ensure the resources for the work stations are available



Resources needed: A short video on density: <https://vitalmaths.com/videos/all-videos/item/166-density>. Materials for weight work stations:

For work station 1: A balancing scale; a litre container of water; some weights; some groceries (with weight labels).

For work station 2: (Empty) containers of household goods of different weights between 20g and 1kg.

For work station 3: Different kinds of scales and some part-filled packets and containers of common household goods.



Learning activity 1: Introducing the lesson (10 minutes)

1. Provide an introduction to the use of work stations using the information in the textbook.
2. Provide an overview of the four stages children go through when learning to measure.
3. Identify the attribute of mass and weight that is different from length, area, capacity and volume.
4. Show the short video on density.
5. Ask student teachers to explain density in their own words.



Assessment

Assess student teachers based on how well they can describe density in their own words.



Possible student teacher responses

1. Density describes how compact a substance is.
2. Density is the amount of matter that is present in a given volume.
3. Density is calculated by dividing mass by volume.



Learning activity 2: Work stations (30 minutes)

1. Request that groups work through the work stations. Each group should experience each work station.
2. Facilitate the flow between each station.



Assessment

Walk around to assess student teachers' understanding as they complete the work stations.



Possible student teacher responses

Work station 1: Student teachers should find that the weight of 1 litre of water is about 1 kg.

Work station 2: The answers depend on the items you brought to class, with the exception to the following answers to part c:

- i. How many grams in a kilogram? 1000 g
- ii. How many grams in half a kilogram? 500 g
- iii. How many grams in a quarter of a kilogram? 250 g

Work station 3: The answers depend on the items you brought to class.



Learning activity 3: Assignment (5 minutes)

1. Ask student teachers to work in their groups to create balancing scales from available resources.



Assessment

You may assess student teachers' work by asking them to demonstrate their scale during the next lesson.



Possible student teacher responses

Student teachers' scales should work effectively. For example, the heavier object should sink down while the lighter object rises.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Have practical ideas to teach measurement of weight including choosing and reading scales;
 - Have practical ideas about how to teach the relationship between grams and kilograms;
 - Have practical tools for students to measure and to compare weights;
 - Have ideas about how to convey the concept of density of materials; and
 - Have ideas about how to practically teach the relationship between litres and kilograms of water.
2. Ask student teachers to turn to their partner and explain a method they would use to teach weight in their classrooms.
3. Remind student teachers that they explored work station activities designed for primary school students to help them explore and discover concepts. These activities provided student teachers with the learning opportunity to think about what students have to understand and what they are likely to find difficult or new.



Extension and differentiation activities

Learning activity 1: Make sure student teachers can understand and differentiate between mass, weight and density. Spend a few minutes explaining if needed.

Learning activity 2: Explain each work station well before the activity. If you find that a group still has difficulty understanding an activity, walk over and explain it to them again.

Learning activity 3: You may ask volunteers to share some ideas of materials that can be used for the scale (for example, bags or cups, a stick, string).

3.2.4. Volume

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain the measurement systems for volume that are used in Myanmar;
- Explain the approaches to teach the measurement of volume of liquids;
- Describe practical tools to make and mark out for measurement; and
- Explain how best to teach volume of three-dimensional spaces.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation and discussion; ‘work stations’; individual work; group assignment



Preparation needed: Ensure the materials for work stations are available



Resources needed: Materials for volume work stations:

For work station 1: Containers with different capacities: 1 litre, 500 ml, 330 ml and 200 ml; a jug that show litres and a jug that show millilitres; water and a funnel.

For work station 2: Common household goods with different capacities.

For work station 3: Jugs; measuring cylinders; beakers for measuring capacity / volume.



Learning activity 1: Introducing the lesson (5 minutes)

1. Provide a summary of the lesson covering a short discussion on conversion of units, a further set of work stations, this time investigating volume and an assignment to design a set of work stations to teach these concepts in Myanmar or imperial units in a local context.



Assessment

Ask a few student teachers to explain volume in their own words.



Possible student teacher responses

- Volume is the amount of space that an object occupies.
- Volume is also the amount of substance inside an object (water, cooking oil, petrol).



Learning activity 2: Introduction to imperial units (5 minutes)

1. Present a conversion table of units from metric to imperial units
2. Discuss where these units are used in Myanmar



Assessment

Assess student teachers based on whether they can identify some uses of these units from real life.



Possible student teacher responses

Student teacher responses could include any measure used in Myanmar (metric or imperial).
For example:

- Miles or kilometres for distance travelled by road;
- Metres for a football field; and
- Gallons or litres of fuel purchased for a car.



Learning activity 3: Volume work stations (30 minutes)

1. Introduce volume concept. Describe that the lesson is designed to give student teachers ideas about how to teach the topic.
2. Provide instructions to ensure a smooth process through the work stations.
3. Request that groups work through the work stations. Each group should experience each work station.
4. Facilitate the flow between each station.



Assessment

Assess learning through responses to questions at the work stations.



Possible student teacher responses

Work station 1A:

1. Ensure that groups correctly identify the 1 litre container.
2.
 - a. The 1 litre container has twice the capacity of the 500 ml container.
 - b. The 500 ml container can be filled twice with water from the 1,000 ml container.
 - c. $2 \times 500 \text{ ml} = 1 \text{ litre}$ and $1 \text{ litre} = 1,000 \text{ ml}$.
3.
 - a. The 1 litre container has 3 times the capacity of the 330 ml container.
 - b. The 300 ml container can be filled 3 times with water from the 1,000 ml container.
 - c. There could be a little water left.
 - d. Yes, it confirms 2c.
4.
 - a. The 1 litre container has 5 times the capacity of the 200 ml container.
 - b. The 200 ml container can be filled 5 times with water from the 1000 ml container.
 - c. $5 \times 200 \text{ ml} = 1 \text{ litre}$ and $1 \text{ litre} = 1,000 \text{ ml}$.

Work station 1B:

1. Groups pour water.
2. Groups identify that $1 \text{ litre} = 1,000 \text{ ml}$

Work station 2: Student teachers' responses will depend on the household goods that you brought to the lesson. They should reach these findings:

1. There are 1,000 ml in a litre
2. There are 500 ml in half a litre
3. There are 250 ml in a quarter of a litre

Work station 3: Student teachers' responses will depend on the items that you brought to the lesson.



Learning activity 4: Assignment (5 minutes)

1. Introduce the assignment to be completed for homework.
2. Student teachers should work with their groups to design an activity that teaches volume to primary students.
3. Clarify that creativity is encouraged.



Assessment

The teacher educator will be able to assess learning through the quality of the lesson designs presented by groups.



Possible student teacher responses

Student teachers may be creative in designing the lesson activity. Make sure it is appropriate for primary students and applies what student teachers have learned in this lesson.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Be aware of the measurement systems that are used in Myanmar;
 - Have practical ideas to teach the measurement of volume of liquids;
 - Have ideas of practical tools to make and mark out for measurement; and
 - Have ideas about how best to teach volume of three-dimensional spaces.

2. Ask student teachers to turn to their partner and explain a method they would use to teach volume in their classrooms.
3. Remind student teachers that they explored work station activities designed for primary school students to help them explore and discover concepts. These activities provided student teachers with the learning opportunity to think about what students have to understand and what they are likely to find difficult or new.



Extension and differentiation activities

Learning activity 1: Make sure student teachers can understand the concept of volume. Spend a few minutes explaining if needed.

Learning activity 2: If student teachers are unfamiliar with any of the metric or imperial units, explain them using examples from everyday life in Myanmar.

Learning activity 3: Explain each work station well before the activity. If you find that a group still has difficulty understanding an activity, walk over and explain it to them again.

Learning activity 4: You may ask volunteers to share some ideas of the features of a good lesson activity on volume in class, before student teachers complete the assignment in groups.

3.2.5.

Measuring length in taung and htwa, weight in peithar and kyattha

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Identify in what contexts these Myanmar units of length and weight are used; and
- Adapt a number of the work station lessons from the previous lessons and developed some others to be used to teach measurement with these units.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Group work, developing and testing work station lessons



Preparation needed: Ensure the materials for work station are available



Resources needed: A balancing scale; a container of water; some weights; some instruments to measure length; some groceries; empty containers of household goods of different weights and shapes; different kinds of scales and some part-filled packets and containers of common household goods; conversion table of Myanmar units



Learning activity 1: Introducing the lesson (5 minutes)

1. Provide a summary of the lesson covering a short discussion on conversion of units, a further set of work stations, this time investigating an assignment to design a set of work stations to teach these concepts in Myanmar units in a local context.



Assessment

Assess student teachers understanding of the use of work stations by asking them to describe the work stations from the previous lesson and why these are useful for students.



Possible student teacher responses

In the previous lesson, student teachers worked with containers of different capacities to compare and understand volume. Through this activity, students should understand and be able to compare millilitres and litres.



Learning activity 2: Introduction to Myanmar units (10 minutes)

1. Present a conversion table of Myanmar units in length and weight.
2. Discuss where these units are used in Myanmar.



Assessment

Assess student teachers based on whether they can identify some uses of these units from real life.



Possible student teacher responses

Student teacher responses could include any instance in which these units could be used in Myanmar. For example:

- ‘*Taung*’ is used to measure length, for example when building a house
- ‘*Htwa*’ is used to measure length as well
- ‘*Peithar*’ is used to measure weight, for example a person’s body weight
- ‘*Kyattha*’ is used to measure weight as well



Learning activity 3: Developing work stations (30 minutes)

Developing work station lessons for Myanmar measures

1. Instruct student teachers to work in their groups to begin an assignment to be completed for homework and presented at the next lesson.
 - You are teaching in a rural school in Myanmar. Develop a set of 5 work stations for lessons for either length or weight. These can model the examples presented in the previous lessons but can also be your own creation. This time use Taung and Htwa for length and Peithar and Kyattha for weight.
 - The activities at the work stations should take consideration of the level of students and what you have learned about how children learn to measure.
 - Imagine what resources will be available in this setting.
 - The measurements must be realistic and the exercises must use the Myanmar units. They must teach measurement of length or weight and not be focused on conversion from one system to another.

Table 3.7. Table of length units

Table of length units				
Unit		Metric	Imperial/US	Ratio to previous
Burmese	Romanised			
ဆံခြည်	<i>sanchi</i>	79.375 μm	$3 \frac{1}{8}$ thou/mil	
နှမ်း	<i>hnan</i>	0.79375 mm	$31 \frac{1}{4}$ thou/mil	10
မုယော	<i>mayaw</i>	4.7625 mm	$\frac{3}{16}$ in	6
လက်သစ်	<i>let thit</i>	1.905 cm	$\frac{3}{4}$ in; one digit	4
မိုက်	<i>maik</i>	15.24 cm	6 in; one shaftment	8
ထွာ	<i>htwa</i>	22.86 cm	9 in; one span	1.5
တောင်	<i>taung</i>	45.72 cm	$1 \frac{1}{2}$ ft; one cubit	2
လံ	<i>lan</i>	1.8288 m	6 ft; one fathom	4
တာ	<i>ta</i>	3.2004 m	$10 \frac{1}{2}$ ft	1.75
ဥသဘ	<i>out-thaba</i> (from Pali usaba)	64.008 m	70 yd	20
ကောသ	<i>kawtha</i> (from Pali kosa)	1.28016 km	0.795455 mi	20
ဂါဂုတ်	<i>ga-wout</i> (from Pali gavuta)	5.12064 km	3.18182 mi; about one league	4
ယူဇနာ	<i>yuzana</i> (from Pali yojana)	20.48256 km	12.7273 mi	4

Table 3.8. Table of mass units

Table of length units				
Unit		Metric	Imperial/US	Ratio to previous
Burmese	Romanised			
ရွဲလေး	<i>yway lay</i>	136.078 mg	2.1 grain	
ရွဲကြီး	<i>yway gyi</i>	272.155 mg	4.2 grain	2
ပဲသား	<i>petha</i>	1.02058 g	15.75 grain	3.75
မူးသား	<i>mutha</i>	2.04117 g	31.5 grain	2
မတ်သား	<i>mattha</i>	4.08233 g	63 grain	2
ငါးမူးသား	<i>nga mutha</i> ⁷	8.16466 g	0.288 oz	2
ကျပ်သား	<i>kyattha</i> ⁸	16.3293 g	0.576 oz	2
အဝက်သား	<i>awettha</i>	204.117 g	7.2 oz	12.5
အစိတ်သား	<i>aseittha</i>	408.233 g	14.4 oz	2
ငါးဆယ်သား	<i>ngase tha</i>	816.466 g	1.8 lb	2
ဝပိသာ	<i>peittha</i> ⁹	1.63293 kg	3.6 lb	2
အချိန်တစ်ရာ	<i>achein taya</i>	163.293 kg	360 lb	100



Assessment

Assess student teachers based on the work stations that they present.

⁷ Literally “five mutha”, but in fact it is only four times one mutha.

⁸ Traditionally known as *tical* in English.

⁹ Traditionally known as *viss* in English.



Possible student teacher responses

Student teachers' work stations should be appropriate for primary students and should build the concept of either length or weight.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Have identified in what contexts these Myanmar units of length and weight are used; and
 - Have adapted a number of the work station lessons from the previous lessons and developed some others to be used to teach measurement with these units.
2. Ask student teachers to turn to their partner and explain how work stations are useful in primary classrooms.
3. Remind student teachers that they developed work station activities designed for primary school students to help them explore and discover measurement. They should complete these activities with their groups as homework and be ready to present them.



Extension and differentiation activities

Learning activity 1: Make sure student teachers can understand work stations and why they are useful. Spend a few minutes explaining and discussing if needed.

Learning activity 2: If student teachers are unfamiliar with any of the units, explain them using examples from everyday life in Myanmar.

Learning activity 3: Walk around and discuss groups' ideas for work stations with them. Make sure they are on the right track, and support them as needed.

Learning activity 4: You may ask volunteers to share some ideas of the features of a good lesson activity on volume in class, before student teachers complete the assignment in groups.

3.2.6. Area of quadrilaterals

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Explain how to derive the area of parallelograms, rhombuses and trapezia from the area of rectangles;
- Teach Grade 5 students to find the area of parallelograms, rhombuses and trapezia from the area of rectangles;
- Explain the importance of linking topics and formulae when teaching Mathematics; and
- Explain the value of visual images when teaching Mathematics.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Working with diagrams on square grids; working with paper folding and cutting; working with tangrams; watching and interpreting videos



Preparation needed: Be prepared to show the Vital Videos:

Area of a trapezium <https://vitalmaths.com/videos/all-videos/item/180-area-of-a-trapezium>

Area of a rhombus <https://vitalmaths.com/videos/all-videos/item/183-area-of-a-rhombus>



Resources needed: Paper; marker pens; scissors; photocopy copies of tangram sets for each pair of student teachers. Alternatively you could ask student teachers to fold their own set of tangrams: see the following two websites for how to do this

<https://www.uen.org/lessonplan/view/11079>

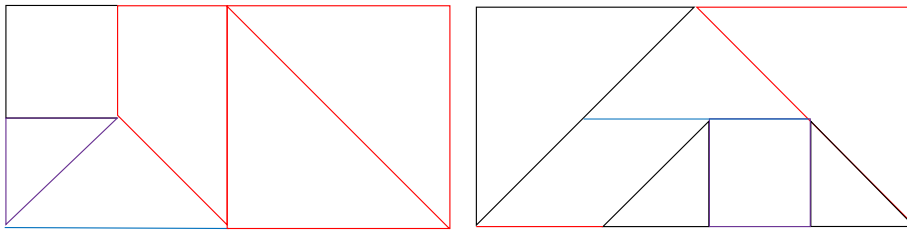
<https://www.youtube.com/watch?v=96nqx2eia0s>



Learning activity 1: Introducing the lesson (20 minutes)

1. Handout a set of tangrams to each pair of student teachers. Two ways to arrange the pieces into a rectangle are shown below.
2. Ask half of the class to reconstruct one of the rectangles and the other half of the class to reconstruct the other rectangle from their tangram pieces.

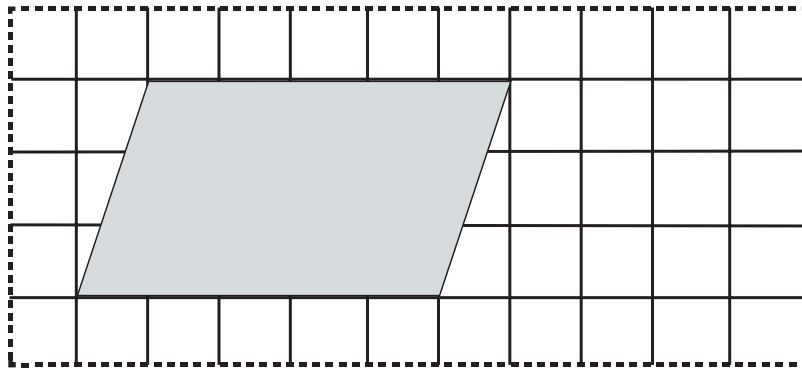
Figure 3.23. Using tangram pieces to make a rectangle (1)



3. Then ask student teachers to move just one piece and change their rectangle into a parallelogram.
4. Ask student teachers why the rectangles and parallelograms have the same area? They need to be able to see that the same pieces are used in all of the shape, it is just one triangle that has been repositioned.
5. Ask student teachers what the formula for the area of a rectangle is?
6. If they state it as length times breadth, confirm this and explain that the sides of a rectangle form right angles with each other, so another way of stating the area of a rectangle is to say it is the base of the rectangles multiplied by the height of the rectangle.

7. Ask student teachers to compare the bases and the heights of the rectangles and parallelograms.
8. Ask for suggestions for the area of a parallelogram. Do not yet explain that the area of a parallelogram is base time \times (perpendicular) height. First let student teachers work with the second example below.
9. Another way to see the relationship between the area of a parallelogram and a rectangle is to show the shapes on a square grid background.
10. Ask student teachers to look at the parallelogram on a grid in their textbook. Explain that each unit is 1 cm. Ask them to find the area of this parallelogram.

Figure 3.24. A parallelogram



11. They should adapt the parallelogram to a shape (with the identical area) whose area they can calculate.



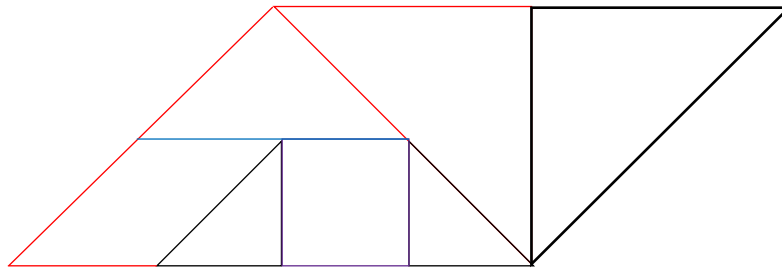
Assessment

Assess student teachers' understanding by watching their work with tangrams and apply their findings to find the area of the parallelogram in their textbook.



Possible student teacher responses

Student teachers should rearrange the tangram rectangle to be a parallelogram:

Figure 3.25. Using tangram pieces to make a rectangle (2)

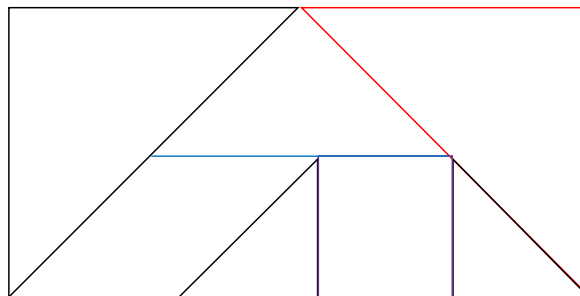
Student teachers should identify the following during the activity:

1. The area of a rectangle is length x breadth or base x height.
2. With the paper parallelogram, student teachers should show that they can cut a triangle off one end of the parallelogram and move it to the opposite end of the parallelogram to make a rectangle with the same area.
3. The area of the parallelogram in the textbook is $6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$.
4. The area of a parallelogram is also base x height.



Learning activity 2: from areas of rectangles to areas of trapezia (15 minutes)

1. Ask student teachers to arrange the pieces of their tangram to make rectangles again.

Figure 3.26. Using tangram pieces to make a rectangle (3)

2. Then ask student teachers to move just one piece and change their rectangle into a trapezium.
3. Check again that student teachers understand why these rectangles and the trapezia have the same the area.
4. Ask student teachers to try to write out a formula for the area of the trapezia.
It is important that they engage with the fact that the parallel sides of the trapezia are not the same length. If student teachers are not able to come with a formula, leave it until after they have seen the video clip below.
5. Many trapezia, are not isosceles trapezia. So how can one move from the area of rectangles to area of these trapezia.
6. Show student teachers the following video: <https://vitalmaths.com/videos/all-videos/item/180-area-of-a-trapezium>
7. Ask student teachers to discuss with a partner how they can use the visuals in this video if they don't have ICT in their classroom.
8. Invite student teachers to share their ideas with the class.



Assessment

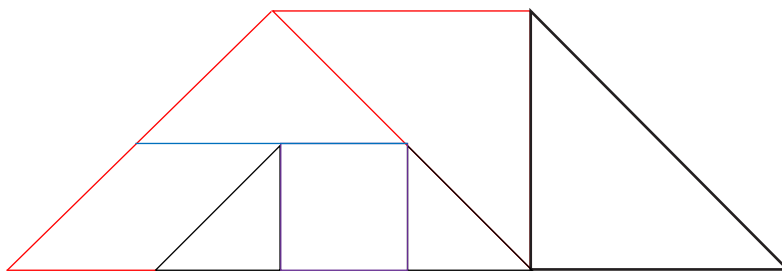
Assess student teachers' understanding by watching their work with tangrams. Ask them to share ideas of how they use the visuals in the trapezoid video if they do not have ICT in their classroom.



Possible student teacher responses

Student teachers should rearrange the tangram rectangle to be a trapezium:

Figure 3.27. Using tangram pieces to make a trapezium



Student teachers should identify that the area of a trapezium is $A = \frac{1}{2} (a + b)h$, where a and b are the parallel sides and h is the height.

Student teachers should identify some ways to present trapezium visuals to primary students. One example is using a paper trapezium, and cutting it apart to show area as it was done in the video.



Learning activity 3: Areas of rhombuses (10 minutes)

1. Ask student teachers to give the definition of a rhombus. Ask if they can guess the formula for area of a rhombus using the information covered in this lesson so far.
2. Show student teachers the following video: <https://vitalmaths.com/videos/all-videos/item/183-area-of-a-rhombus>
3. Ask student teachers to discuss with a partner how they can use the visuals in this video if they do not have ICT in their classroom.
4. Invite student teachers to share their ideas with the class.



Assessment

Assess student teachers' understanding based on their responses.



Possible student teacher responses

1. A rhombus is a parallelogram with 4 equal sides.
2. The formula for area of a rhombus is the same as that of parallelogram: base x height.
3. The rhombus has another formula for its area: $\frac{1}{2} \times$ (product of diagonals)
4. Both formulas for area could be shown in a classroom using a paper rhombus, cutting it into parts and piecing it together as a rectangle.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand how to derive the area of parallelograms, rhombuses and trapezia from the area of rectangles;
 - Teach Grade 5 students to find the area of parallelograms, rhombuses and trapezia from the area of rectangles;
 - Know the importance of linking topics and formulae when teaching Mathematics; and
 - Understand the value of visual images when teaching Mathematics.
2. Ask student teachers to turn to their partner and explain an effective method of teaching area in a primary classroom.
3. Remind student teachers that they discussed how to teach the area of shapes using tangrams, videos, diagrams and paper shapes. All of these can be effective methods of teaching area in a primary classroom.



Extension and differentiation activities

Learning activity 1: If student teachers complete the activity and have difficulty understanding the relationship between the area of a parallelogram and rectangle, you may demonstrate and explain again using a tangram or paper parallelogram and rearranging it to a rectangle.

Learning activity 2: If student teachers complete the activity and have difficulty understanding the relationship between the area of a trapezium and rectangle, you may demonstrate and explain again using a tangram or the video.

Learning activity 3: If student teachers complete the activity and have difficulty understanding the relationship between the area of a rhombus and rectangle, you may demonstrate and explain again using a paper rhombus or the video.

3.2.7.

Angle measure part 1

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Teach the dynamic concept of angle as an amount of turning; and
- Plan lessons on angle measurement that minimise common student misconceptions about angles.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1 Demonstrate capacity to teach subject-related subject concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Discussion; practical; reading



Preparation needed: Ask student teachers to read this section of the textbook before the lesson



Resources needed: Sticks



Learning activity 1: Introducing the lesson – common misconceptions (25 minutes)

1. Divide student teachers into four groups.
2. Ask student teachers to read the section 'Common misconceptions around angle measure' in the textbook. There are four common misconceptions, labelled a-d.

- Each group should prepare give a presentation about one of the misconceptions. Their presentations should describe the student’s misconception and what their response would be as a teacher.



Assessment

Assess student teachers’ understanding of the misconceptions and their ability to address these based on their presentations.

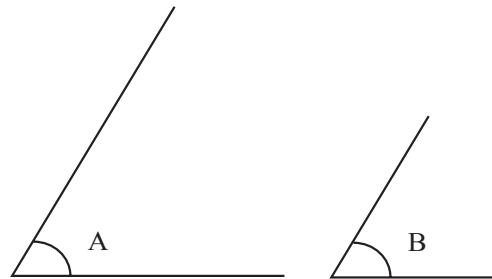


Possible student teacher responses

- 'Angle B is the smaller angle.'

Here the student assumes that if the arms of the angle are longer, then the angle is bigger. Student teachers can show that these angles are equal by tracing one angle and then superimposing this on the other angle to confirm that they are the same size. Alternatively, they could measure the angles.

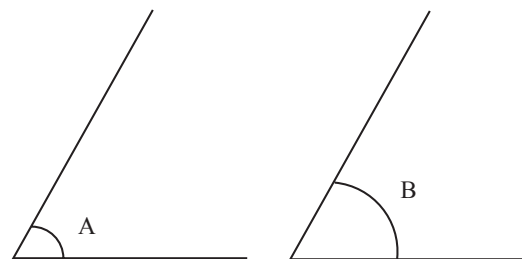
Figure 3.28. Comparing angles (1)



- 'Angle B is the larger angle.'

Here the student assumes that if the area inside the arc is bigger, then the angle is bigger. Student teachers can show that these angles are equal by tracing one angle and then superimposing this on the other angle to confirm that they are the same size. Alternatively, they could measure the angles.

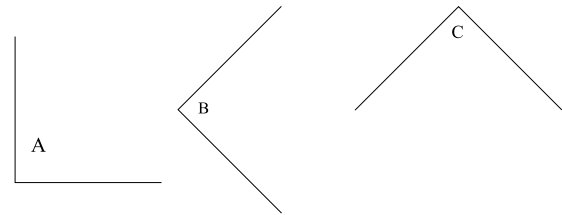
Figure 3.29. Comparing angles (2)



c. 'Angle A is the only right angle.'

Here the student can only recognize a right angle in one orientation. He /she wrongly assumes that a right angle can only face in one direction. Student teachers can show that these angles are equal by tracing one angle and then superimposing this on the other angle to confirm that they are the same size. Alternatively, they could measure the angles.

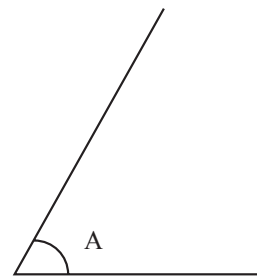
Figure 3.30. Comparing angles (3)



d. 'Angle A measures 130° '

Student teachers might say that the student has read the degrees off the wrong arc. However, the more important issue is that the student has not considered that the angle is less than 90° , so it cannot be 130° . It is important that students can estimate the size of angles relative to key angles such as right angles, straight angles and revolutions.

FIGURE 3.31. Angle A

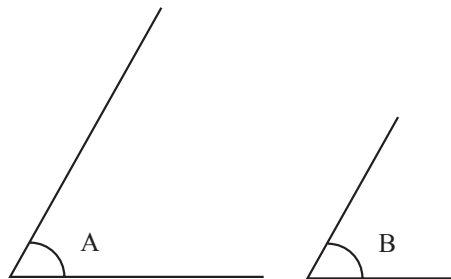


Learning activity 2: Dynamic view of angles (20 minutes)

1. Explain to student teachers that there are two ways of viewing angles, one is a static view and one is a dynamic view. Remind student teachers that students will work with right angles from Grade 2. They work with the static notion of right angles.
2. Give student teachers this definition of angle: 'Given two intersecting lines or line segments, the amount of rotation about the point of intersection (the vertex) required to bring one into correspondence with the other is called the angle between them.' (<http://mathworld.wolfram.com/Angle.html>)
3. Demonstrate this definition using two sticks and rotating the one so that it is superimposed upon the other one.

4. In this activity, either work with student teachers in a passage, a large hall or outside. Student teachers will turn their bodies as an introduction to a dynamic view of angle: to focus on angle as an amount of turning. It is an activity which can be done with students to help them to develop a feel for the size of angles.
5. Ask student teachers to line up in two rows at arms-length from each other. Student teachers should turn to face a partner in the opposite row. Explain that you will give instructions for one of each pair to move but that BEFORE he / she moves he/ she should tell you where his / her hands will point after the turn.
 - a. Partner 1 holds her / his hands out in front of them, pointing to partner 2. Partner 1 makes a full turn. Predict first before turning. Where will her / his hands point at the end of the turn? Turn to check.

Figure 3.32. Angle A and B



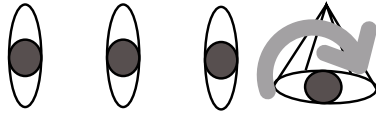
- b. Partner 2 holds her / his hands out in front of them, pointing to partner 1. Predict first before turning. If partner 2 makes a half turn, what part of her / his body will be facing partner 1? Turn to check.

Figure 3.33. Turning angles (1)



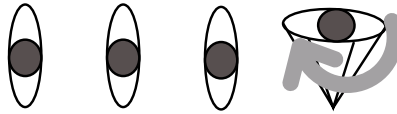
- c. Partner 1 holds her / his hands out in front of them, pointing to partner 2. Predict first before turning: if partner 1 makes a quarter turn to the left, what part of her / his body will be facing partner 2? Turn to check.

Figure 3.34. Turning angles (2)



- d. Partner 2 holds her / his hands out in front of them, pointing to partner 1. Predict first before turning: if partner 2 makes a quarter turn to the right, what part of her / his body will be facing partner 1? Turn to check.

Figure 3.35. Turning angles (3)



6. Ask three student teachers to volunteer. One person to ‘turn the angles’ and two people to mark out the outer points of the angle. The rest of the class will draw the angles each time.
- All three volunteers take up a position. The ‘turner’ holds her arms out so that they point to one ‘end’ of the angle. She then turns to point to the other ‘end’ of the angle.
7. The rest of the class draws angle 1.
- The turner then takes a long stick. She repeats the movement holding the big stick outstretched. Has the size of the angle changed?

8. The rest of the class draws angle 2.

- The instructions below are given to the people at the ‘end points of the angle’.
After each set of instructions
 - The other student teachers first predict how the size of the angle will change.
 - Then the turner moves.
 - Then the other student teachers draw the angle.
- Take four steps away from the turner.
- Take six steps towards the turner.
- One moves nearer, the other moves further away.
- One takes 4 steps to the right
- The other takes 2 steps to the left.



Assessment

Watch student teachers as they perform the exercise to assess their understanding of the exercise and the dynamic concept of angles.



Possible student teacher responses

These body turns aim to develop students understanding of factors that affect the size of an angle. For example, the length of the arms of the angle does not affect the size of the angle. When the people at the end points move away from the turner, the size of the angle might or might not change. It depends whether they move along the ‘arms’ or not. See the illustration below.

Figure 3.36. Size of angles

The figure consists of three vertically arranged diagrams, each showing a change in an angle's size. In each diagram, a central figure (a smiley face or a frowny face) is connected to two other figures (stick figures) by dashed lines representing the sides of an angle. The vertex of the angle is marked with a black dot.

Top Diagram: Shows two identical angles. The left angle has a smiley face vertex and two stick figure sides. The right angle also has a smiley face vertex and two stick figure sides. The sides of the right angle are longer than those of the left angle, but the angle's size is the same.

The size of the angle does not change.
 Check this by tracing over angle on left hand side and superimposing it on the angle on the right-hand side.

Middle Diagram: Shows two angles with frowny face vertices. The left angle has two stick figure sides. The right angle also has two stick figure sides, but the vertex is closer to the sides, making the angle smaller.

The size of the angle changes: the angle gets smaller.
 What will happen if the student teachers walk towards the turner?

Bottom Diagram: Shows two angles with frowny face vertices. The left angle has two stick figure sides. The right angle has two stick figure sides, but the vertex is further from the sides, making the angle larger.

The size of the angle changes: the angle gets bigger.
 What will happen if the other student walks some paces to his left?

In explaining their predictions through drawing and discussions, student teachers will need to think about their mathematical reasoning more consciously. This provides you with the chance to focus on the mathematically important aspects of angle. It also allows you to develop student teachers' mathematical reasoning ability and to develop their mathematical language.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Teach the dynamic concept of angle as an amount of turning; and
 - Plan lessons on angle measurement that minimise common student misconceptions about angles.
2. Ask student teachers to turn to their partner and explain the dynamic definition for angle and how they would demonstrate this definition to primary students.
3. Remind student teachers that they explored a more dynamic definition of angle, which is based on the amount of turning. They demonstrated this definition using a pair of sticks, an activity that could be done with primary students.
4. If they have not already done so, ask student teachers to read all of Lesson 3.2.7 in the Student Teacher Textbook.



Extension and differentiation activities

Learning activity 1: If student teachers have difficulty identifying ways to address the misconceptions among primary students, you may help them by asking guiding questions or giving example responses.

Learning activity 2: If student teachers have difficulty completing this activity, demonstrate a few steps yourself first, using a student teacher as a partner.

3.2.8.

Angle measure part 2

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Name angles;
- Link dynamic and static notions of angles; and
- Teach Grade 4 students how to use a protractor.



Competencies gained: A4.1. Demonstrate understanding of the structure, content and expected learning outcomes of the Basic Education Curriculum; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Paper folding; using angle turners; drawing and labelling angles; designing lessons on using a protractor



Preparation needed: Ensure the resources below are available



Resources needed: You will need for each pair of students: Pieces of scrap paper to fold, 2 straws and a pin or 2 strips of paper and either a drawing pin or a split pin

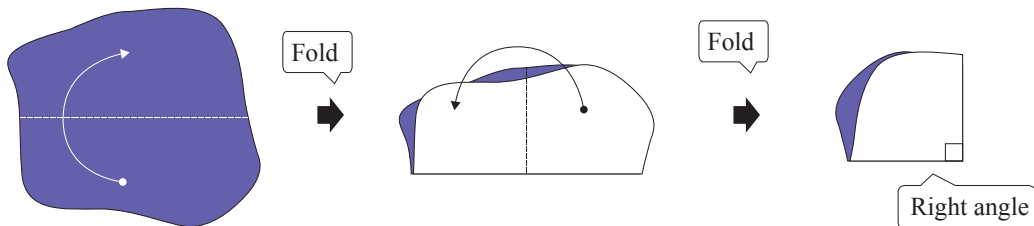


Learning activity 1: Introduction to the right angle as a reference angle (10 minutes)

1. When measuring, it is always useful to have some reference measures.
When measuring angles, there are three angles that are useful reference points, right angles, straight angles and revolutions.
2. Hand a piece of paper to each student. The paper should be unlined and without any right-angled corners.
3. Ask student teachers to work in pairs and following the instructions from page 80 of the Grade 2 Maths textbook to fold a right angle.

Figure 3.37. Folding a right angle

Let us fold a paper of any shape.



The corner you have made is called a **right angle**.

4. Ask student teachers to unfold the right angle to show a straight angle.



Assessment

Ask a few student teachers to describe 'right angle' in their own words.



Possible student teacher responses

A right angle is 90 degrees. It is formed by perpendicular lines. It looks like the corner of a book or a room.

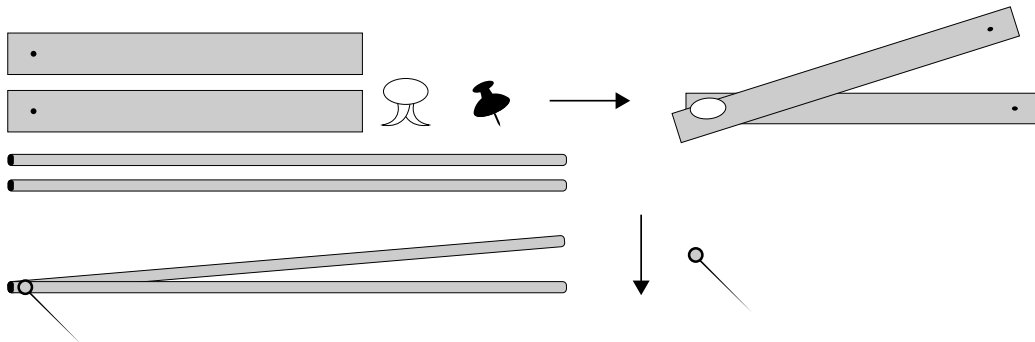


Learning activity 2: Linking dynamic and static angles (20 minutes)

1. Ask student teachers to work in pairs. Each pair of student teachers should make their own angle turner. These can be made from: two strips of cardboard or paper joined with a split pin or drawing pin or two straws joined with a pin.
2. Student teachers should work in pairs. They should take turns to make the angles described in part 2 of the textbook with their angle turner.
3. The one student shows the angle; the other student names the angle. Then they swap. Both student teachers in the pair should draw and label each angle using the terms in part 3.

Steps for making an angle turner:

Figure 3.38. An angle turner (1)



Assessment

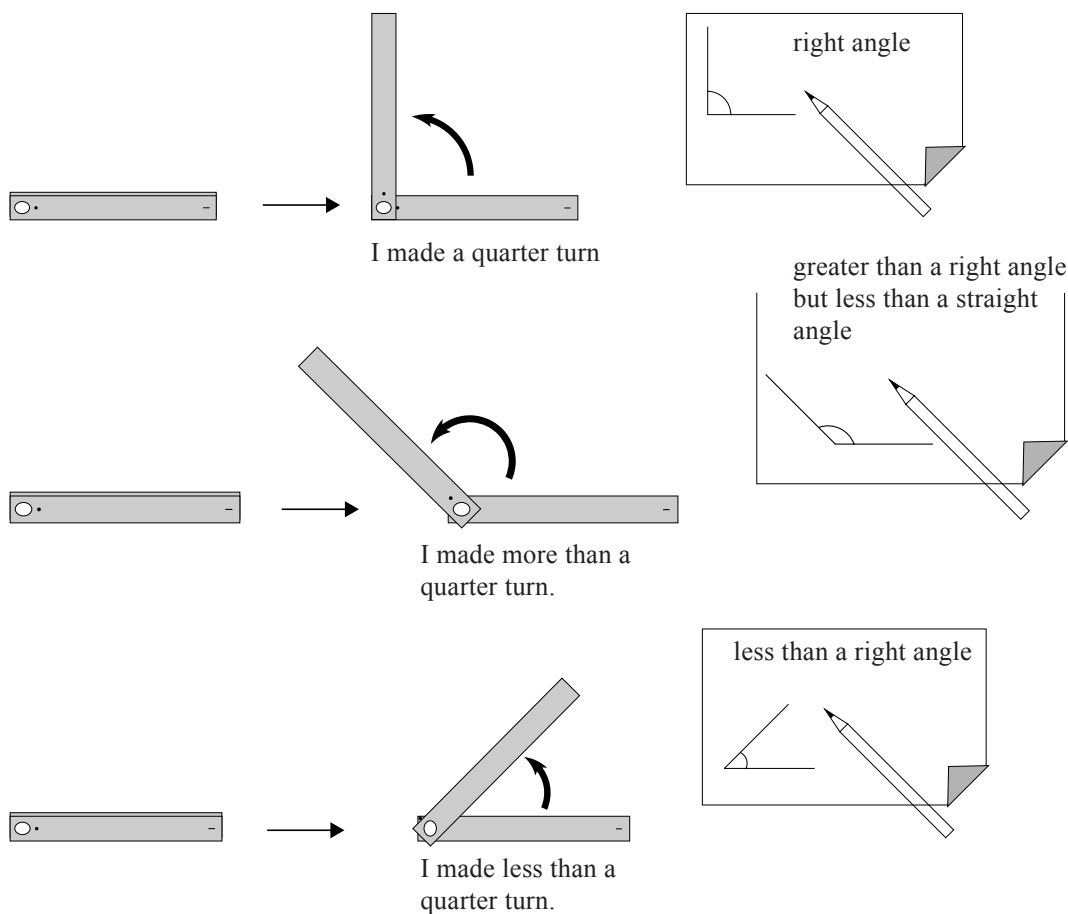
Assess students by observing them as they work in pairs.



Possible student teacher responses

Example responses:

Figure 3.39. An angle turner (2)



Learning activity 3: Linking names of angles with degrees (10 minutes)

1. Check whether student teachers know the names of angles.
 - Check that student teachers know that a full turn or revolution is 360° .
 - Let student teachers deduce how many degrees in a straight angle and a right angle.
 - Let them list the possible angle sizes of acute angles, obtuse angles and reflex angles.
2. Talk about the importance of students estimating angle sizes before measuring angles with protractors.



Assessment

Assess student teachers' understanding of angles based on their responses to questions.



Possible student teacher responses

These are some example questions and answers:

1. How many degrees are in a straight angle? (180°)
2. How many degrees are in a right angle? (90°)
3. What are the possible sizes of an acute angle? (more than 0° and less than 90°)
4. What are the possible sizes of an obtuse angle? (more than 90° and less than 180°)
5. What are the possible sizes of an reflex angle? (more than 180° and less than 360°)



Learning activity 4: Assignment (5 minutes)

Ask student teachers to work in their groups and design a short lesson to teach Grade 4 students to use protractors to read angles.



Assessment

Assess student teachers based on the lesson activities that they present.



Possible student teacher responses

Student teachers' work stations should be appropriate for 4th grade students and should build on their background knowledge of angles to use a protractor.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Name angles;
 - Link dynamic and static notions of angles; and
 - Teach Grade 4 students how to use a protractor.
2. Ask student teachers to turn to their partner and explain what they have learned about teaching primary students angles.
3. Remind student teachers that they explored the dynamic definition of angle in the previous lesson, and tied it to the static notion of angles in this lesson. It is best to teach students the dynamic definition first, then teach them the angle labels (such as 'acute'). Students should only be introduced to a protractor after they have a good understanding of these basic concepts.



Extension and differentiation activities

Learning activity 1: Make sure student teachers can identify right angles and explain to students what they look like. Revise as a class if needed.

Learning activity 2: Walk around and help student teachers as they work in pairs if needed. If you find that several pairs have difficulty with part of the activity, go over it as a whole class.

Learning activity 3: If student teachers need to refresh their knowledge of angles, you may draw a few angles on the board and have them identify the type (acute, obtuse, right, straight, reflex).

Learning activity 4: Make sure student teachers understand the assignment before leaving class.

**Review questions: Possible student teacher responses**

Question 1: What are some appropriate methods to introduce measurement to a Grade 1 primary student?

Answer: According to the grade-wise curriculum, students do not cover standard units in Grade 1. They may compare lengths of different objects, or measure length using arbitrary units such as pencil lengths.

Question 2: What are some appropriate methods to teach units of measurement to a Grade 2 primary student?

Answer: According to the grade-wise curriculum, students begin learning units in Grade 2. They should be introduced to them in a concrete way. For example, they can explore volume by comparing the amount of water held in containers of different sizes.

Question 3: How should angles be introduced to primary students at first?

Answer: They should be introduced to the dynamic definition first which views an angle as a rotation. After that they can be introduced to the static definition.

3.3. The language of Mathematics

Three lessons are included in this sub-unit. In the first lesson, we consider the language we use in Mathematics and how to ensure that students are not intimidated or alienated by language. Most often problems are posed in language even if it is an instruction like ‘simplify’. The lesson explores the kinds of problems that primary school students need to work with and categorises them. The second lesson focuses on problem-solving up to Grade 3. Through looking at sample problems from summative Grade 3 assessments, student teachers will be able to work with these kinds of problems for learning in the class or for assessment. The third lesson provides for more practice in using these kinds of questions but at the Grade 5 and 6 levels.

3.3.1.

Language of Mathematics, learning of Mathematics and problem-solving in the early grades

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Describe the importance of considering the language of Mathematics particularly when teaching early grade students;
- Explain the levels through which Mathematical learning develops for young students through the early grades; and
- Explain the problem types that are useful to help early grade students develop problem-solving skills.



Competencies gained: A1.1. Demonstrate understanding of different theories of how students learn relevant to their age; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation; individual work; group discussion



Preparation needed: A PowerPoint presentation covering the key points could be useful



Resources needed: None



Learning activity 1: Introducing the lesson (10 minutes)

1. Highlight that in this lesson, we will be focused on early grade students and on mathematical language development, levels of mathematical understanding and types of problems that are helpful for developing their problem-solving skills. The lesson will be in the format of a lecture presentation and discussion.
2. Ask student teachers to turn and discuss this question with a partner: Why is it important for students to develop mathematical language skills?



Assessment

Walk around and listen to student teachers as they discuss with their partner.



Possible student teacher responses

There are no incorrect/correct answers. This assesses student teachers' current understanding of the importance of developing mathematical language skills:

- Mathematics has special words that students must know to understand and apply Mathematics.
- Mathematics is useful in daily life, and the words are often used in everyday language (for example, 'plus' and 'times').



Learning activity 2: Presentation on the language of Mathematics (15 minutes)

1. Present the key points of the language of Mathematics, using the information in the textbook.
2. Check for understanding by posing questions for discussion
3. Particularly raise a discussion around language and Mathematics in Myanmar.



Assessment

Pose some questions for discussion. For example:

- Why is it important to introduce the key words for a topic?
- What do you think the challenges are with Mathematics language development for primary students in Myanmar?



Possible student teacher responses

Importance of introducing key words:

- Students need to learn the meaning of words, and they often struggle with Mathematics words because they are in English or they are not used in everyday life.
- Some everyday words have a different, mathematical meaning.

Potential challenges:

- Most students do not speak English, and the Mathematics terms are English.
- There are a lot of Mathematics words to learn.
- Students do not read well yet, and need practice reading ‘story sums’.



Learning activity 3: Developing mathematical understanding (20 minutes)

1. Request that student teachers read the section ‘Developing mathematical understanding in the early grades’
2. Ask student teachers to work with a partner to identify the key points of the text.
3. Ask each pair to present a key point.
4. Lead a discussion on the highlights of this section.



Assessment

Assess student teachers by listening to the key points they present.



Possible student teacher responses

1. The same Mathematics topics occur repeatedly in the early grades, but students' confidence increases and they must use more advanced strategies to solve problems.
2. Number concept develops over the course of the early grades.
3. Learners progress from solving simple addition problems with symbols, to being able to solve addition problems by counting, to adding abstractly, and finally to adding 2-digit numbers.
4. The teacher can help students develop number sense by designing appropriate activities and encouraging different solution methods.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand the importance of considering the language of Mathematics particularly when teaching early grade students;
 - Understand the levels through which Mathematical learning develops for young students through the early grades; and
 - Understand the problem types that are useful to help early grade students develop problem-solving skills.
2. Ask student teachers to turn to their partner and explain what they have learned about Mathematics teaching and learning during the early grades.
3. Remind student teachers that they explored why Mathematics language is important for young students, and how their Mathematics skills are built over the course of their study in the early primary years.
4. Request that student teachers study the 'Problem types for Grades 1 to 3' in preparation for the exercise in the next lesson.



Extension and differentiation activities

Learning activity 1: It is okay if student teachers do not yet have meaningful responses to the posed question. This topic will be explored in the next activity of the lesson.

Learning activity 2: You may ask student teachers to share concrete ideas of how they will build the Mathematics vocabulary of their students.

Learning activity 3: You may ask student teachers to share concrete ideas of how they will design activities to build their students' number concept.

3.3.2. Problem-solving up to Grade 3

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Describe the kinds of problem-solving questions that are useful to extend mathematical thinking for students to Grade 3;
- Categorise problem types and develop similar examples for Grade 3 students; and
- Explain the level Grade 3 students should be able to attain.



Competencies gained: A1.1. Demonstrate understanding of different theories of how students learn relevant to their age; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation; discussion; pair work



Preparation needed: A PowerPoint presentation covering the problem categories



Learning activity 1: Introducing the lesson (5 minutes)

1. Highlight that in this lesson we will focus on early grade students and the categories of problems appropriate up to the Grade 3 level. That student teachers working in pairs will be expected to review a series of problems pitched at Grade 3, to categorise these and to change them to better suit the Myanmar context where necessary. Student teachers will then have to develop a set of six problems at this level.

2. Ask a few questions to assess student teachers' understanding of the assigned reading, 'Problem types for Grades 1 to 3'.



Assessment

Pose some questions for discussion. For example:

- What is the main point of this section?
- What are some things to remember when posing problems to students?
- What are some examples of problem types?
- Which problem types do you think will be most effective in building your students' number sense?



Possible student teacher responses

Main point of this section:

- Students should be introduced to the same types of problems repeatedly during the early grades.

Things to remember:

- Questions should be posed orally.
- Use the same types of problems repeatedly, but change the context and make them more challenging by using bigger numbers.

Examples of problem types:

- Grouping, sharing, proportional sharing, repeated addition, rate, grids, addition and subtraction.



Learning activity 2: Categorising problems (10 minutes)

1. Present the ‘Categorising problems’ table (in Lesson 3.3.2 of the textbook) in some detail.
2. Check for understanding by posing questions for discussion.
3. You may wish to get the student teachers to study this table after you have covered a few examples.



Assessment

Pose some questions for discussion. For example:

- Which grades of instruction is this table aimed at?
- Do you think this table is useful? Why or why not?
- In which grades are ‘summation’ problem types appropriate?



Possible student teacher responses

- The table is for Grades 1-6.
- The table is useful in guiding teachers to create different types of problems that build students’ number concept over time.
- ‘Summation’ type problems are appropriate for all primary grades, but the difficulty of the problem will change.



Learning activity 3: Categorising Grade 3 problems (30 minutes)

1. Request that student teachers in pairs to categorise the Grade 3 problems in the textbook.
2. Explain that they should categorise each problem if they are able. Explain that some of the problems do not fit into any of the categories identified. They should decide if and how the problem can be adapted to a Myanmar context.
3. Explain that they should then develop six Grade 3 level problems. Each of the problems should be in a different category.



Assessment

Assess that student teachers have grasped the key points based on their observations of the ten given problems and the six problems they create.

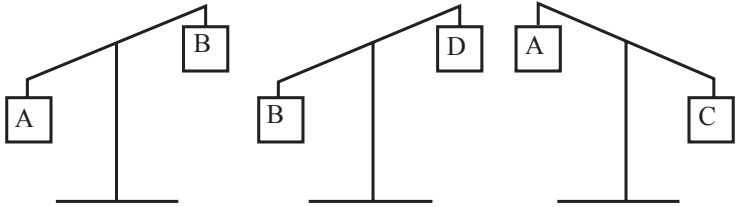
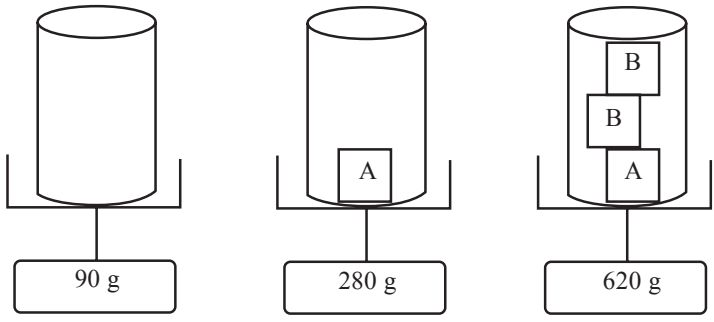








Possible student teacher responses

The problems that are given in the Student Teacher Textbook follow, with the problem types identified. Student teachers may identify different problem types. Their answers may still be accurate but ensure that student teachers are able to defend their decisions. Answers to the first 10 questions are given here. There is no need to cover all 21 questions in class.

Figure 3.40. Problem-solving for Grade 3 students

	Grade 3 problem	Problem Type
1	When a number is divided by 7 or 9, there is a remainder of 2. What is this number? (1) 51 (2) 61 (3) 65 (4) 83	Grouping
2	Mrs Smith has 322 chocolate bars. She puts 8 chocolate bars into each goodie bag. What is the smallest number of goodie bags she will need to contain all the chocolate bars? (1) 41 (2) 40 (3) 5 (4) 4	Sharing
3	String A is longer than String B. String C is the same length as String B. String D is longer than String A. Which is the longest string? (1) A (2) B (3) C (4) D	None of the given problem types

<p>4</p>	<p>Look at the picture carefully.</p>  <p>Which object is the heaviest?</p> <p>(1) A (2) B (3) C (4) D</p>	<p>None of the given problem types</p>
<p>5</p>	<p>Mrs Sim baked some muffins and packed them into boxes for her friends. Each box can hold 6 chocolate muffins and 4 banana muffins. She packed a total of 48 chocolate muffins. How many banana muffins did she pack altogether?</p> <p>(1) 8 (2) 12 (3) 24 (4) 32</p>	<p>Grouping</p>
<p>6</p>	<p>Calvin put 324 sea shells equally into 9 containers. He gave 6 containers of sea shells to Bernard. How many sea shells did Bernard receive?</p>	<p>Sharing</p>
<p>7</p>	<p>Look at the picture below.</p>  <p>(a) Find the mass of one piece of Object A. (b) Find the mass of one piece of Object B.</p>	<p>Comparison by difference</p>

8	<p>In a test, every student had to answer 20 questions. 4 marks were given for every correct answer and 2 marks were taken away for every wrong answer. Samuel answered all 20 questions and scored 44 marks. How many question did he answer correctly?</p> <table border="1" data-bbox="320 389 1046 638"> <thead> <tr> <th>Guess</th> <th>Marks for right questions</th> <th>Marks for wrong questions</th> <th>Marks scored</th> <th>Check</th> </tr> </thead> <tbody> <tr> <td>1</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>2</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>3</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>4</td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Guess	Marks for right questions	Marks for wrong questions	Marks scored	Check	1					2					3					4					None of the given problem types
Guess	Marks for right questions	Marks for wrong questions	Marks scored	Check																							
1																											
2																											
3																											
4																											
9	<p>The sum of two numbers is 500. The bigger number is 268. Find the difference between the two numbers.</p>	Summation																									
10	<p>Sue had \$10. She bought a slice of pie and a carton of milk for lunch. How much money did she have left?</p> <table border="1" data-bbox="308 890 1066 1170"> <tr> <td data-bbox="308 890 663 1170"> <p>Pie</p>  <p>\$5.50</p> </td> <td data-bbox="663 890 1066 1170"> <p>Carton of milk</p>  <p>\$2.40</p> </td> </tr> </table>	<p>Pie</p>  <p>\$5.50</p>	<p>Carton of milk</p>  <p>\$2.40</p>	Increase and decrease																							
<p>Pie</p>  <p>\$5.50</p>	<p>Carton of milk</p>  <p>\$2.40</p>																										

Student teachers’ examples of Grade 3 level problems should be at an appropriate level and can be similar to the questions above and in the ‘Categorising problems’ table in the textbook.



Check student teachers’ understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Understand the kinds of problem-solving questions that are useful to extend mathematical thinking for students to Grade 3;
 - Be able to categorise problem-types and develop similar examples; and
 - Understand the level Grade 3 students should be able to attain.
2. Ask student teachers to turn to their partner and explain one of the problem types they examined during this lesson.

3. Remind student teachers that they explored the Mathematics problem types that are used in early grades. The same types are used throughout the early grades but the difficulty level increases.



Extension and differentiation activities

Learning activity 1: Make sure student teachers understand the content of the reading assignment and review it if needed.

Learning activity 2: You may select a problem type and ask student teachers to quickly write a Mathematics problem of this type. Invite a few volunteers to share their problems with the class.

Learning activity 3: If student teachers have difficulty writing their own problems, refer them to the examples in the textbook and support them as needed.

3.3.3.

Problem-solving up to Grade 6

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Discuss the differences between problem-solving and routine calculations;
- Examine different processes for solving problems;
- Describe the kinds of problem-solving questions that are useful to extend mathematical thinking for students from Grade 4 to Grade 6;
- Categorise problem-types and develop similar examples for Grade 4 to Grade 6 students; and
- Understand the level Grade 6 students should be able to attain.



Competencies gained: A1.1. Demonstrate understanding of different theories of how students learn relevant to their age; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation; discussion; pair work



Preparation needed: A PowerPoint presentation covering key points of the lecture notes could be used



Learning activity 1: Introducing the lesson (5 minutes)

1. Highlight that in this lesson, we will be focused on problem-solving to Grade 6 level. Explain that we start by examining when problem-solving strategies are needed and then review a diagram that provides some approaches teachers can use to help learning at the various stages in the process of problem-solving. Explain that then the lesson will focus on a worksheet of Grade 6 Level problems that student teachers working in pairs will be expected to review.
2. This lesson builds on the previous lesson. Review and check for understanding of the previous lesson by asking questions.



Assessment

Pose some questions for discussion. For example:

- What was the topic of the previous lesson?
- What did you learn during the pair activity in the previous lesson?



Possible student teacher responses

Topic of the previous lesson:

- Categorising problems; and
- Creating problems appropriate for Grade 3 students in different categories.

What was learned in the pair activity:

- How to create problems from different categories that are appropriate for Grade 3 students.



Learning activity 2: Problem-solving (10 minutes)

1. Present the key points of ‘What is problem-solving’.
2. Work through the problem on Thura and Thiri in the textbook. Solve it as a class with algebra.
3. Explain that at the Grade 6 level algebraic skills are not developed, but they can solve problems like these. Problems like this one are ‘non-routine problems’. Teachers can support students to work through such problems.



Assessment

Ask a few student teachers to describe ‘problem-solving’ in their own words.



Possible student teacher responses

Examples:

- Problem-solving involves using higher order thinking skills to work through a problem.
Problem-solving is a process used to work through a problem that is not routine.



Learning activity 3: Helping students solve non-routine problems (10 minutes)

1. Present the key points of Questions and procedures to help students with non-routine problems (in the diagram in the Student Teacher Textbook).
2. Ask a few questions to assess student teachers’ understanding of the diagram. Invite volunteers to share their responses with the class.



Assessment

Pose some questions for discussion For example:

- How can a teacher help a student to get started on a problem?

- How can a teacher help a student work through a problem?
- How can a teacher help a student check their solution to a problem?



Possible student teacher responses

These are example answers. Student teachers' responses may be different but should reflect an understanding of the diagram.

Getting started:

- Remind them of a similar problem they solved.
- Encourage them to organise the information in the problem in a list or table.

Working through a problem:

- Act out the problem.
- Encourage them to use concrete objects or draw a picture.
- Encourage them to work backwards by trying an answer.

Checking the solution:

- Ask if the solution is reasonable. Does it make sense?



Learning activity 4: Grade 6 problems (20 minutes)

1. Request that student teachers in pairs to categorise the Grade 6 problems in the Student Teacher Textbook.
2. Explain that they should categorise each problem if they are able. They should then decide if it is a routine problem. They should identify whether the problem is appropriate for the Myanmar context, and discuss how they would assist students to think through the problem.
3. Explain that they should then develop six Grade 6 level problems.



Assessment

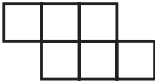
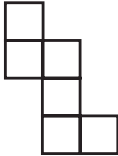
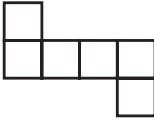
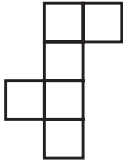
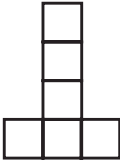
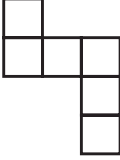
Assess that student teachers have grasped the key points based on their observations of the ten given problems and the six problems they create.

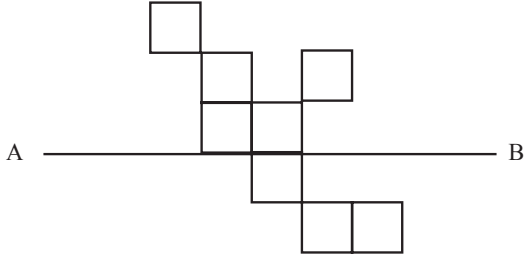


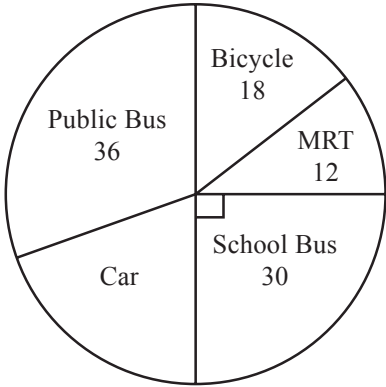
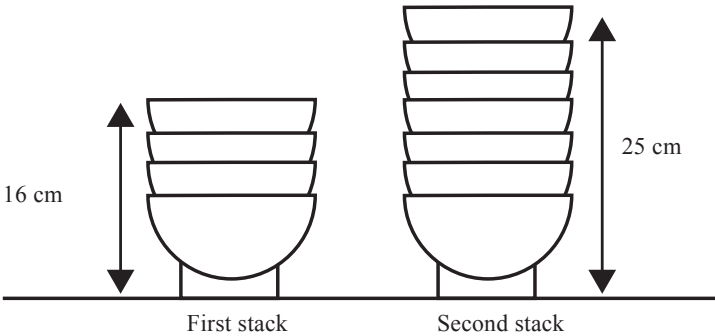
Possible student teacher responses

The problems that are given in the textbook follow with the problem types identified. Student teachers may identify different problem types. Their answers may still be accurate but ensure that student teachers are able to defend their decisions.

Figure 3.41. Problem-solving for Grade 6 students

	Grade 6 problem	Responses
1	<p>Which of the following figures are nets of a cube?</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <p>A</p> </div> <div style="text-align: center;">  <p>B</p> </div> <div style="text-align: center;">  <p>C</p> </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="text-align: center;">  <p>D</p> </div> <div style="text-align: center;">  <p>E</p> </div> <div style="text-align: center;">  <p>F</p> </div> </div> <p>(1) A, B, C and D (2) A, B, E and F (3) B, C, D and E (4) C, D, E and F</p>	<p>None of the given problem types.</p> <p>A routine operation cannot be used.</p> <p>Learners can be assisted by imagining each shape as a piece of paper. Can it be wrapped around a cube?</p>

<p>2</p>	<p>The figure below show 8 squares. What is the smallest number of squares that must be added so that line AB will be the line of symmetry?</p>  <p>(1) 7 (2) 6 (3) 5 (4) 4</p>	<p>None of the given problem types.</p> <p>A routine operation cannot be used.</p> <p>Learners can be encouraged to draw a picture.</p>
<p>3</p>	<p>The length of each side of a square is an odd number. What is a possible perimeter of the square?</p> <p>(1) 16 cm (2) 20 cm (3) 24 cm (4) 32 cm</p>	<p>None of the given problem types.</p> <p>A routine operation can be used.</p> <p>Learners can be encouraged to draw a picture. They can be encouraged to guess and check.</p>
<p>4</p>	<p>Hannah spent $\frac{4}{5}$ of her pocket money to buy 8 pens. She wanted to buy another 8 similar pens but found that she was short of \$12. What was the price of 1 pen?</p> <p>(1) \$1.20 (2) \$1.50 (3) \$2.00 (4) \$4.00</p>	<p>Sharing.</p> <p>A routine operation can be used.</p> <p>Learners can be encouraged to organise information in writing.</p>
<p>5</p>	<p>There are 182 chickens and cows in a farm. For every 4 chickens, there are 3 cows. How many more chickens than cows are there?</p>	<p>Sharing, Treating groups as units.</p> <p>A routine operation can be used.</p> <p>Learners can be encouraged to organise information in writing.</p>

<p>6</p>	<p>Find the value of $5n - 4 = \frac{3n}{2}$ when $n = 4$</p>	<p>Sharing.</p> <p>A routine operation can be used.</p> <p>The teacher can give the student hints or ask guiding questions, such as 'What is the order of operations?'</p>
<p>7</p>	<p>The pie chart below shows the number of pupils travelling to school by various modes of transport.</p>  <p>What percentage of the pupils travel by car?</p>	<p>Rate.</p> <p>A routine operation cannot be used.</p> <p>The teacher can give the student hints or ask guiding questions, such as 'How can you find how many students travelled by car?'</p>
<p>8</p>	<p>Some identical bowls are stacked vertically to save space. In the figure below, the height of the first stack of 4 bowls is 16 cm. The height of the second stack of 7 bowls is 25 cm. Find the height of one such bowl.</p> 	<p>Summation.</p> <p>A routine operation cannot be used.</p> <p>The teacher can encourage students to guess and check.</p>
<p>9</p>	<p>Ali has \$840 more than Baba. If Ali gives $\frac{7}{9}$ of his money to Baba, Ali will have $\frac{1}{5}$ as much money as Baba. How much money does Ali have at first?</p>	<p>Increase and decrease, Comparison by ratio.</p> <p>A routine operation cannot be used.</p> <p>Learners can be encouraged to organise information in writing.</p>

10	There were 60 children on board a bus at first. 4 boys and 2 girls alighted from the bus. The ratio of the number of boys to the number of girls then became 4:5. Find the number of boys on board the bus at first.	<p>Comparison by ratio.</p> <p>A routine operation cannot be used.</p> <p>The teacher can give the student hints or ask guiding questions, such as 'How many students were left on the bus after 6 got off?'</p>
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Student teachers' examples of Grade 6 level problems should be at an appropriate level. They should encourage students to apply their problem-solving skills.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Discuss the differences between problem-solving and routine calculations;
 - Examine different processes for solving problems;
 - Understand the kinds of problem-solving questions that are useful to extend mathematical thinking for students from Grade 4 to Grade 6;
 - Be able to categorise problem-types and develop similar examples; and
 - Understand the level Grade 6 students should be able to attain.
2. Ask student teachers to turn to their partner and explain the importance of encouraging problem-solving skills.
3. Remind student teachers that they explored how problems that are not routine can be solved using problem-solving skills. Remind them that it is important to build students' problem-solving skills, and they can refer to the diagram for support on how to do so.



Extension and differentiation activities

Learning activity 1: Make sure student teachers understand the content of the previous lesson and review it if needed.

Learning activity 2: You may ask student teachers to reflect on the algebra problem and look for a way to solve it without algebra. Encourage them to think like a Grade 6 student.

Learning activity 3: You may ask student teachers to apply this diagram to the algebra problem solved in the previous learning activity. How could they encourage a Grade 6 student to think about and solve this problem?

Learning activity 4: If student teachers have difficulty writing their own problems, discuss and support them as needed.



Review questions: Possible student teacher responses

Question 1: Why is it important to design Mathematics problems at the appropriate level?

Answer: Students are still developing their mathematics vocabulary and ability during primary school. The problems must be appropriate for their level of understanding.

Question 2: What are some tools and resources that you can use to design mathematics problems?

Answer: The table for categorising problems, which gives problem types that can be used for Grades 1-6.

Question 3: What are some tools and resources that you can use to support students as they solve problems?

Example answer: The diagram 'Questions and procedures to help students with non-routine problems'.

3.4. Mathematics in life

This sub-unit focuses on real-life applications of Mathematics through the use of scenarios in lessons. Scenarios describe imaginary situations that are realistic and could occur in everyday life within the context they are taught. Students may apply various Mathematics skills and knowledge in a given scenario, including measurement, operations and statistics. This sub-unit explores how scenarios can be applied appropriately in a primary classroom.

3.4.1. Planning an agriculture project

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Use the skills with measurement of length, weight, volume, time and area and graphs to design a lesson around an agricultural or food project; and
- Develop similar practical lessons to demonstrate how numeracy skills and Mathematical thinking are used in real life situations.



Competencies gained: A2.1. Demonstrate understanding of appropriate use of a variety of teaching and learning strategies and resources; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation; group assignment; discussion; presentation



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; pens



Learning activity 1: Introducing the lesson (5 minutes)

1. Introduce the lesson. Explain that a scenario is a situation that, although imaginary, should be as realistic as possible. Explain that the lesson is designed to give an example of how teachers can develop lessons that encourage Mathematical thinking and application of Mathematical skills that have been developed, by using real life situations. Explain that this lesson will focus on a range of measurement calculations, design and choice made after doing calculations.



Assessment

Pose a few questions to the class to check for understanding.



Possible student teacher responses

Example questions and responses:

- What is a scenario? (A situation that is imaginary but should be realistic)
- Why do you think scenarios are useful in Mathematics lessons? (They give students the opportunity to apply the skills they have learned to real-life situations.)



Learning activity 2: Scenario group work (35 minutes)

1. Present the scenario in the textbook: A school has asked groups of students to design farming projects to raise money for the school. Two groups, A and B, are designing projects that are described in the textbook.
2. Assign each groups to one of the student projects in the scenario (group A or group B). Make sure an even number of groups select A and B.
3. Ask groups to work through the scenario. They should answer the question at the end of their group's project description.
4. Groups should also make points on a flipchart on: 'Possible changes to the scenario that could make it most realistic to a local context.'
5. Get group representatives to present and (agree to changes in the scenario that make it most realistic).



Assessment

Walk around to assess student teachers' ability to interpret the scenario and answer the questions while they are working. Assess their presentations for whether they can identify changes to make the scenario more appropriate for the local context.



Possible student teacher responses

Responses to the scenario are given below. Student teachers may have used different steps to solve the problem. Their answers may be slightly different based on the steps they took. That is okay, and different processes of problem-solving should be encouraged and shared.

Group A:

1. Student teachers should calculate the area of indoor and outdoor space need for the chickens. They should then design the house and chicken yard, and estimate the amount of materials needed.
 - a. The total indoor space is $\frac{1}{3}$ sq. m. x 15 chickens = 5 sq. m.

- b. The total outdoor space is 1 sq. m. x 15 chickens = 15 sq. m.
 - c. The total space needed for the chickens is $5 + 15 = 20$ sq. m.
 - d. Student teachers should identify locally available materials in reasonable quantities.
2. Student teachers should calculate the food needed each month and the monthly cost.
 - a. Each chicken eats 100 g per day, so 15 chickens eat $15 \times 100 = 1,500$ g.
 - b. In one month, the chickens eat around $1,500 \times 30 = 45,000$ g = 45 kg.
 - c. The amount of feed the chickens will need each month is $45/25 = 1.8$ bags
 - d. The cost of feeding the chickens each month is $\text{Ks } 12,000 \times 1.8 = \text{Ks } 21,600$.
 3. Student teachers should calculate how many eggs the chickens will produce in a week and month on average.
 - a. If 1 chicken produces 320 eggs/year, 15 chickens produce $320 \times 15 = 4,800$ eggs/year
 - b. In 1 week, the chickens produce $4800/52 = 92$ eggs
 - c. In 1 month, the chickens produce $4800/12 = 400$ eggs
 4. Student teachers should calculate the weekly and monthly income for the eggs:
 - a. In 1 week, $92/12 = 7.7$ dozen eggs are produced, which can be sold for $\text{Ks } 2,600 \times 7.7 = \text{Ks } 20,020$
 - b. In 1 month, $400/12 = 33.3$ dozen eggs are produced, which can be sold for $\text{Ks } 2,600 \times 33.3 = \text{Ks } 86,580$

Group B:

- Student teachers should find the area of the field:
 - The parallel sides of the trapezium are 80 m and 120 m, and the height is 50 m.
 - Apply the formula: $A = \frac{1}{2} (a + b)h = \frac{1}{2} (20 + 120)50 = \frac{1}{2} (140)50 = 3,500$ sq. m.
- Student teachers should find the perimeter of the field:
 - Apply Pythagoras' theorem to find the unknown sides of the trapezium. They are equivalent, if each side is given by c, then:

- $20^2 + 50^2 = c^2$
- $400 + 2,500 = c^2$
- $2,900 = c^2$
- $53.9 \text{ m} \approx c$
- Add the four sides of the trapezium: $P = 120 + 80 + 53.9 + 53.9 = 307.8 \text{ m}$
- Student teachers should calculate the yield of the field in tons of rice.
 - Convert the area of the field to hectares: $3,500/10,000 = 0.35$ hectares
 - A field of this size will produce on average $0.35 \times 3.89 = 1.36$ tonnes of rice
- Student teachers should calculate approximately how many bags of rice can be produced.
 - Convert the yield to kg: $1.36 \times 1,000 = 1,360$ kg
 - If rice is sold in 49 kg bags, then $1,360/49 = 27.8$ bags of rice will be produced
- Student teachers should estimate the income:
 - If each bag of rice is sold for Ks 52,000 the income is $52,000 \times 27.8 = 1,445,600$

Student teachers may have a variety of ideas for making the scenario more appropriate for the Myanmar context. There are no incorrect/correct answers.



Learning activity 3: Scenario assignment (5 minutes)

1. Ask each group to create a new scenario that is realistic and that use as much Mathematical thinking and operations for primary school students as possible. This should be completed for homework for presentation at the next lesson.



Assessment

Student teachers will be assessed based on the scenario they create and present to the class.



Possible student teacher responses

Student teachers' scenarios should be realistic for primary students in the local context. They should apply the mathematical skills in the curriculum to real-life scenarios.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Have used the skills with measurement of length, weight, volume, time and area and graphs to design a lesson around an agricultural or food project; and
 - Develop similar practical lessons to demonstrate how numeracy skills and Mathematical thinking are used in life.
2. Ask student teachers to turn to their partner and explain how scenarios are useful in a Mathematics lesson.
3. Remind student teachers that they explored a scenario in which they applied primary Mathematics skills to a real-world situation. Scenarios are an opportunity for students to apply the Mathematics they have learned. They also motivate students to learn Mathematics by showing them how Mathematics topics are applicable in the real world.



Extension and differentiation activities

Learning activity 1: Make sure student teachers understand what a scenario is and why it is useful in a Mathematics class. Spend a few minutes explaining if needed.

Learning activity 2: Support groups as they work through the scenarios. If they finish early, ask them to identify the specific Mathematics skills that are used here and identify an appropriate level in the grade-wise curriculum for this activity.

Learning activity 3: Make sure student teachers understand the assignment before they leave class.

3.4.2. Planning a class excursion

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Use the skills with measurement of length, weight, volume, time and area to plan a class excursion; and
- Develop similar lessons to demonstrate how numeracy skills and Mathematical thinking is used in real life situations.



Competencies gained: A2.1. Demonstrate understanding appropriate use of a variety of teaching and learning strategies and resources; and B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation, group assignment, class discussion, internet research



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; pens; access to the internet for student teacher groups



Learning activity 1: Introducing the lesson (5 minutes)

1. Outline that during the class, groups will present their new scenarios, will work through the scenario ‘Planning a class trip’ and will begin research for data on climate in Myanmar.
2. Review the importance of using scenarios in Mathematics lessons.



Assessment

Pose a few questions to the class to check for understanding.



Possible student teacher responses

Example questions and responses:

- What is a scenario? (A situation that is imaginary but should be realistic)
- Why are scenarios useful in Mathematics lessons? (They give students the opportunity to apply the skills they have learned to real-life situations.)



Learning activity 2: Group presentations (20 minutes)

1. Give groups time to present the scenarios that they created for their assignment. Allow for critical discussion on their quality based on whether the scenario is realistic and the level and amount of Mathematical thinking and operations used.



Assessment

Assess student teachers based on their presentations.



Possible student teacher responses

Student teachers’ scenarios should be realistic for primary students in the local context. They should apply the mathematical skills in the curriculum to real-life scenarios.



Learning activity 3: Considering the class trip scenarios (15 minutes)

1. Present the key parameters for the scenario and request that groups design their scenario. There is a rough scenario in the textbook, but student teachers should make it appropriate for the local context. Remind them to be aware of the mathematical thinking, modeling and operations used.
2. Observe groups, ask probing questions and guide them where necessary.
3. Request that a representative make presentation if there is time.



Assessment

Assess student teachers based on their scenario. It should be appropriate for the context and involve mathematical thinking, modeling and operations.



Possible student teacher responses

There are no incorrect/correct scenarios, but a good scenario will consider the following:

- Realistic for the context and contains local details;
- Encourages imagination; and
- Encourages students to model Mathematics and use operations that they are familiar with.



Learning activity 4: Assignment on researching climate (5 minutes)

1. Explain that in the next lesson student teachers will have to assemble a set of posters with graphs illustrating climate, seasons, rainfall and temperature patterns across Myanmar.
2. Request that they read the assignment for Lesson 3.4.3 and complete the scenario and poster with their groups. They should be prepared to present these in the next lesson.



Assessment

Assess student teachers based on their posters and presentations in the next lesson.



Possible student teacher responses

Student teachers should use real data on the seasons, climate, temperature and rainfall patterns in Myanmar. They should show an understanding of Mathematics concepts and their application to these real-world issues.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Use the skills with measurement of length, weight, volume, time and area to plan a class excursion; and
 - Develop similar lessons to demonstrate how numeracy skills and Mathematical thinking is used in life.
2. Ask student teachers to turn to their partner and explain the mathematical skills they applied to the scenario during today's lesson.
3. Remind student teachers that they explored a scenario in which they applied primary Mathematics skills to a real-world situation. They will work through another scenario for homework.



Extension and differentiation activities

Learning activity 1: Make sure student teachers understand what a scenario is and why it is useful in a Mathematics class. Spend a few minutes explaining if needed.

Learning activity 2: If there is enough time, you may guide a brief discussion after each presentation. You may ask the class to identify one thing that the group did well, and one way in which their scenario could be improved.

Learning activity 3: Student teachers may design scenarios that are more or less detailed based on the amount of time remaining in the lesson. Encourage them to add specific details to the scenario and problems that require operations to solve.

Learning activity 4: Make sure student teachers understand the homework before they leave class.

3.4.3. Graphing climate

Expected learning outcomes



By the end of the lesson, student teachers will be able to:

- Use skills with measurement, data and graphs to design a lesson around climate in Myanmar;
- Develop graphic posters as a teaching and learning resources for classroom display; and
- Develop similar practical lessons to demonstrate how numeracy skills and Mathematical thinking are used in real life situations.



Competencies gained: B1.1. Demonstrate capacity to teach subject-related concepts and content clearly and engagingly.



Time: One period of 50 minutes



Learning strategies: Presentation; discussion



Preparation needed: Ensure the resources below are available



Resources needed: Flipchart paper; coloured pens; poster-size white cardboard; cardboard of different colours; scissors; glue; access to the internet



Learning activity 1: Introducing the lesson (5 minutes)

1. Explain that the key focus of this lesson is to develop quality classroom posters that represent the climate of Myanmar in a graphical form. That the time will be taken up producing and presenting these posters.



Assessment

Ask a few questions to assess whether student teachers completed the homework assignment and whether they encountered challenges.



Possible student teacher responses

Example questions and responses:

- What are some topics that you researched for homework? (Rainfall, temperature, seasons, climate zones)
- Did you have any challenges finding the information? If so, what were the challenges? (No, relevant information was available on the internet for all topics.)
- Do you think a scenario like this would be useful for a primary student? Why or why not? (Yes, it would be useful for a Grade 6 student to apply what they have learned about measurement, time and graphics.)



Learning activity 2: Discussing data (20 minutes)

1. Request that a representative from each group describe the data they were able to find on the internet, the key facts they gleaned from this research and how this data was presented on the sites they visited. Inquire whether they were able to read and understand the data as it was presented.
2. Facilitate discussion after each presentation.



Assessment

Assess student teachers based on the findings in their presentations.



Possible student teacher responses

Student teachers should give realistic and useful findings based on what they found on the internet. They should address all four of the areas they were to research: rainfall, temperature, seasons, and climate zones.



Learning activity 3: Create and exhibit posters (20 minutes)

1. Now request that student teachers spend time developing their graphical display that could be used on a classroom wall as a teaching tool.
2. Indicate what stationery is available to them to do this (scissors, glue, cardboard, coloured card and coloured pens, sticky tape, drawing pins).
3. Observe the production and advise teams where appropriate.
4. Ask groups to display their posters. Spend a few minutes discussing what groups did well and what could be improved.



Assessment

Assess whether student teachers are able to translate data into useful and appropriate graphic presentation in their classroom posters. Comment on the quality of these.



Possible student teacher responses

Student teachers should use graphs and tables to display the information they found. The information should be easy to read and interpret. These are some examples that they could use:

- Rainfall patterns displayed on a line or bar graph, with the average rainfall for each month indicated.
- Temperature patterns displayed on a line graph, with the average rainfall for each month indicated.
- Seasons displayed in a table or chart showing the months that fall into each season. Pictures can be used to show the characteristics of each season (such as an umbrella for rainy, or a sun for dry and hot).
- Climate zones could be displayed on a map of Myanmar.



Check student teachers' understanding (5 minutes)

1. Remind student teachers of the learning outcome of this lesson:
 - Have used skills with measurement, data and graphs to design a lesson around climate in Myanmar;
 - Have developed graphic posters as a teaching and learning resources for classroom display; and
 - Be able to develop similar practical lessons to demonstrate how numeracy skills and Mathematical thinking are used in life.
2. Ask student teachers to turn to their partner and explain the mathematical skills they applied to the scenario they completed for homework and presented in this lesson.
3. Remind student teachers that they explored a scenario on weather and climate, which required them to use their knowledge of measurement, tables and graphs.



Extension and differentiation activities

Learning activity 1: Make sure student teachers understood the assignment and collected the information. You may give them a few minutes to discuss with their groups and finalise their data if needed.

Learning activity 2: If there is enough time, you may ask groups to identify the Mathematics skills they used while collecting the data. For example, they would apply their knowledge of measurement to interpret rainfall patterns.

Learning activity 3: If groups find the task challenging, you may support them by suggesting graphs to be used for each type of data (such as a line graph for temperature data).



Review questions: Possible student teacher responses

Question 1: Why is it important to use scenarios in a Mathematics class?

Answer: Scenarios give students the opportunity to apply their Mathematics skills and knowledge. They will also understand how the Mathematics they have learned applies in the real world which motivates them to learn Mathematics.

Question 2: What are some Mathematics skills that you could have students practise by using a scenario in class?

Answer: Many skills can be practised in a real-world scenario including operations, measurement, and statistics.

Unit Summary



Key messages

3.1. explored number systems, and student teachers solved problems in base 5 to understand how primary students feel when they begin learning Mathematics in base 10. The unit then discussed fractions, identifying some common misconceptions primary students may have and how these can be addressed.

3.2. explored appropriate methods of introducing measurement to primary students. Student teachers identified that students should be introduced to measurement with non-standard units in Grade 1, and standard units in later grades. The last two units covered angles, identifying some common misconceptions and how to address these.

3.3. described the importance of introducing children to the language of Mathematics. It described the types of problems that can be used across the primary grades, and gave specific examples and activities with problems appropriate for grades 3 and 6.

3.4 explored applications of Mathematics to real-world scenarios. Student teachers should understand the importance of using scenarios in a Mathematics classroom and have ideas of how to bring Mathematics to life for their students.



Unit reflection

The key learning for you to reflect on in this unit, and start to think about how you can apply in your teaching are:

- The challenges that primary students may face in learning the number system and operations;
- Common misconceptions about fractions and how to address them;
- How to introduce measurement to primary students at different levels;
- The importance of introducing the language of Mathematics to primary students;
- How problem-solving should be applied at different levels of primary; and
- How to use scenarios in a Mathematics classroom.

What are some ideas you have for teaching the number system to primary school students? It is important to build their number concept through different types of activities involving counting, comparing and estimating.

When teaching measurement to primary school students, it is advisable to compare different amounts and use non-standard units first, then introduce standard units in the metric or imperial system.

During this unit, we looked at how problems are designed differently for a Grade 3 student from those designed for a Grade 6 student. Grade 3 students should be able to solve simple word problems that involve an operation. Grade 6 students should be able to solve problems that require problem-solving skills and may not be solved through a routine operation.

Mathematics is enlivened by using scenarios in a primary classroom so students can apply the skills they are learning in real-life. Scenarios are often related to planning, and can have a focus on any topic relevant in Myanmar. These include agriculture, trade, or planning an event. Students might apply their knowledge of measurement, operations, or statistics.



Further reading

3.1.

Sapire, I., Shalem, Y., & Reed, Y. (2015). *Assessment for Learning: Using learners' test data for professional development*. Johannesburg: University of Witwatersrand and Saide.

3.2.

Booker, Bond, Briggs & Davey. (1998). *Teaching Primary Mathematics*. Longman. Melbourne.

Hogben, L. (1955). *Man must Measure: The wonderful world of Mathematics*. London: Rathbone books.

Outhred, L & Mitchelmore, MI & McPhail, D & Gould, (2003). "Count Me into Measurement A Program for the Early Elementary School." In Clements(D) *Learning and Teaching Measurement: 65th Yearbook of National Council of Teachers of Mathematics*. Reston V.A. NCTM

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2011). *Teaching Mathematics: Foundation and Middle years*. Melbourne: Oxford University Press.

The Open University. (n.d.). *Project Mathematics Update: Preparing to Teach Angle*. Great Britain: The Open University.

van de Walle. Karp & Bay-Williams. (2013). *Elementary and Middle School Mathematics: teaching developmentally*. Boston: Pearson.

Wilson & Rowland. (1993). Unit 8 in Jensen, R.J. (ed.) *Research Ideas for the Classroom*. National Council of Teachers of Mathematics, Inc., Reston, VA. Macmillan.

3.3.

Polya, G. (1990). *How to solve it*. Melbourne: Penguin.

Glossary

Terms	Elaborations
Code	A complicated system of letters, figures, or symbols used to represent other letters.
Cognitive development	The way that children develop mentally.
Curriculum	A description of what, why, how and how well students should learn in a systematic and intentional way (IBE/UNESCO, 2013).
Direct measurement	Standard units (such as metric or imperial units) are used to measure.
Enquiry-based learning	A student-centred approach to learning that is based on problem-solving.
Error	Something done wrong, which is pervasive and can be repeated in different contexts.
Grade-wise curriculum	A view of the curriculum that helps us see the scope and sequencing of a topic by grades
Heuristics	Strategies, or the methods and rules of discovery, used to solve Mathematics problems.
Indirect measurement	No units are used to measure objects; they can be measured using comparisons or a non-standard unit.
Language of Mathematics	The set of words and phrases which are specific to Mathematics, or which have a different meaning when used in Mathematics.
Learning Outcome	What specifically a student should learn as the result of a learning programme or lesson
Mathematical thinking	A particular way of thinking in Mathematics, which is both logical and abstract.
Misconception	An alternative understanding that people may have about a concept.
Mistake	An incorrect answer that has been obtained as a result of a 'slip'.
Number concept	An understanding of numbers and simple operations that provides the base upon which Mathematics concepts are built.
Pattern	A design or set of items that is repeated, following a set of rules.
Place value	The value that a digit has based on its position in a number.
Scaffolding	Support provided to students by a skilled teacher and then slowly withdrawn this when the support is no longer needed.

Terms	Elaborations
Scenario	A situation that is imaginary but realistic, which can be used a Mathematics lesson.
Syllabus	A document which outlines the aims, selection and sequence of contents to be covered, mode of delivery, materials to be used, learning tasks and activities, expected learning objectives or outcomes, and assessment/evaluation schemes of a specific course, unit of study or teaching subject (IBE/UNESCO, 2013).

Bibliography

Booker, Bond, Briggs & Davey. (1998). *Teaching Primary Mathematics*. Longman. Melbourne.

Outhred, L & Mitchelmore, M, McPhail, D & Gould (2003). “Count Me into Measurement A Program for the Early Elementary School.” In Clements, D. *Learning and Teaching Measurement: 65th Yearbook of National Council of Teachers of Mathematics*. Reston V.A. NCTM

van de Walle. Karp & Bay-Williams. (2013). *Elementary and Middle School Mathematics: teaching developmentally*. Boston: Pearson.

Wilson & Rowland. (1993). Unit 8 in Jensen, R.J. (ed.) *Research Ideas for the Classroom*. National Council of Teachers of Mathematics, Inc., Reston, VA. Macmillan.

Grade-wise Mathematics Curriculum. (n.d.).

Hogben, L. (1955). *Man must Measure: The wonderful world of Mathematics*. London: Rathbone books.

International Bureau of Education & UNESCO. (2013). *Glossary of Curriculum Terminology*. Retrieved from http://www.ibe.unesco.org/fileadmin/user_upload/Publications/IBE_GlossaryCurriculumTerminology2013_eng.pdf

Larson, L. C. (1983). *Problem-Solving through Problems*. New York: Springer-Verlag Publishers.

Mensa Education and Research Foundation. (2019). Fabulous Fibonnaci. Retrieved from Mensa for Kids website: <https://www.mensaforkids.org/teach/lesson-plans/fabulous-fibonacci/>

- Piaget, J. (1964). *Cognitive Development in Children: Development and Learning*. In *Journal of Research in Science Teaching*.
- Polya, G. (1957). *“How to Solve it”*. Garden City, New York: Doubleday.
- Polya, G. (1990). *How to solve it*. Melbourne: Penguin.
- Posamentier, A. S., & Krulik, S. (1998). *Problem-solving strategies for efficient and elegant solutions: A resource for the Mathematics teacher*. Thousand Oaks, Calif.: Corwin Press.
- Sapire, I., Shalem, Y., & Reed, Y. (2015). *Assessment for Learning: Using learners’ test data for professional development*. Johannesburg: University of Witwatersrand and Saide.
- Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2011). *Teaching Mathematics: Foundation and Middle years*. Melbourne: Oxford University Press.
- The Open University. (n.d.). *Project Mathematics Update: Preparing to Teach Angle*. Great Britain: The Open University.
- The World Bank. (2016). *Myanmar: Analysis Of Farm Production Economics*. Retrieved from <http://documents.worldbank.org/curated/en/509581468181132091/pdf/100066-ESW-P144951-Box394886B-PUBLIC-MM-Farm-Production-Economics-online-version.pdf>
- Tsukahara, S. (1994). *Heuristics in Secondary Mathematics: Introduction to Heuristics. (In Japanese)*. Japan: Toyokan publishing.
- UNESCO. (2001). *Understanding and Responding to Children’s Needs in Inclusive Classrooms- A Guide for Teachers*.
- UNESCO. (2009). *Towards inclusive education for children with disabilities: a guideline*. Retrieved from <https://unesdoc.unesco.org/ark:/48223/pf0000192480>

Vaidyanathan, S. (2019). Teaching Coding to Kids: What Programming Language Should We Use? Retrieved from EdSurge website: <https://www.edsurge.com/news/2019-03-11-teaching-coding-to-kids-what-programming-language-should-we-use>

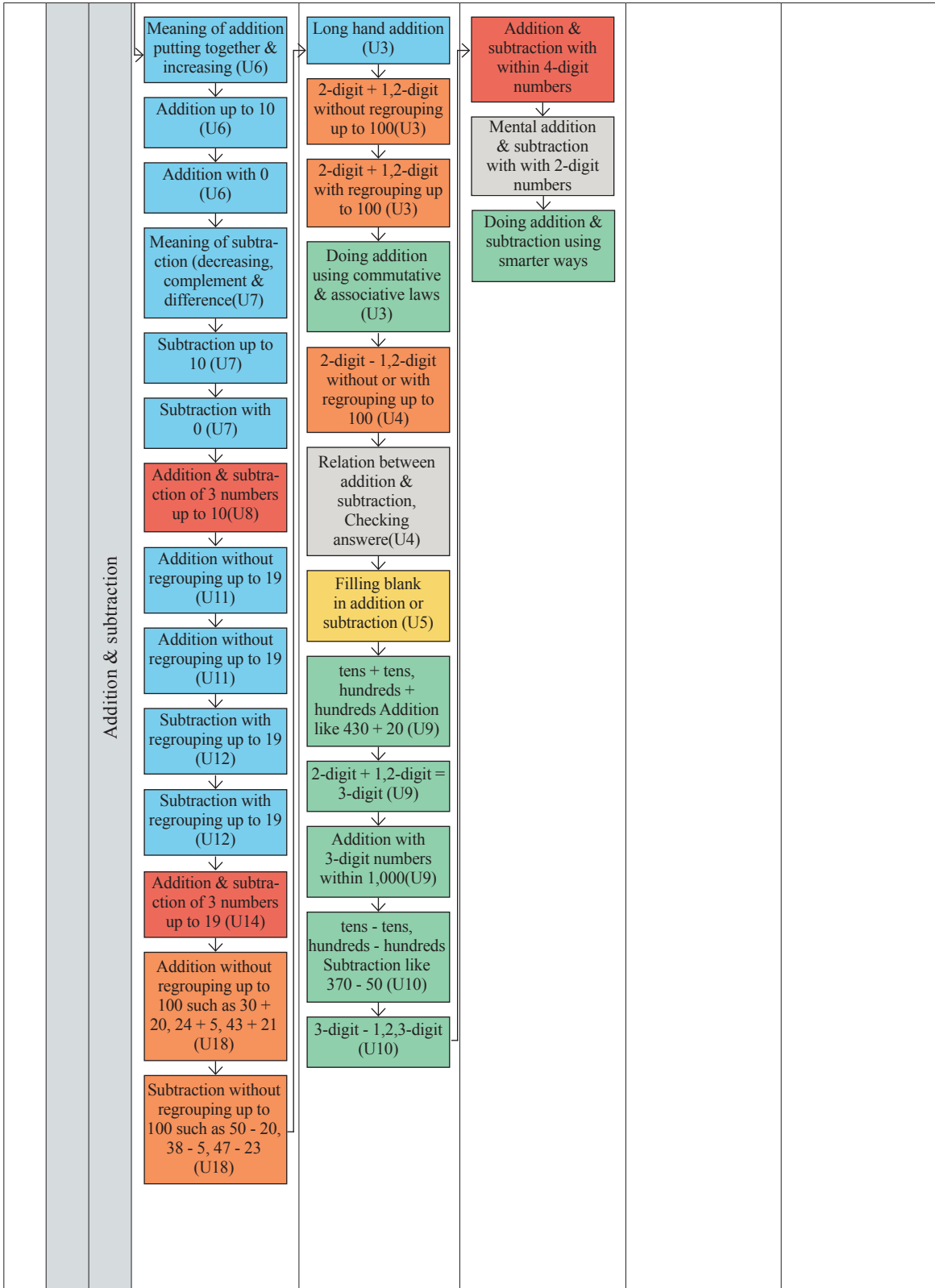
Veldhuis, M. & Zhao, X. (2018). The effectiveness of Mathematics teaching in primary schools. *Research in Mathematics Education*.

Annexes

Annex 1: Lesson 1.2.2.

Grade-wise Mathematics curriculum

		Grade 1	Grade 2	Grade 3	Grade 4	Grade 5													
Numbers	Whole numbers	Counting, reading & writing of numbers up to 10 (U1)	Counting, reading & writing of numbers up to 1,000 (U8)	Counting, reading & writing of numbers up to 100,000	Counting, reading & writing of numbers up to 100,000,000	Overview of decimal systems of whole numbers and decimals													
		Counting & arranging numbers up to 10 (U1)	Decimal system for 3-digit numbers (U8)	Decimal system for 5-digit numbers	Comparing & arranging numbers up to 100,000,000 including number line	10 times, 100 times, $\frac{1}{10}$ times, $\frac{1}{100}$ times & movement of decimal point													
		Meaning, reading & writing of Zero (U3)	Comparing & arranging numbers up to 1,000 including number line (U8)	Comparing & arranging numbers up to 100,000 including number line	10 times 100 times & divided by 10	Meaning of even & odd numbers													
		Counting, reading & writing of numbers up to 20 (U11)		10 times 100 times & divided by 10	Reading numbers up to 1,000,000,000	Meaning of & how to find multiples, common multiples & least common multiples													
		Comparing & arranging numbers up to 20 (U11)			Meaning of rounded numbers	Meaning of & how to find factors, common factors & greatest common factors													
		Number line up to 20 (U11)			Rounding off, down & up numbers														
		Counting by 2 and 5 (U11)			Calculation using rounded numbers														
		Counting, reading & writing of numbers up to 100 (U19)																	
		Decimal system for 2-digit numbers (U19)																	
		Comparing & arranging numbers up to 100 (U19)																	
		Number line up to 100 (U19)																	
		Composition & decomposition of numbers up to 10(U2)																	
		Ordinal numbers from 1 st to 5 th (U16)																	
			<table border="1"> <tr> <td>Blue</td> <td>→ Topic in old grade 1</td> </tr> <tr> <td>Orange</td> <td>→ Topic in old grade 2</td> </tr> <tr> <td>Green</td> <td>→ Topic in old grade 3</td> </tr> <tr> <td>Red</td> <td>→ Topic in old grade 4</td> </tr> <tr> <td>Yellow</td> <td>→ Topic in old grade 5</td> </tr> <tr> <td>Gray</td> <td>→ Topic not in old Primary</td> </tr> <tr> <td>Red dots</td> <td>→ Undecided Topic (excluded for now)</td> </tr> </table>	Blue	→ Topic in old grade 1	Orange	→ Topic in old grade 2	Green	→ Topic in old grade 3	Red	→ Topic in old grade 4	Yellow	→ Topic in old grade 5	Gray	→ Topic not in old Primary	Red dots	→ Undecided Topic (excluded for now)		
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Gray	→ Topic not in old Primary																		
Red dots	→ Undecided Topic (excluded for now)																		



	Multiplication		<p>Meaning of multiplication (U14)</p> <p>Multiplication table of 2,5,3,4 (U14)</p> <p>Multiplication table of 6,7,8,9,1 (U14)</p> <p>Multiplication table of 10(U14)</p> <p>Multiplication with (U14)</p>	<p>Commutative & distributive laws</p> <p>Multiplication table up to 20</p> <p>Multiplication with 10 & 100</p> <p>Multiplication of 3 numbers</p> <p>Associative law</p> <p>Long hand multiplication</p> <p>2,3-digit x 1-digit</p> <p>2,3,4,5-times</p> <p>1,2-digit x tens</p> <p>2,3-digit x 2-digit</p>		
	Division		<p>Meaning of division (Sharing & Quotative)(U17)</p> <p>Divisions whose divisor & quotient are 1-digit (U17)</p> <p>Division by 10 (U17)</p> <p>Division of 0 (U17)</p> <p>Checking answer of division (U17)</p>	<p>Simple division with quotient of 2-digit number (e.g. $48 \div 2$)</p> <p>Simple division of 2-digit numbers with remainders</p> <p>Checking answers of division (with remainder)</p> <p>Long hand division</p> <p>2,3-digit + 1-digit</p>	<p>Tens, hundreds, thousands + tens, hundreds</p> <p>2,3,4-digit + tens</p> <p>2,3,4-digit + 2-digit</p> <p>3,4-digit + hundreds</p> <p>3,4-digit + (2-digit x 10) (e.g. $1856 + 390$)</p> <p>3,4-digit + 3-digit</p>	

					<p>Order of calculation with brackets, +, -, x, ÷</p> <p>↓</p> <p>Summary of commutative, associative & distributive laws of + & x</p> <p>↓</p> <p>Doing calculation using laws (e.g. $13 \times 8 + 13 \times 2$)</p>	
Fractions	Four operations					
	Concept of fractions	Simple fraction such as $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ (U20)	<p>Meaning of fractions (up to 10)</p> <p>↓</p> <p>Identifying equivalent fractions</p> <p>↓</p> <p>Meaning of proper, improper & mixed fractions</p> <p>↓</p> <p>Conversion between improper & mixed fractions</p> <p>↓</p> <p>Comparing & arranging fractions with same denominator in order</p>	<p>Converting to equivalent fractions</p> <p>↓</p> <p>Comparing & arranging fractions with denominators one of which is a multiple of the other</p>	<p>Expressing a fraction as a division of two whole numbers</p> <p>↓</p> <p>Reduction of fraction to lowest term & common factor</p>	
	Addition & subtraction		<p>Addition & subtraction of fractions with same denominator within 1</p>	<p>Addition & subtraction of fractions with denominators one of which is a multiple of the other</p>	<p>Addition & subtraction of fractions with different denominators</p>	
	Multiplication				<p>Meaning & calculation of fraction x whole number</p> <p>↓</p> <p>Meaning & calculation of whole number x fraction</p> <p>↓</p> <p>Meaning & calculation of fraction x fraction</p>	

	Division					Meaning & calculation of fraction + natural number
Decimals	Concept of decimals				Meaning & decimal system of decimals up to 100 th place	Decimal system of decimals up to 1,000 th place
					Comparing & arranging decimals up to 100 th place	Rounding off decimals to nearest 10 th & 100 th place
					Relation between decimals & fractions	Rounding off division of two whole numbers to nearest 10 th & 100 th place
					Comparing decimal & fraction	
	& subtract				Addition & subtraction of decimals up to 100 th place	
	Multiplication					Meaning & calculation of decimal x whole number ↓ Meaning & calculation of whole number x decimal ↓ Meaning & calculation of decimal x decimal within 1000 th place ↓ Multiplying a number more than 1 & less than 1
	Division					Meaning & calculation of decimal + natural number

		Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	
Geometry	Planegeometry	Lines & angles		Meaning & Drawing of straight line (U13) ↓ Meaning & Drawing of angle (U13) ↓ Meaning & right angle (U13)		Meaning, property & drawing of parallel & perpendicular lines ↓ Drawing rectangles & squares by drawing parallel & perpendicular lines	
		Polygons	Identifying rectangular, triangular & circular shapes (U16)	Meaning of triangle & quadrilateral (U13) ↓ side & vertex (U13) ↓ Meaning, property & grid-drawing of rectangle & square (U13) ↓ Meaning, property & grid-drawing of right triangle (U13)	Meaning, property & drawing of isosceles & equilateral triangles ↓ Making patterns using the figures learnt up to Grade 3	Meaning, property & drawing of parallelograms ↓ Meaning, property & drawing of rhombuses & trapeziums ↓ Meaning of diagonals ↓ Property of parallelograms, rhombuses & trapeziums with diagonals ↓ Making patterns using the figures learnt in Grade 4	Meaning, property & drawing of pentagons & hexagons ↓ Making patterns using the figures learnt up to Grade 5 ↓ Sum of interior angles of triangle ↓ Sum of interior angles of quadrilateral ↓ Sum of interior angles of polygon ↓ Drawing polygons under given conditions
		Circles				Meaning, property & drawing of circles ↓ Meaning of centre, radius & diameter	
		Sym-metry			Identifying Symmetry figures ↓ Identifying lines of symmetry ↓ Making symmetric figures and patterns ↓ Drawing symmetric figures on grid paper		

Solid geometry	Solid figures	Identifying shapes of boxes, cans & balls (U5)		Meaning & property of cubes & cuboids ↓ Faces, edges & vertices ↓ Constructing cubes & cuboids using nets		

		Grade 1	Grade 2	Grade 3	Grade 4	Grade 5		
Measurement	Basic SI units	Length	Direct comparison of lengths (U9) ↓ Indirect comparison of lengths (U9) ↓ Measuring lengths with arbitrary units (U9)	Measuring lengths with cm & mm (U7) ↓ Relation between cm & mm (U7) ↓ Simple calculation with cm & mm (U7) ↓ Measuring lengths with m & cm (U16) ↓ Relation between m & cm (U16) ↓ Simple calculation with m & cm (U16)	Distance with km ↓ Relation between km & m ↓ Simple calculation with km & m	Finding perimeter of polygons		
			Weight		Comparison of weights of objects with balance (U19)	Measuring weight with kg & g ↓ Relation between kg & g ↓ Simple calculation with kg & g ↓ Relation between \square , $k\square$, $c\square$ & $m\square$		
				Time	Time with o'clock & half past (U13)	Time with o'clock, minutes AM & PM (e.g. 5:48 AM) ↓ Time duration with hour, minute (e.g. 2 hours and 27 minutes)(U2) ↓ Relations between day & hour, hour & minute (U10)	Relation between minute & second ↓ Time & time duration with minute & second ↓ Finding times & time during by calculation	

SI derived units	Area				<p>Finding area of rectangular shapes using cm^2 and m^2 (formula)</p> <p>↓</p> <p>Relation between cm^2 & m^2</p> <p>↓</p> <p>Finding area of rectangular shapes using km^2</p> <p>↓</p> <p>Relation between m^2 & km^2</p> <p>↓</p> <p>Finding area of shapes like L shape</p>	<p>Finding area of parallelograms (formula)</p> <p>↓</p> <p>Finding area of triangles (formula)</p> <p>↓</p> <p>Relation between height & area</p> <p>↓</p> <p>Finding area of trapeziums (formula)</p> <p>↓</p> <p>Finding area of rhombuses (formula)</p>
	Volume		<p>Comparison of volumes of liquid (U18)</p> <p>↓</p> <p>Measuring volumes with L & mL (U18)</p> <p>↓</p> <p>Simple calculation with L & mL (U18)</p>			<p>Finding volumes of cubes and cuboids with cm^3 & m^3 (formula)</p> <p>↓</p> <p>Relation between cm^3 & m^3</p> <p>↓</p> <p>Relation between cm^3 & L, m^3 & L, cm^3 & mL</p> <p>↓</p> <p>Relation between length & volume</p> <p>↓</p> <p>Finding volume of shapes consisting of 2 cuboids</p>
	Angle		<p>Meaning & Drawing of angle (U13)</p> <p>↓</p> <p>Meaning of right angle (U13)</p>		<p>Measuring & drawing angles using protractor up to 360°</p> <p>↓</p> <p>Drawing triangles with given sides & angles</p>	

	British units	Length			<p>Measuring lengths with inch & feet</p> <p>↓</p> <p>Relation between inch & feet</p> <p>↓</p> <p>Simple calculation with inch & feet</p> <p>↓</p> <p>Measuring lengths with yard & feet</p> <p>↓</p> <p>Relation between yard & feet</p> <p>↓</p> <p>Simple calculation with inch & feet</p>		
		Weight			<p>Measuring weights with pounds & ounce</p> <p>↓</p> <p>Relation between pounds & ounce</p> <p>↓</p> <p>Simple calculation with pounds & ounce</p>		
	Myanmar units	Length			<p>Measuring lengths with Taung & Htwa</p> <p>↓</p> <p>Relation between Taung & Htwa</p> <p>↓</p> <p>Simple calculation with Taung & Htwa</p>		
		Weight			<p>Measuring weights with Peithar & Kyathar</p> <p>↓</p> <p>Relation between pounds & Peithar & Kyathar</p> <p>↓</p> <p>Simple calculation with Peithar & Kyathar</p>		

		Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Mathematical relations	Data collection & arrangement		<p>Reading pictograms (U1)</p> <p>↓</p> <p>Making tables & graphs with from data(U1)</p>	<p>Making table from data using tallies</p> <p>↓</p> <p>Making bar graph form table</p> <p>↓</p> <p>Making bar graph form data</p> <p>↓</p> <p>Integrating two tables into one table</p>	Making and reading line graph	
	Data analysis			<p>Simple analysis using bar graph</p>	<p>Interpreting line graph in terms of change with time</p>	<p>Meaning of & how to find average</p> <p>↓</p> <p>Analysing given data using average</p>

Annex 2: Lesson 2.13. Lecture note

George Polya's four phases of problem-solving

In order to teach students problem-solving and critical thinking skills, teachers need to be understand the theories and approaches have been developed by leading thinkers in the field and be able to use them in the classroom.

George Polya (1887 -1985) was a Hungarian mathematician. He described problem-solving principles and strategy in four interrelating phases. His thinking is particularly useful for teachers of Mathematics. It helps teachers pose problems and guide students through the stages. Problem-solving develops mathematical thinking and so is a very important aspect of teaching and learning the subject. Polya's phases are:

1. Understanding the problem;
2. Devising a plan;
3. Carrying out the plan; and
4. Looking back.

The first phase, does not only entail understanding the meaning of the sentence stating the problem. This phase also requires students to clarify what conditions are given and what they have to find. Many students tend to overlook the conditions and misunderstand what they have to do.

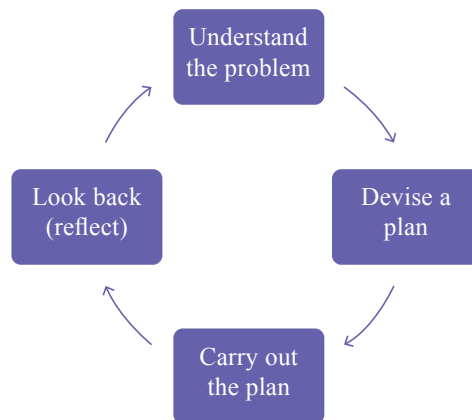
In the second phase the given conditions are analysed. Students use their knowledge and the experiences they have had in the past to develop a strategy to solve the problem. During this phase, students are required to think about whether they have experienced solving problems before that are the same or similar. In addition, they need to think about what other 'tools' such as theorems they know and can use to solve the problem. Student must also ensure that they are using all the conditions and data available in the procedure they follow. They must also decide on what procedure to follow to solve the problem and whether they can solve the problem.

In the third phase, students carry out their plan. It is not a simple process because they have to check whether each step of their approach is correct or not. And they have to show proof.

In phase 4, students should re-examine their solution and the steps they took to reach the solution. Many, at this point, move on to the next problem after they obtain a solution (regardless of whether it is correct or not). Their understanding of the concept is likely to be shallow. They could think about whether or not their approach is correct whether there are other approaches that they could have considered and what the solution means. Then they may be able to generalise from this problem to other like it or learn a specific lesson.

Polya's four phases of problem-solving

Solution Pathways = Problem-Solving
Polya's Four Steps to Problem-Solving



Source: Polya, George. *How To Solve It*. 2nd ed. (1957). Princeton University Press.

Polya's phases for problem-solving are represented in the diagram above.

In the diagram, there are some backward arrows. This indicates for example, if an unexpected answer comes out in phase 3, either the plan has a problem or you have not yet understood the problem sufficiently. If mistakes are found when checking the answer in phase 4, you should check the calculation procedure and the plan itself.

Teachers are responsible for developing their students' approach to solving mathematical problems. If they help students to develop Mathematical skills and teach them to think through problems in these four steps, teachers are making a significant contribution to their students' learning.¹⁰

¹⁰ Polya, G. (1957). *How to Solve It*. Garden City, New York: Doubleday

Annex 3: Lesson 2.1.4. - Lecture note

Heuristics for problem-solving

The strategies used for mathematical problem-solving is called ‘mathematical thinking’. Another word used for this is ‘heuristics’. Heuristics is particularly relevant to Phase 2 of Polya’s four phases when, after understanding the problem, we have to think of a way through the problem.

However, ‘heuristics’ should not be understood simply as an equivalent term for ‘strategy’. Polya (1957)¹¹ describes heuristics as the methods and rules of discovery. We may employ a number of strategies when solving a problem.

We could consider the pattern of a given sequence, we could recall the definition of a term used in the problem or we could prove the proposition indirectly. (These are some examples) Through a number of trials and errors, we finally discover a way to solve the problem. All the strategies we use for the discovery are what we call ‘heuristics’.

Heuristics are often discussed in mathematics education at the secondary level, rather than at the primary level. However, the strategies shown below can be a great help even for primary students to promote their mathematical thinking skills.

We will consider four strategies in the thinking through the attached problems.

1. Back to definition
2. Drawing a figure
3. Finding a pattern
4. Solving a simpler analogous problem

These are just four strategies of many. The educators below have outlined in their work the strategies, or heuristics, for solving mathematical problems they see as important. Larson (1983), Tsukahara (1994) and Posamentier and Krulik (1998)¹² have examined the strategies (listed in the table below) in their work.

¹¹ Polya, G. (1957). *How to Solve It*. Garden City, New York: Doubleday

¹² Larson, L. C. (1983). *Problem-Solving through Problems*. New York: Springer-Verlag Publishers.

Tsukahara, S. (1994). *Heuristics in Secondary Mathematics: Introduction to Heuristics*. (In Japanese) Toyokan Publishing.

Posamentier, A.S., & Krulik, S. (1998). *Problem-solving strategies for efficient and elegant solutions: A resource for the Mathematics teacher*. Thousand Oaks, Calif.: Corwin Press.

Table 3.1. Heuristics

Larson (1983)	Tsukahara (1994)	Posamentier and Kruli (1998)
<ul style="list-style-type: none"> • Searching for a pattern • Drawing a figure • Formulating an equivalent problem • Modifying the problem • Choosing effective notation • Exploiting symmetry • Dividing into cases • Working backward • Arguing by contradiction • Pursuing parity • Considering extreme cases • Generalisation 	<ul style="list-style-type: none"> • Analogy • Working backwards • Back to definition • Reformation • Indirect proof • Fewer variables • Symmetry • Logical reasoning • Specialisation and generalisation • Inductive thinking 	<ul style="list-style-type: none"> • Working backwards • Finding a pattern • Adopting a different point of view • Solving a simpler analogous problem • Considering extreme cases • Making a drawing • Intelligent guessing and testing • Accounting all possibilities • Organising data • Logical reasoning

Annex 4: Lesson 2.1.4. - Activity 1

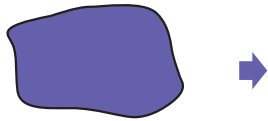
Example 4: Using a simpler analogous problem

Q. Calculate 1.2×7 (Think how to teach a student who knows how to calculate a whole number but has not learned how to calculate a decimal)

Annex 5: 2.1.4. - Learning activity 2

Exercise 1: Back to definition

Q. Make parallel lines by folding paper. Recall the definition first.



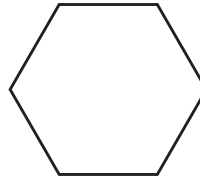
Definition:

Exercise 2: Drawing a figure

Q. Some students are standing in line. Nwe Nwe is the fifth from the left, and the second from the right. How many students are there in the line?

Exercise 3: Finding a pattern

Q. How many diagonals are there in a hexagon?



Exercise 4: Using a simpler analogous problem

Q. Calculate 200×300 (Think how to teach a student who knows multiplication table up to 10 and knows how to make a group of 10).

Annex 6: 2.2.1. – Lecture note

Research in mathematics teaching and learning

Mathematics is an important subject at school. Through studying Mathematics, children learn to think logically. Numeracy skills are important to function effectively in the world of work and in commerce. Mathematics skills are important for studying after school in many careers.

Many students find some parts of Mathematics difficult to understand and teachers sometimes do not know alternative ways to help with this understanding. If students fail to grasp key concepts at the primary phase, they are left without a firm Mathematics foundation on which to build. There is much research about teaching and learning of various aspects of the mathematics curriculum, what students find difficult and alternative strategies for teaching and learning to improve understanding.

This research has been done by academics at universities but also by teachers in schools. Researching teaching and learning should be done by teachers so that they are able to understand the difficulties students have and try different approaches and see what works and what not.

The research has shown that there are typical errors students make and misconceptions they have formed.

In this lesson, we will look at some of the specific errors students make and misconceptions they have when adding and subtracting.

Errors, mistakes and misconceptions

There are three words that are used a great deal when teachers speak about students' mathematical work. These are the words '*misconceptions*', '*errors*' and '*mistakes*'.

'Errors' are pervasive (found everywhere) and are often repeated in different contexts. Teachers shake their heads and wring their hands over students who keep doing the same things wrong over and over again even though these have been addressed and corrected repeatedly in class. Teachers need to realise that understanding where these errors are coming from and why they are being made is a very valuable starting point for improving teaching and for improving students' understanding of concepts.

The reason why students make errors is that their incorrect thinking (misconceptions), results in inaccurate or incomplete knowledge structures. 'Misconceptions' is a term used to describe the alternative understandings that people may have about concepts. These understandings may be either incorrect or incomplete. Teachers are able to identify students' misconceptions through understanding why they have made the errors that they see.

'Mistakes', on the other hand, are incorrect answers that have been obtained as a result of 'slips' such as pressing an incorrect calculator key or calculating $3 \times 3 = 6$. A mistake is quickly realised and corrected because there is no conceptual misunderstanding associated with the mistake. It is important that teachers are able to distinguish between mistakes (easily fixed) and errors (very difficult to correct), so that they do not become frustrated with their students.

What can you learn from thinking about the errors that your students make?

Take some time to look at some errors your students have made and think about whether these can give you any insight into the difficulty they were having when they made the errors. If you can identify conceptual misunderstanding, you can help the student to understand mathematics better and develop the confidence to do mathematics. This will make your job easier. It will also make you a better teacher as you will be able to teach in ways that help students avoid developing these misconceptions.

Analysing student errors can help you to think about ways of teaching the mathematical concepts that might prevent students developing those errors.¹³

¹³ Sapire, I. Shalem, Y. and Reed, Y. 2015. *Assessment for Learning: Using learners' test data for professional development*. Johannesburg: University of Witwatersrand and Saide.

Place value and number bonds

A student needs to understand that the numerical value of a digit is based on the digit's position in the number. This is the value it has because of its place in the number, its **place value**.

In addition, the student needs to have developed some arithmetic tools such as the knowledge of **number bonds** (up to 20) so that she/he is able to undertake the calculations required. A number bond is the rule in Mathematics that two numbers add up to a given total. The number bonds for 8 are 8+0, 1+7, 2+6, 3+5 and 4+4. The number bonds for 20 are 20+0, 19+1, 18+2, 17+3 and so on.

In addition, the system we use for calculating is a decimal one. Students need to understand what to do when adding to numbers equalling 10 or more.

Problems with place value in addition

Primary students need to grasp that in the number 548, the 5 represents five hundred, the 4 represents forty (four tens) and the 8 represents eight units ('ones'). The value each has is based on its position.

If students have not grasped this concept, they are likely to make mistakes when adding and subtracting in numbers in 10's, 100's or 1,000's

- When adding $435 + 33$, the student who does not understand the place value concept may write

$$\begin{array}{r}
 435 \\
 + 33 \\
 \hline
 765
 \end{array}$$

- Or the student may be tempted to add another column. In doing the sum $363+156$, they may write

$$\begin{array}{r} 363 \\ + 156 \\ \hline 4119 \end{array}$$

- When the sum in a column is more than '10' and it requires the student to carry 10s or 100s to the next column, some students may mix the digits and reverse them.

For example

$$\begin{array}{r} 162 \\ + (3)73 \\ \hline 415 \end{array}$$

- They may get the order correct and then forget to add the amount.

$$\begin{array}{r} 162 \\ + (1)73 \\ \hline 135 \end{array}$$

- When students move on to numbers that contain 100s, they may find it difficult to understand that the hundreds column exists even if the numbers to calculate do not have hundreds. They may write

$$\begin{array}{r} 63 \\ + 75 \\ \hline 38 \end{array}$$

Errors and misconceptions with subtraction

- Because students often hear that with subtraction, they should subtract the smaller number from the bigger one, they may get confused

54 – 37 may be calculated as

$$\begin{array}{r} 54 \\ - 37 \\ \hline 23 \end{array}$$

To understand this mistake, the student needs to understand that 54 is 50+4 or it can be 40+14. (This is the basis of regrouping and is critical for adding and subtracting) This means now that 14 is larger than 7 and the answer is 17.

- The student might also have not grasped the idea of regrouping and may calculate 54 – 37 as follows

$$\begin{array}{r} 5(1)4 \\ - 37 \\ \hline 27 \end{array}$$

- If there is a zero used as a ‘place holder’ in a number, students can get confused. And the calculation could look like:

$$\begin{array}{r} 405 \\ - 173 \\ \hline 372 \end{array}$$

Annex 7: 2.2.1. - Lecture note

'No two children are the same'.¹⁴

How often have you said that? Take any class of students in your school. They may be the same age, but are they all alike? No! How do they differ from one another? Some are tall; others are small. Some are shy; others are forward. Some learn quickly; others are slow.

'If you were asked to write down what makes children different' you might have written: some like sports; others do not; some are good singers; others are not; some read well others do not; some are naughty, others are not. The list can go on and on.

Including children with special needs

There are some children with impairments

Some children are born with impairments such as eyes that do not see well; arms and legs that are deformed, or a brain that is not developing in a typical way. Some children can be left with impairments after childhood illnesses like measles and cerebral malaria or from accidents such as burns and bad falls. Often these children are called 'disabled' or 'handicapped'.

There are some children who have been deprived

Some children's growth and development is impaired because their environment causes them harm or does not support their well-being. They may not have enough food or a good diet; they may live in poor housing and are prone to illnesses; they may be beaten; their parents may have separated; they are refugees or survivors of war. Sometimes they live on the streets. They may abuse drugs.

¹⁴ The following notes comprise extracts from UNESCO. (2001). *Understanding and Responding to Children's Needs in Inclusive Classrooms - A Guide for Teachers*

Many children do not go to school

In many countries, these children do not go to school. Various reasons are given for this. Families do not know of their child's right to education or they chose to spend their scarce money on their other children.

The school cannot cope with children who have additional needs and they are not allowed to enrol.

The children come to school but soon dropout. They attend special schools instead.

But more children from deprived backgrounds or with disabilities are attending their local pre-schools, primary or secondary schools. Most countries have laws which state that all children have to be educated.

All children can learn

All children can learn but if they do not go to school, their chances to learn are much reduced. All children have the right to learn with their peers in local schools.

Many children encounter problems at some time in their lives. Some problems quickly pass, but others require ongoing help. More special schools are not the solution. They are often at a distance from the family home and separate the children from their peers.

Increasing the opportunities for all to learn at schools in their communities

The main reason for promoting the attendance in ordinary schools of children with disabilities or from deprived backgrounds is to increase their opportunities to learn through interaction with others and to promote their participation in the life of the community.

The impact of exclusion of children

Often these children are excluded from society. They might be hidden away at home if they look different because of fear and superstition. Or poverty forces families to live in city slums with few amenities. Often their needs are not recognised and they are thought to have little to contribute to their community.

But this exclusion reduces children's opportunities to learn, grow and develop. They are disadvantaged twice over. Attending local school is the main way of ensuring that all children are included in society.

Children's learning does not just take place in schools

They learn from their families, through contact with peers and friends, and through participation in all the diverse activities that occur in communities. But attending school helps to promote these other forms of learning as well.

Through Education for All, it should be possible to enable all human beings – including the disabled – to develop their full potential, to contribute to society and, above all, to be enriched by their difference and not devalued. In our world constituted of differences of all kinds, it is not the disabled but society at large that needs special education in order to become a genuine society for all. (Federico Mayor, former director-general of UNESCO)

The challenge for teachers

Teachers have a particular responsibility for ensuring that all children participate fully in society.

A teacher's job is not easy. You may have big classes of 40 and more students – all of them individuals! Having children with disabilities or from disadvantaged backgrounds in your class often means more work, but it need not be so.

Teachers can manage differences among the children if:

- They recognise children's strengths and weaknesses and plan your lessons accordingly.
- They know how children's learning can be affected by disabilities and deprivation and use teaching strategies for overcoming these difficulties.
- They have confidence in your abilities as a teacher to plan lessons for individuals and adapt the curriculum to suit the needs of all children.
- They get help and support from colleagues, parents and other professionals, such as community and health workers.
- They believe that all children have the right to education and that all can learn.

The purpose of this discussion

- To inform teachers what they can reasonably do to adapt the classroom and school environment to overcome the barriers to learning faced by children with impairments.
- To describe strategies teachers can use to respond to the diversity of children in their classrooms.
- To show some ideas how the curriculum can be adapted to individual needs.
- To encourage teachers to work with families and with other personnel in health and social services and in the community.

Key matters to consider

Teachers should consider the following matters when developing approaches to including all students.

Nine golden rules

Include everyone
Communicate
Manage classrooms
Lesson planning
Individual plans
Individual help
Assistive aids
Manage behaviour
Work together

The value of inclusive education to students comes from mixing and sharing with other children.

Teachers need to encourage this to happen as experience suggests that children with special needs can become isolated within classes and schools. Teachers may need to explain to other children the reasons why some children cannot talk behave differently and so on.

Diversity should be recognised and respected. Let the students discover for themselves how they work together with their peers. Similar work may need to be done at parent-teacher meetings. The biggest obstacle to inclusion is usually negative attitudes.

Children may not be accustomed to other children who look and behave differently.

Parents may also be worried about ‘lowering the standards’ if children with disabilities and other special needs are included in ordinary classrooms.

Teachers are instrumental

Teachers are instrumental in developing positive attitudes among students, parents and of course other teachers. Children who use assistive devices such as hearing aids or depend on special equipment can tell the class about it and demonstrate its use. Encourage children to ‘befriend’ students with special needs. It is important for social inclusion that children play and work together. They can also assist them at toilets; moving between classroom and at break times.

Within the class, develop opportunities for ‘peer tutoring’. More able students can assist the less able in class-work. Also make sure that children with special needs can also make their meaningful contribution in school work so that they do not become depended and objects of ‘help’. Devise learning games than students can play together in class.

Groups

Groups of children can play those with special needs to master reading and number skills. Set the class activities to complete as a group; so that all the children can contribute to its completion and gain credit for achieving it. Think of how children with special needs can join in games and sports. For example, a blind child can be partnered with a sighted child in running competitions. Or create a game in which all children move around blindfolded.

Promote the talents of all children

Promote the talents of all children by encouraging their participation in school activities, such as singing, dancing and drama. Involve all children in all school activities; for example in cleaning and cooking chores and as class monitors. It is a good sign when you see children playing happily together at break times and if they tell you about visiting each other.

Communicate clearly

Teachers need to make their communication very clear when teaching. All students are different, some learn best by seeing, other by hearing or doing. A good communicator always uses various channels or ways to communicate and repeats essential contents by using different learning activities in the classroom and elsewhere.

Teachers should be clearly seen by all the students stand rather than sit at the desk. Talk clearly and project your voice (slightly raised) but not shouting. Keep the words simple and the sentences short. Alert the students to important messages: ‘listen carefully’ and make eye contact with them if it is culturally appropriate. Repeat important messages. Use gestures and facial expressions alongside.

Use simple language

Use simple language to get your meaning across. These are especially useful when organising or managing the class. But they should also be used when explaining and teaching. Check with the students that they have understood. Ask them to repeat what you have said or to say in their own words what you have told them.

Encourage the children to indicate

Encourage the children to indicate if they have not understood by raising their hands and asking you questions. They should also do this when they do not understand what other students have said. Encourage the children to show you and to gesture if you cannot make out what they are trying to say.

Often classmates or brothers and sisters of a child with a disability are able to tell you what the child is trying to communicate. Get their help.

Learn the local sign language

Learn the local sign language and to fingerspell if you have children with hearing impairments. All the class can learn some signs or sign language so that they too can communicate with their peers. Contact your National Association for Deaf Persons for information about the training courses they run. Augment your verbal communication with pictures, drawings and writing.

Managing classrooms

The layout of classrooms

The layout of classrooms can help or hinder children's learning. Perhaps there is not much teachers can do to change their classroom. But here are some ideas.

Children with special needs need to sit close to the teacher and the chalkboard. Try to arrange the room so that children can move freely, especially if some have mobility or visual problems. Likewise, some children might need extra light while some might have light sensitive eyes. The children's desks or tables can be arranged in groups so that they can easily work together and help each other.

If space permits, try to set aside an area of the classroom so that you can work with certain children on a one-to-one basis or in small groups for short periods. This area could be screened off using a moveable screen to reduce the distractions for the child.

Have a variety of activities

Have a variety of activities which children can use if they have completed their work ahead of others. This could include a small library of books, worksheets and games.

Display charts and posters at children's eye-level rather than high up on the walls. Use large writing, pictures and symbols so that these are easily seen and understood by all children. You can also add different textures and real objects for touching to help children with visual difficulties.

Some learning is better done outside of classrooms. Using money to buy food can be done in a pretend way in the classroom but even better if children have the opportunity to practice in real settings. Likewise, lessons about plants and animals could be done in the school grounds or neighbouring farms.

Children with visual and hearing impairments may find it more difficult to learn if classes are held out-of-doors. Schools may have to arrange for teachers to have suitable rooms if they have such children in their class particularly when they are just starting school.

Have varied lessons

Group work facilitates participation of all students and is an excellent way of responding to individual needs. With special needs students, you may need to give them a lesson suited to their needs and leave them to do work on this while you teach the rest of the class.

However, each lesson should consist of different types of activities: whole class activities, group work, pair work, individual tasks. Use different grouping (mixed / ability / interest groups) and change groups frequently to avoid labeling and ‘streaming’.

Adjust the pace of lessons

You may need to adjust the pace of the lesson and the amount of material you cover in the time available. It is better to be selective rather than trying to cover too much material. As well as individual lesson plans, you also need a plan for the school day. This will note the alternative activities for children with special needs. It can be helpful to share this plan with students at the beginning of lessons.

Teachers collaborating

Teachers should not be expected to work alone

Here are some ways in which teachers have received advice and support.

- Ask other teachers in your school for advice. This may not be common practice nor is it a sign of failure!

- You can learn a great deal from one another. A group of staff could meet once a week for 30 minutes. Each week, a teacher takes it in turn to briefly summarise a specific problem: this could be about finding an appropriate method to a teaching task; managing behaviour, and so on. The other teachers brainstorm possible ways of dealing with this. The teacher should then select the ones which seem to offer most promise. Head teachers have an important role to play in developing initiatives such as these.
- Take time to observe how other teachers manage their classes and invite colleagues to watch your teaching. Together you can discuss what you felt went well and areas for improvement.
- The school could send teachers on available training courses. Afterwards, they could make a presentation to the staff and share any resources they obtained.
- Find out if there are any disability specialists available in the locality, for example, staff in a community based rehabilitation project, or in special schools. Try to meet them to obtain their advice. It is good if they can visit you at school to see the child there and observe your teaching.
- In some countries, the education authorities have provided ‘resource’ teachers to assist students with special needs. They may be based in one school or they cover a cluster of schools. They can work with classroom teachers in curriculum adaptations and teaching methods. They may also work individually with some students or groups of students. Teachers should try to observe these sessions and work together with the resource teachers in planning how to incorporate these lessons into their classrooms.
- There are a growing number of books and magazines about supporting students with special needs. Schools might establish a small library for teachers to consult.

The challenges for Mathematics teachers

With a good overview of the issues, challenges and some ideas for teachers to make their classrooms inclusive Mathematics teachers need to learn and implement.

Annex 8: 3.1.1. - Lecture note

A brief history of the development of the number system

From early in the history of human development, people were able to recognise shapes, sizes and quantities. We can identify five steps that created the need and spurred people on to develop counting systems.

Step 1 in the development of a counting system: the need to measure and order according to size.

As societies developed, they began to need to order objects and spaces according to size. People needed to measure. This was the first step towards developing counting systems.

Step 2 in the development of a counting system: the need to compare quantities.

Most societies began by comparing numbers of objects to the number of fingers on one hand, two hands or all their fingers and toes. Some societies compared objects to their number of hands or numbers of hands and feet.

As people moved from hunter-gatherer societies to more settled farming societies, they had a need to keep a record of their numbers of live-stock and their amounts of produce. Sometimes their numbers of fingers and toes were not sufficient. People often put aside special collections of stones or sticks or shells to represent their number of possessions. This process of noting one finger for each object or one stone for each possession is called one-to-one correspondence.

Step 3 in the development of a counting system: an awareness of order

When we count whole numbers, there is an order to numbers. Each number is one more than the number before it.

Step 4 in the development of a counting system: numbers are abstractions

When you count 1-sheep, 2-sheep, 3-sheep and so on, the numbers are a property of the sheep. In order to develop a number system, you need to understand that numbers are independent of one another. They can stand on their own. Counting and the development of a number system is not possible without this concept.

Recording numbers: moving to large numbers

It became important for people to record numbers, especially as trading developed. In many societies in China, Japan, Tibet, India, North and South America and Africa, people kept records of numbers by tying knots in cords. In other societies, people cut grooves or notches in bone or wood. This process of marking lines in wood or bones is known as tallying.




When people developed written records of numbers, the shape of the numbers was often influenced by their previous physical records. Societies that had kept records with knots, stones or seeds often used dots as a written symbol. Societies that keep tallies on bones or wood often used lines as a written symbol. The materials used for recording numbers also influenced the shapes of written numbers. For example, the Babylonians used wedge and semi-circular shapes because they wrote by pressing into clay.

Development of counting system (1)

	one	two	three	four
Egyptians				
Babylonians	∟	∟∟	∟∟∟	∟∟∟∟
Mayans	•	••	•••	••••


























For small numbers, societies initially used one mark per object and repeated it for the number of objects (see table above). It is easy to read small numbers expressed like this but it is more difficult to read large numbers expressed in this way. For example, it is easy to see that II is two but it is difficult to read llllllllllll. The latter symbolisation also takes up too much space. This led to people developing grouping shapes. Recognising the grouping made it easier than reading the string of symbols in a line.

Development of counting system (2)

	Representations of the number seven
Egyptians	
Babylonians	
Mayans	

To make recording even easier, societies invented different symbols for different groups of objects.

Development of counting system (3)

Egyptians	      1 10 100 1000 10000 100000
Babylonians	    1 10 60 600
Roman	       1 5 10 50 100 500 1000
Aztec	    1 20 400 6000
Mayans	    1 5 20 400

Keeping a record of large numbers is possible with special collections of stones and so on. But this does not always help when you want to estimate or judge the size of large numbers. If numbers are grouped in some way, it makes it easier to judge the size of or estimate of large numbers. People all over the world began to group numbers. This was often done by grouping according to their number of hands, or hands and feet, or fingers on one hand or fingers on both hands or all their fingers and toes. Many counting systems across the world grouped their counts in 10s, 5s, 20s; grouping into 2s or 4s was less common. Some societies grouped numbers into 60s.

Recording numbers is easier if you invent a collection of symbols. Many societies used one symbol for units. This symbol was generally repeated as the number of units increased. They developed another symbol for the grouping for example, 5, 10, 20 which was also often repeated as the numbers of that group increased, and yet other symbols for each power of that group.

Development of counting system (4)

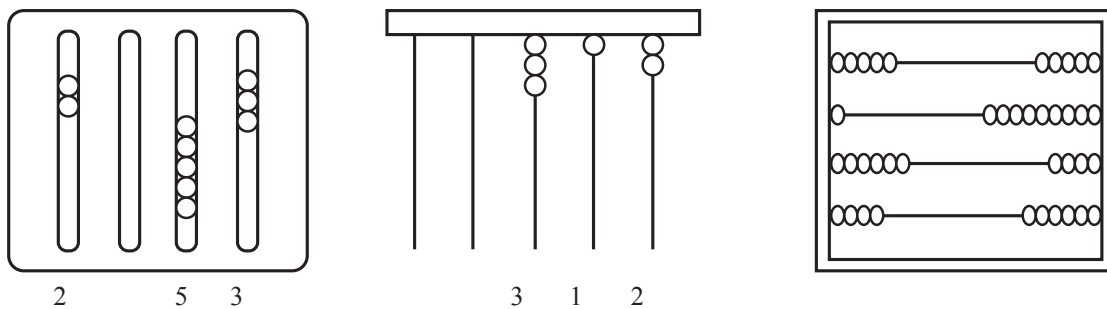
For example, the Egyptians would have represented

twenty as $\cap\cap$ thirty as $\cap\cap\cap$ forty as $\cap\cap\cap\cap$ etc
 thirty five as $\cap\cap\cap\text{||}$
 two hundred as 99
 three hundred and forty two as $\text{999}\cap\cap\cap\text{||}$

From counting to calculating

Marks on sticks, knots on cords, and pebbles in bags were ways to record numbers. But they could also be used to calculate. More effective calculating tools began to be developed, for example, a flat surface with groves for units and each power of the base, systems of upright sticks which held pieces seeds, shells, stones or beads. These were all precursors to the closed frame abacus.

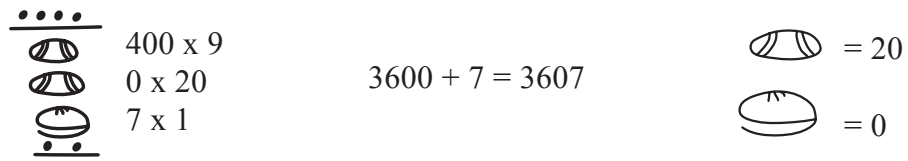
Development of counting system (5)



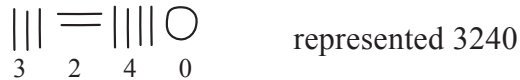
Calculating with numbers: The development of zero

We use numbers to calculate. But early on in the development of number systems, calculations were done in people's heads or on tools such as those described above and the answers were recorded in numbers.

If you use a counting board or abacus to calculate, you need a symbol to represent an empty column or row. The Babylonians developed a symbol for nothing in that place. It was only used in the middle of numbers and not at the end of numbers. This is equivalent to us being able to write 101, but not 110 or 1 110 and so on. They would distinguish between 1, 10, 100 and so on. by the contexts. Mayans used a shell symbol for zero which they used at the beginning and end of numbers. Chinese also used a zero in the middle and end of numbers.

Development of counting system (6)

The Chinese stick system also used a zero



Our decimal number system is derived from a number system that developed in India. They grouped numbers in tens and powers of 10 like we do, and used zero. This is a very powerful system as one only needs 10 symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0, to represent any number. It allows one to calculate easily with numbers and has allowed important conceptualisations around fractions.

Annex 9: 3.1.6. – Lecture note

What do teachers need to know?

People working in teacher education often ponder the question ‘What do teachers need to know?’ Lee Shulman introduced the term pedagogical content knowledge (PCK) which he described as the blend of content and pedagogy that forms the basis of teachers’ professional understanding. Ball, Thames and Phelps (2005) have extended Schulman’s notion of PCK, in order to answer the question ‘Is there a professional knowledge for teaching which is tailored to the work that teachers do with curriculum, materials, instruction and student teachers?’ This question points to an important idea – that professional knowledge for teaching consists of something very specific, some kind of knowledge that a pure content specialist (for example, a mathematician rather than a mathematics teacher) does not need to develop.

Ball et al sub-divided content knowledge and pedagogical content knowledge into six categories, as shown below.

SMK and PCK

Subject matter knowledge (SMK)		Pedagogical content knowledge (PCK)	
Common content knowledge (CCK)	Horizon content knowledge (HCK)	Knowledge of content and student teachers (KCS)	Knowledge of content and curriculum (KCC)
Specialised content knowledge (SCK)		Knowledge of content and teaching (KCT)	

Source: Ball et al.’s (2008) classification of teachers’ knowledge resources.

These knowledge domains can be described as follows:

Common content knowledge (CCK)

This is general subject matter knowledge.

Specialised content knowledge (SCK)

This is mathematical knowledge specific to teaching which according to Ball et al (2008) general mathematicians may not need.

Horizon content knowledge (HCK)

This is knowledge of mathematics beyond the level that is being taught.

Knowledge of content and student teachers (KCS)

This is teacher's knowledge of how students of different ages and different social contexts come to know particular mathematical topics. This is learnt through experience in the classroom.

Knowledge of content and curriculum (KCC)

This is knowledge of what content is appropriate for which standard, and how best to pace it through the school year. It includes an understanding of the rationale for the selection of certain topics and how they are covered by learning support material.

Knowledge of content and teaching (KCT)

This links CCK, SPK, KCS and KCC. Teachers use their knowledge of teaching to decide the sequence and planning of lesson content, or which aspects of students' contributions to take up and which to ignore.

Using student's errors productively as learning and teaching opportunities.

Some mistakes that students make are just careless slips. Students can correct these easily. Other mistakes that students make are based on a misunderstanding (or incomplete understanding) of the mathematical concepts. These are called misconceptions. The student believes that he / she is doing the mathematics correctly. There are some misconceptions that are common amongst students everywhere in the world.

If teachers become aware of student's misconceptions, it helps them to adjust their teaching to minimise the chances of these misunderstandings arising. A starting point for teachers is to assume that students had reasons for doing what they did and to try to unpack these reasons before moving on to help students understand where and why they went wrong, and to develop the confidence to approach the problem differently. Student errors can be used to improve both learning and teaching.

Mathematicians also analyse errors in the course of their work: mathematics will self-correct their calculations, solutions or thinking. Teachers, for the most part, are analysing errors in students' work. They have to think about what students have done, whether it is correct for the problem and whether the method will work in general (this involves subject matter knowledge). In order to respond to student errors productively, a teacher needs firstly to know the mathematical content.

Teachers also need to think about why students may have made the errors that they did, and how best to engage with students to help them think more productively about the problem and solutions and to build their mathematical confidence. The activity in error analysis goes beyond identifying the actual mathematical error to understanding the way students may have been reasoning when they made the error. This requires thinking diagnostically about students' reasoning when they made the errors. The emphasis in diagnostic reasoning is on the quality of the teachers' attempts to provide an explanation for how students were reasoning mathematically. Once errors have been diagnosed, the task becomes addressing the students' misconceptions. In some situations, limited conceptualisation or misconceptions of the topic may have been laid down early in the learning of the topic. Contexts of learning (such as age and background) affect students' understanding. For these reasons, teachers need to develop a range of explanations so that they are able to address differences in students' current knowledge and the readiness to move to more complex understandings. Some explanations are more easily understood, while others, though more complex, are more accurate. Once the errors have been diagnosed, they may need to be explained differently to different students. How does this link with types of knowledge for teaching? How are these types of knowledge linked in practice?

'Recognising a wrong answer is common content knowledge (CCK), whereas sizing up the nature of an error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialised content knowledge (SCK). In contrast, familiarity with common errors and deciding which of several errors student teachers are most likely to make are examples of knowledge of content and student teachers (KCS)'

(Ball, Thames and Bass, 2008)

Sometimes teachers respond to student errors in ways which undermine students' confidence in their ability to learn mathematics (or even their confidence in themselves). Working with student errors is not a routine activity, it requires not only a knowledge of the mathematics, but also and understanding of how to respond to students' errors in ways that help the student to both learn the correct mathematics and to develop an attitude that they can learn mathematics.

Annex 10: 3.2.1. - Lecture note

Principles of measuring

Before young students learn to measure with standard units and use calibrated instruments, they need a set of skills and knowledge:

- A sense of the size of each unit used and of the sub-units;
- Skills to position the instruments and themselves in relation to the instruments; and
- Knowledge on how to read the instruments, the numbered and unnumbered intervals.

They also need to learn about the principles of measurement. All of the principles of measurement can be learned while measuring informally. If students first work with direct and informal measurement, they will be able to explore principles. Then they can progress to reading standard calibrated instruments.

When we measure, we give a numerical value to a physical attribute or property of an object. We can then compare and order objects based on this numerical value.

Booker, Bond, Briggs and Davies (1998) outline the principles of (indirect) measurement

Always state the unit of measurement when you say 'how many units big' something is.

Never change the unit of measure halfway through measuring something.

Unless you, as an example, are saying that your book is 2 pencils and 2 paper clips long, or that it is 25 cm and 5 mm long.

Comparison of measurements is possible when using the same unit.

It is also possible to compare measurements of different units (say metric, Imperial or Myanmar units) with knowledge of conversion ratio between units.

If a unit is large, the number of that unit will be small in comparison to the number of a smaller unit.

For example, if you measure the width of the classroom will be fewer strides long or more handspans long, because handspans are shorter than strides.

Units are chosen for convenience.

Finding out more about the history of measurement helps students understand this. As students get experience with measurement, they should be encouraged to think about which units are appropriate to measure what.

An understanding of all these principals can be developed with regular practice and discussion of the ideas of measurement.

Stages in learning to measure

Booker, Bond, Briggs and Davies (1998) and Wilson and Rowland (1993) have identified the stages through which young students need to progress in order to grasp measurement. Teachers should consider these stages when introducing students to a new attribute to measure.

Van de Walle, Karp & Bay-Williams (2013), Siemon, Beswick, Brady, Clark, Faragher, Warren (2011) and Outhred, Mictchelmoré, McPhail & Gould (2003) suggest that classroom practice where students make measuring instruments could be considered as another stage.

Knowing what is being measured

Young children take time to understand what they are measuring. They do not initially grasp that length is different to mass. It is even more difficult to understand that longer objects do not necessarily have a greater mass. When students understand the difference between various attributes of an object, they will be able to compare the same attributes of different objects using direct measurement.

Comparison is important.

If students place objects next to or on top of one another, they are able to see which one is longer, shorter, wider, taller, fuller or emptier.

Measuring attributes using informal or non-standard units.

If students can choose an informal unit to measure with, they count how many of the informal units they need. If students have a wide variety of measuring experiences using non-standard units, they will get plenty of opportunities:

- to estimate before they measure;
- to choose units to suit different situations; and
- to understand that levels of accuracy depend on the context.

Measuring attributes using standard units.

Here students need to learn to choose, use and read appropriate instruments. (This includes reading the measurement where interval line is numbered and where it is not numbered.) to estimate measurements before measuring to convert between units

Applications.

Now the young student is able to measure and solve real world problems. They should even be able to develop formulae for calculations.

Intermediate students are able to understand that attributes of objects such as length, weight, volume, time and temperature are different. They also understand how to use formal units to measure these. When designing a lesson for these, students teachers are likely to start at the fourth stage. However, where new concepts are introduced such as angle, area and volume the teacher should still ensure that students understand the concepts (and so may need to go through the earlier stages)

Annex 11: 3.2.3. - Worksheet

Work station 2

At this work station, you will find common household goods with different weights.

1. Pretend each box or container is full. Work as a group.
2. Place the containers in order from the one the lightest to the heaviest.
3. Now divide the items into the following seven groups:
 - a. Items that weigh more than 1 kilogram.
 - b. Items that weigh 1 kilogram.
 - c. Items between a kilogram and 1 kilogram
 - i. How many grams in a kilogram?
 - ii. How many grams in half a kilogram?
 - iii. How many grams in a quarter of a kilogram?
 - d. Items that weigh half a kilogram.
 - e. Items that weigh between a quarter of a kilogram and half a kilogram.
 - f. Items that weigh a quarter of a kilogram.
 - g. Items that had a mass of less than a quarter of a kilogram.

Measurement of weight

Weight	Example 1	Example 2	Example 3
Items that were more than 1 kilogram			
Items that were 1 kilogram			
Items that were between 1 kilogram and $\frac{1}{2}$ a kilogram			
Items that were $\frac{1}{2}$ a kilogram			
Items that were between $\frac{1}{2}$ kilogram and $\frac{1}{4}$ a kilogram			
Items that were $\frac{1}{4}$ a kilogram			
Items that had a mass of less than $\frac{1}{4}$ a kilogram			

Work Station 3

At this work station, you will find different kinds of scales and some part-filled packets and containers of common household goods.

1. Which scales would you use to measure:
 - a. The weight of the rice left in the packet?
 - b. The weight of the sugar left in the packet?
 - c. The weight of the beans left in the packet?
 - d. Your own weight?
 - e. 20 g of something
 - f. 450 g of something
 - g. 2.8 g of something

Fill in your answers in the table below.

Measurement of weight 2

I want to measure	Which analogue mass meter will I use?	The reason why I choose this mass meter
The weight of the rice left in the packet?		
The weight of the sugar left in the packet?		
The weight of the beans left in the packet?		
My own weight		
20 g of something		
450 g of something		
2,8 kg of something		

2. The table below is designed to help you to read analogue scales. Use this flow diagram to read the weight of the sugar, the rice and the beans. Work in pairs. One person reads the instructions, the other answers the questions. Fill in the weight in the table below. Then swap roles.

Table 3.8. Reading analogue scales

	Weight of rice	Weight of beans	Weight of sugar
Estimated weight			
Step 1.			
Step 2.			
Step 3.			
Step 4.			
Step 5.			

Annex 12: 3.2.4. - Worksheet

Work station A

At this work station, you will find containers with different capacities: 1 litre, 500 ml, 330 ml and 200 ml.

1. Find the container that shows, in writing, that it can hold 1 litre. Fill this container with water.
2. Find the container that shows, in writing, it can hold 500 millilitres.
 - a. Estimate how many times greater the capacity of the 1 litre container is than the capacity of the 500 ml container.
 - b. Fill the 500 ml container from the litre container. Pour the water out, and fill it again. Count how many times you can do this before the litre of water is used up.
 - c. Complete:

..... × 500 ml = 1 litre

So 1 litre = ml
3. Find the container that shows, in writing, that it can hold 330 millilitres.
 - a. Estimate how many times greater the capacity of the 1 litre container is than the capacity of the 330 ml container.
 - b. Fill the 330 ml container from the litre container. Pour the water out, and fill it again. Count how many times you can do this before the litre of water is used up.
 - c. Is there any water left over in the litre bottle?
 - d. Does the exercise tend to confirm the results that you got in 2c?
4. Find the container that shows, in writing, that it can hold 200 millilitres.
 - a. Estimate how many times greater the capacity of the 1 litre container is than the capacity of the 200 ml container.
 - b. Fill the 200 ml container from the litre container. Pour the water out, and fill it again. Count how many times you can do this before the litre of water is used up.

c. Complete:

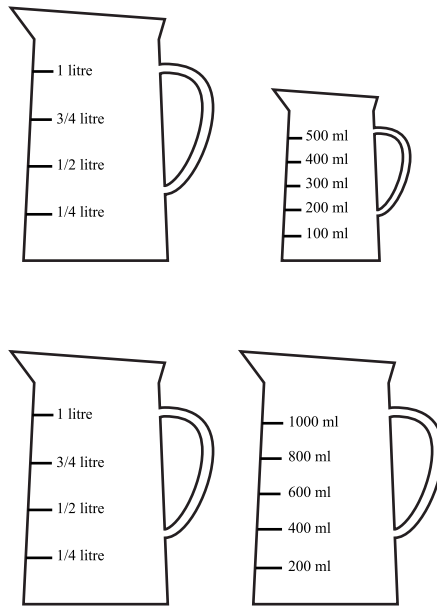
..... \times 200 ml = 1 litre

So 1 litre = ml

Work station B

At this work station, you will find a jug that show litres and a jug that show millilitres.

Figure 12.1.

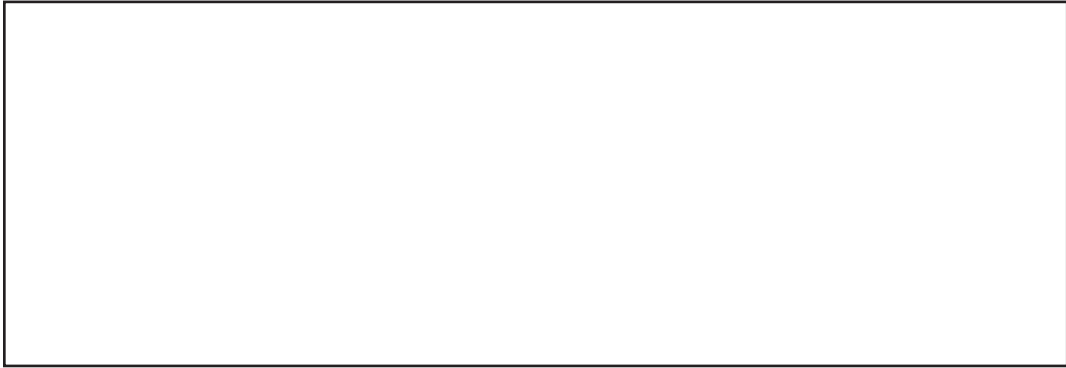


You will also find water and a funnel.

1. Fill the jug that shows litres.
2. Pour water from the litre jug into the jug that shows millilitres.
 - a. Can this jug hold 1 litre of water?
 - i. If so, complete the following: 1 litre = ml
 - ii. If not write down, how many millilitres you have poured into this jug. Then empty the jug and fill it from the litre jug. Repeat this until you have used the full litre of water. Write a number sentence to show how many millilitres equals one litre
 = ml = 1 litre

Reflection on work stations A and B.

1. Which work station, A or B, required more thought? Discuss your reasons.



2. Which work station, A or B, was easier to do? Discuss your reasons.



3. Which work station, A or B, is more likely to help students to remember the relationship between litres and millilitres. Explain your answer.



Work station 2: Sequencing by using the relationship between litres and millilitres

At this work station, you will find common household goods with different capacities.

1. Place the containers in order from the one that can hold the most to the one that can hold the least.
2. Now divide the items into the following seven groups:
 - a. Containers that hold more than 1 litre
 - b. Containers that hold 1 litre
 - c. Containers that hold between $\frac{1}{2}$ a litre and 1 litre.
 - i. How many millilitres in a litre?
 - ii. How many millilitres in half a litre?
 - d. Containers that hold half a litre.
 - e. Containers that hold between a quarter of a litre and half a litre.
 - i. How many millilitres in half a litre?
 - ii. How many millilitres in a quarter of a litre?
 - f. Containers that hold a quarter of a litre.
 - g. Containers that hold less than a quarter of a litre.
 - h. Complete the table below, by filling in at least three examples in each column.

Measuring Capacity (1)

Capacity	Example 1	Example 2	Example 3
Holds more than 1 litre			
Holds 1 litre			
Holds between 1 litre and $\frac{1}{2}$ a litre			
Holds $\frac{1}{2}$ a litre			
Holds between $\frac{1}{2}$ litre and $\frac{1}{4}$ a litre			
Holds $\frac{1}{4}$ a litre			
Holds less than $\frac{1}{4}$ a litre			

Work station 3: Read measuring instruments calibrated in millilitres and litres

At this work station, you will find jugs, measuring cylinders and beakers for measuring capacity / volume.

1. Imagine that you fill each container to its top mark. Do not fill them, just imagine.
 - a. How much would each container hold? Write these amounts in the table below.
2. Now use the flow diagram on the next page to read the volume of liquid in each instrument. Work in pairs. One person reads the instructions, the other answers the questions. Fill in the volume in the table below. Then swap roles.

Measuring Capacity

3. Compare the capacity and volume of liquid in each instrument.

Instrument	Maximum capacity if filled to the top mark
A	
B	
C	
D	
E	
F	
G	

Measuring Capacity (3)

	Instrument A	Instrument B	Instrument C	Instrument D	Instrument E	Instrument F	Instrument G
Step 1.							
Step 2.							
Step 3.							
Step 4.							
Step 5.b.							

Annex 13: 3.2.5. - Worksheet

Myanmar units of length and weight

Consider which of these units are commonly used and where they are most common in Myanmar.

Myanmar units of length

Table of length units				
Unit		Metric	Imperial/US	Ratio to previous
Burmese	Romanised			
ဆံခြည်	sanchi	79.375 μm	3 $\frac{1}{8}$ thou/mil	
နှမ်း	hnan	0.79375 mm	31 $\frac{1}{4}$ thou/mil	10
မုယော	mayaw	4.7625 mm	$\frac{3}{16}$ in	6
လက်သစ်	let thit	1.905 cm	$\frac{3}{4}$ in; one digit	4
မိုက်	maik	15.24 cm	6 in; one shaftment	8
ထွာ	htwa	22.86 cm	9 in; one span	1.5
တောင်	taung	45.72 cm	1 $\frac{1}{2}$ ft; one cubit	2
လံ	lan	1.8288 m	6 ft; one fathom	4
တာ	ta	3.2004 m	10 $\frac{1}{2}$ ft	1.75
ဥသဘ	out-thaba (from Pali usaba)	64.008 m	70 yd	20
ကောသ	kawtha (from Pali kosa)	1.28016 km	0.795455 mi	20
ဂါဂုတ်	ga-wout (from Pali gavuta)	5.12064 km	3.18182 mi; about one league	4
ယူဇနာ	yuzana (from Pali yojana)	20.48256 km	12.7273 mi	4

Myanmar units of weight

Table of length units				
Unit		Metric	Imperial/US	Ratio to previous
Burmese	Romanised			
ရွဲလေး	yway lay	136.078 mg	2.1 grain	
ရွဲကြီး	yway gyi	272.155 mg	4.2 grain	2
ပဲသား	petha	1.02058 g	15.75 grain	3.75
မူးသား	mutha	2.04117 g	31.5 grain	2
မတ်သား	mattha	4.08233 g	63 grain	2
ငါးမူးသား	nga mutha ^[N 1]	8.16466 g	0.288 oz	2
ကျပ်သား	kyattha ^[N 2]	16.3293 g	0.576 oz	2
အဝတ်သား	awettha	204.117 g	7.2 oz	12.5
အစိတ်သား	aseittha	408.233 g	14.4 oz	2
ငါးဆယ်သား	ngase tha	816.466 g	1.8 lb	2
ဝပိသာ	peittha ^[N 3]	1.63293 kg	3.6 lb	2
အချိန်တစ်ရာ	achein taya	163.293 kg	360 lb	100

Annex 14: 3.2.8. - Lecture Summary

Summary of angles

	Amount of turning	Angle name	Size in degrees
	Full turn	Revolution or full angle	360°
	Half turn	Straight angle	180°
	Quarter turn	Right angle	90°
	Less than a quarter turn	Acute angle	Between 0° and 90°
	Between a quarter turn and half a turn	Obtuse angle	Between 90° and 180°
	Between half a turn and a revolution	Reflex angle	Between 180° and 360°

Annex 15: 3.3.1. - Lecture note

The language of mathematics

The language of mathematics is different to language that is used in other contexts.

- Maths has special words. Students need to learn the meaning of these words and the concepts they represent. When teachers are teaching a topic of Mathematics, they should introduce and explain the key words that are to be used and check that students understand the meaning.
- Sometimes there is more than one word to describe something in Mathematics. Students may not be aware that, for example, there are a range of words that mean 'add'. (Sum, compute and so on.) As teachers we are not always aware when we use different words that have the same meaning. We need to clarify this to students.
- Some words have one meaning in a Mathematical context and another in another context. For example in English, the words like difference, face, table, column, fraction, regular, area have an everyday meaning and a Mathematical meaning. Students usually need to have this explained to them.
- Often the way sentences are structured is different to when you speak ordinary English (or another language and when you write a mathematical question.

Primary students are first challenged to problem solve through modelling when confronted with 'story sums' or 'word problems' where they have to use their thinking skills and the mathematical skills they have learnt with numbers or measurement, model through all students need to be helped with the language of 'story sums'.

- All students need to be taught mathematical language: learning new mathematical words; learning that some words have a different meaning in a mathematical context; learning and practicing the way sentences are structured in mathematics.
- All students need to learn to read and need to practice reading.

- Many students need support in learning in a second or even third language.

Students in Myanmar do not speak English as a first language and yet often the language and terms used for mathematics is English.

The language of ‘story sums’ can present a challenge not only to students who are learning in a second or third language. Some students struggle with mathematical language in their home language. Some students struggle to read in any language including their home language.

As teachers we need to think about the language, we use in the Mathematics lessons. We need to be conscious of building students vocabulary around mathematics. We need to help students to understand the structure of sentences in mathematics texts.

Use the internet or a mathematics dictionary if you need to check on the meaning of any word.

Developing mathematical understanding in the early grades

Although the volume of content included in the curriculum increases over the early grades to Grade 3, the topics largely stay the same. This is because the concepts and skills have to develop over time and students do not necessarily have the cognitive ability to move beyond. The real progression takes place by:

- Students’ increased confidence in solving problems, shown by increased willingness to think abstractly, justify their thinking and listen critically to others;
- Students’ use of more advanced strategies when solving problems; and
- Higher and higher number ranges in which students can operate using abstract methods.

An excellent example of how learning takes place in the primary phase is of how number concept develops. Number concept development is a key feature of early Mathematics.

The development of number concept proceeds through four levels after the child has learnt to count. At Level 1, the student cannot use a number abstractly as a number, but has to ‘create’ the number through marks, counters or fingers before it can be used: $5 + 3$ would be calculated as IIIII + III and then counting all the marks. After much counting and solving numerical problems, the student naturally develops.

At Level 2, the student can use numbers abstractly as numbers and can for example count on to solve $5 + 3$: five, six, seven, eight. At this level, they cannot yet break down numbers or devise shortcut strategies.

After more number work, they naturally develop Level 3 understanding. They are now able to change a given problem into smaller, more manageable steps by breaking down and/or changing the numbers, for example, $27 + 35$ may become $20 + 30 \rightarrow 50 + 7 \rightarrow 57 + 5 = 62$. (The arrow has to be used because the = sign would be incorrect in the above sequence.)

At Level 4, students are able to think of a number as ‘so many tens’ and ‘so many units’. Level 4 is therefore a more refined Level 3 and has to be supported by Level 3 understanding. Level 4 understanding is only attained in the next phase. Any attempt to teach Level 4 concepts (hundreds, tens, units) in this phase usually leads to errors and misconceptions and harms students’ development.

The following are typical examples of errors that occur when students have been taught column arithmetic (in these cases vertical addition) before they were at Level 3.

Typical errors in arithmetic

$$\begin{array}{r}
 34 \\
 + 17 \\
 \hline
 15
 \end{array}
 \qquad
 \begin{array}{r}
 164 \\
 \underline{8} \\
 152
 \end{array}
 \qquad
 \begin{array}{r}
 26 \\
 + 37 \\
 \hline
 81
 \end{array}$$

These errors are not due to carelessness. In the first example, the student adds all the digits together. In the second example, the 8 is added to both the tens and the units. In the third example, when the 6 and 7 are added to produce 13, the 1 (‘the small one’) is written down

and the 3 ('the big one') is carried over. In all such cases, the students' explanations of what they are doing echo the teachers' instructions but they do not have the understanding of number which allows them to make sense of the instructions. These errors cannot be remediated by explaining the method or the idea of HTU again — Level 3 understanding (which includes a thorough understanding of place value) develops through engaging in suitable counting and computational activities over a long period of time, and cannot be transmitted from the teacher to the student.

The development process has the following implications:

- The teacher cannot 'teach' number concept; however the teacher can assist students in their development by
 - engaging students in appropriate oral and written activities; and
 - posing word problems so that students are required to use and extend their number knowledge.
- The teacher can confuse and even harm students by trying to teach them concepts and strategies for which they are not ready, for example, 'tens and units' to Level 1 and Level 2 students.
- The teacher should expect students to be at different levels, should welcome and encourage all solution methods. Continued exposure to good activities will mean that students' understanding of number will become increasingly abstract. Students learn about numbers by using numbers.

It is clear that if a teacher does not take the students' levels of number concept development into account, there is a mismatch between what she expects students to do and what students can do. Another mismatch occurs when students try to memorise what the teacher says instead of trying to understand it. This leads to very rigid, inflexible knowledge which is frequently misapplied or which cannot be applied at all, and which is easily forgotten.

On the other hand, when the students are involved in solving problems in context, working with numbers, sorting and building with shapes and objects, comparing and measuring and collecting data, with the teacher there to set up the learning situation, to challenge, ask questions and demand explanations while helping students to express themselves using mathematical language and symbols, then learning with understanding takes place. The concepts acquired in such a way are flexible, adaptable to new situations and can grow.

Students' mathematical development is, therefore, very much dependent on their teacher. They need their teacher:

- To understand and implement the curriculum, because the curriculum is designed to help young students develop a full and flexible understanding of the mathematical concepts and skills appropriate for this phase;
- To take into account students as individuals when designing learning activities, so that both the level and the context of the activity are suitable (for example, does the context of a word problem make sense to the student?); and
- To respect all students' thinking, in the firm belief that given time and encouragement, they will progress.

Problem types for Grades 1 to 3

These are examples of important problem types that the teacher needs to present repeatedly to classes in each grade. The teacher should start with small numbers and increase the number sizes as students' understanding of and familiarity with the problem types grow, and as their number concept develops into the higher number ranges.

The same problem types are therefore used from Grade 1 to Grade 3. The teacher should just keep on making the numbers bigger, and change the contexts to suit the students' knowledge of the world. The problem types should be mixed (addition and subtraction).

Teachers should pose problems orally. When the students can read, provide a written version of the problem as well, but still pose the problem orally. Problems in context can be included in worksheets, but should then be short, straightforward and familiar, and the teacher should ensure that all the students understand them.

Grouping

Grouping, discarding the remainder:

- Hla Hla sells apples in bags of 3 apples each. She has 14 apples. How many bags of 3 apples each can she make up?

Grouping, incorporating the remainder in the answer:

- Kyaw Kyaw wants to take 15 eggs to his grandmother. How many egg boxes that can take 6 eggs each does he need to pack all the eggs?

Sharing

Sharing, discarding the remainder

- Share 14 sweets among 3 friends so that they all get the same number of sweets.

Sharing, leading to fractions

- Share seven chocolate bars among three friends so that they all get the same amount of chocolate bar and there is nothing left over.

Fraction of a collection

- Grandmother gives Aye Aye Ks 1,200. Kiki wants to save a third of the money. How much money must she save?
- This problem type must only be posed **after** students have solved four or five problems of the sharing, leading to fractions type and know the names of fractional pieces.

Putting fractions together

- The netball coach gives half an orange to each player. There are 14 players. How many oranges does she need?
- This problem type must only be posed **after** students have solved four or five problems of the Sharing, leading to fractions type and know the names of fractional pieces.

Proportional sharing

- Cho Cho is smaller than Rhulani. When Cho Cho eats one banana, Rhulani eats two. When Cho Cho eats two bananas, Rhulani eats four. After a few days, they have eaten 12 bananas. How many bananas did Cho Cho eat and how many bananas did Rhulani eat?
- Daw and Maung do a piece of work together. Daw works for three hours and Maung works for one hour. They get paid Ks 6,000. How must they share the money?

Repeated addition

- How many wheels do four bicycles have?

Rate

- Thami drinks three cups of milk every day. How many cups of milk does he drink in a week?

Grids

- Mr Khumalo plants three rows of cabbage plants. There are five plants in a row. How many cabbage plants are there altogether?

Addition and subtraction

There are at least three basic types of addition and subtraction problems and each type can be posed in different ways. The basic types are:

Change

- Thando had five apples. Silo gave her eight apples. How many apples does she have now?
- Thando had 13 apples. She gave five apples to Silo. How many apples does she have now?

Combine

- Sisi has five green and eight blue marbles. How many marbles does she have?
- Sisi has 13 marbles. Five are green and the rest are blue. How many blue marbles does Sisi have?

Compare

- Thabo has 13 bananas. Themba has five bananas. How many more bananas does Thabo have than Themba?

Posing each problem in different ways

Problems have to be posed in different ways. For example, both of these are change problems, but the ‘unknowns’ are in different places in the problem.

- Thandar had some apples. Sithu gave her 8 more apples. Now she has 13 apples. How many apples did Thandar have in the beginning?
- Thandar had 5 apples. Sithu gave her some apples. She now has 13 apples. How many apples did Sithu give her?

Learning activity

Problem situations with different functional relationships

Heila sells meals for Ks 1,200 each. Make a table to help her find the amount for large orders.

Number of meals	1	2	3	4	5	10	20
Cost in Ks							

Use the table to find the cost of 7 meals and 23 meals.

Aung works part-time looking after children. He charges Ks 20,000 for travel costs, and then Ks 500 per hour for the care. Complete this table for him.

Number of meals	1	2	3	4	5	10	20
Cost in Ks	2,500						

Note that Heila's problem and Aung's problem work differently.

The above problem types are given to guide the teacher. Students should not be burdened with the names.

Note that students often use different ways of solving a problem that may not be what the teacher expects. For example, a division problem may be solved by repeated subtraction, addition, or multiplication. Students' methods will change in the course of the year as their understanding of and familiarity with the problem types grow, and as their number concept develops.

Annex 16: 3.3.2. - Lecture note

Categorising problems

The following table outlines the categories of problems that students up to Grade 6 should be able to solve.

Categorizing problems

Problem type	Additional notes	Examples
Summation	A sum	A man buys a specific brand of smart phones for all his stores. He buys the following colors: 126,789 black, 341,567 white and 344,532 silver. How many smart phones players did he buy altogether?
	Missing part of a given sum	Farm workers picked 342,345 pears in the morning. After lunch they picked some more. By the end of the day, they had 866,589 pears. How many pears did they pick after lunch?
Increase and decrease	Calculate the result	The price for some bags of rice is Ks 268,231. Since water leaked into some of the containers, the price is decreased by Ks43,789. What price does a shop owner pay for the rice?
	Calculate the change	A clothing maker earned Ks864,328 during November. During December, the amount decreased to Ks 367,435. How much less money did the tailor make during December than in November?
	Calculate the initial result	A farmer struggles to sell his chickens. He decreases the original price by Ks 10,456. He sells his chickens for Ks 985,787. What was the original price that the farmer wanted for his chickens?

Problem type	Additional notes	Examples
Grouping	<ul style="list-style-type: none"> Grouping problems that are solved with division and/or repeated subtraction. Answers to problems have or do not have remainders 	A rich man gives boxes of balls to a school. Each box has 126 toys. If there are 5,375 toys, how many boxes are needed?
	<ul style="list-style-type: none"> Grouping problems that are solved with multiplication and/or repeated addition. Answers to problems have or do not have remainders 	This year a company gave 523 boxes of balls to children. Each box had 3,126 balls. How many balls did the company give away?
	<ul style="list-style-type: none"> Grouping problems in an array form These problems can be solved with division (or repeated subtraction) or multiplication (repeated addition) 	A farmer wants to plant 6,708 apple trees. He can only make 156 rows to plant these apple trees. How many apple trees must he plant in each row?
Sharing	<ul style="list-style-type: none"> Sharing problems can be solved with division/repeated subtraction Smaller groups of equal size are formed from a given amount. Answers to calculations that have remainders can lead to the concept of fractions (common or decimal fractions). See Grade 4 example. 	A man owns 346 shops. He bought 8,654 radios on sale and shares them equally between these shops. How many radios does each shop get?
Comparison by difference		Thombi spent Ks175,322. Ziggi spent Ks25,789 more than Thombi. How much money did Ziggi spent?
Treating groups as units		Houses in a town need new toilets. 123 toilets will cost the municipality Ks4,132. How much will 17,835 new toilets cost?
Rate	Students calculate the total if given rate per object	A second hand MP3 player cost Ks14,500. How much will 3,445 MP3 players cost?
	Students calculate the rate per object	156 pairs of shoes cost Ks702,000. How much will one pair of shoes cost?
	Students first calculate the rate and then apply it to generate more information	If 12 chairs cost Ks280,800, how much will 2,567 chairs cost?
Comparison by ratio		Zwi collected 132 bottles for recycling. Her friend collected $\frac{5}{6}$ of this number of bottles. How many bottles did the friend collect?

Problem type	Additional notes	Examples
Proportional sharing		Denozo works for 8 days and Chino works for 7 days at a building site. Together they are paid Ks678,000. How should the money be shared fairly between the two to show numbers of days worked?
Meaning of the fraction		Grade 6
Part of a whole where the whole is a single object		Susan eats one half of a chocolate bar. The remainder is equally divided between two friends. How much does each one get? Show your answer with a drawing.
Part of a whole where the whole is a collections of objects		During the holidays, Avril spends $\frac{1}{3}$ of his day watching TV and $\frac{1}{4}$ of his day sleeping. How many hours of a day is left over?
Relationship		The son earns $\frac{20}{100}$ of what his father earns per month. If his father earns Ks18,000 per month, how much does the son earn?
Ratio		$\frac{2}{5}$ cups of milk is needed to make 40 biscuits. How many cups are needed for 2,000 biscuits? Or: Sue has 10 litres of milk. Is this enough milk to make 2,000 biscuits?
Comparator		What is the longest? $\frac{60}{100}$ metres or $\frac{7}{10}$ metres of a strip of material?
Unit of measurement		Zin Zin has $2\frac{2}{10}$ metres of rope. How many baskets can she make with $28\frac{1}{5}$ metre of rope?
Number		On a number line, show the following numbers: 0.1 ; $\frac{8}{10}$; $\frac{2}{5}$; $1\frac{40}{100}$
Fractional parts put together to make a whole (iterative)		On a sports day, 500 children get: $\frac{20}{100}$ of a bottle of drink and $\frac{4}{10}$ of a bar of chocolate. How many bottles and chocolate bars are needed to serve all the children?
Operator		Calculate: $\frac{2}{3} \times 336 = ?$

Annex 17: 3.3.3. - Lecture note

What is problem-solving?

'For a question to be a problem, it must present a challenge that cannot be resolved by some routine procedure. Problem-solving is the process of accepting a challenge and striving to resolve it'.¹⁵

Problem-solving is not just about numbers. Students need to solve problems in every Content Area. Problem-solving is not only doing 'story sums'. Some 'story sums' do require problem-solving, but others can be solved by routine calculations or by applying routine procedures. This means that not all contextualised calculations require problem-solving, only those to which there is no immediate answer or no immediately obvious way to find the answer. Problem-solving requires higher order thinking skills. Regular problem-solving develops higher order thinking skills.

Which questions require problem-solving?

As students move through school, they will learn more and more ways to calculate, their mathematical thinking skills will change and their abilities to check the reasonableness of their answers should improve. This means that some questions require problem-solving in the early grades, but become a routine calculation in higher grades. It is not possible to place all questions into two groups: routine and problem-solving. A routine question at one level may require problem-solving in earlier years.

For example:

Thura and Thiri are carrying 1 kg packets of rice. Thiri says

'If I gave you one of my packets, we would be carrying the same weight of rice. If you gave me one of your packets, I would be carrying twice the weight that you are carrying.'

How many kilograms of rice is each student carrying?

¹⁵ Polya, George (1990) *How to Solve It*. Melbourne: Penguin cited in Booker, G., Bond, D., Briggs, J., & Davey, G. (1998) *Teaching Primary Mathematics*. Melbourne: Longman.

Senior students have the tools to use algebra to solve this problem. This makes the problem routine. Let the weight of Thura's packets be x kg and the weight of Sedick's packets be y kg.

$$(x - 1) \text{ kg} = (y + 1) \text{ kg.}$$

$$(x - 1) = (y + 1)$$

$$x = y + 2$$

$$(x + 1) \text{ kg} = 2(y - 1) \text{ kg.}$$

$$x + 1 = 2y - 2$$

Substitute the solution to the 1st equation ($x = y + 2$) into this 2nd equation

$$(y + 2) + 1 = 2y - 2$$

$$y + 3 = 2y - 2$$

$$5 = y$$

Substitute the value for y , in other word, $y = 5$, into the 1st equation to get the value for x

$$x = y + 2 = 5 + 2 = 7$$

The mass of Thura's packets is 7 kg.

The mass of Thiri's packets is 5 kg.

Check your answer by putting these values into the original statements to see whether your answers are correct.

But Intermediate Phase students do not yet have the mathematical tools for this to be a routine problem. It remains a non-routine challenge. It requires problem-solving for them.

Helping students to work through non-routine problems

The more students practice working with non-routine problems, the more comfortable they will become with non-routine problems. We can help students to learn certain problem-solving strategies. People express these strategies in different ways.

Remember that:

- Understanding the problem can take time. This is not only about understanding the words, but making sure students know what is being asked.
- Students need to feel comfortable with making mistakes. Mistakes are an opportunity to learn. They can learn to ask 'why is that wrong?' or 'where did I make a mistake?'

Students will sometimes start with a strategy that does not help them. This does not mean that they 'got it wrong, or failed'. Learning is about being comfortable with the process of trying. Learning is not about always getting the answer correct the first time you try to solve a problem.

Finding out from other people 'How did you get that?' can help learning a lot. Discussing why some ideas worked, and why some did not, can also lead to learning.

The diagram that follows provides some ideas of questions to ask and procedures to explain to students to help them with non-routine problems.

Annex 18: 3.4.1. - Lecture note

Maths in life

We use our mathematical skills in all kinds of situations in life. We use our skills to think through problems, to represent data and to make calculations and choices. It is important that students understand this and are able to use mathematical skills outside the classroom to do calculations and solve problems. In the following three lessons, you will be provided with examples of lesson designs that give your students an opportunity to do this.

So that these kinds of lessons are most effective, the scenarios that are presented should be as realistic as possible. They should also provide maximum opportunity to practice mathematical thinking and operations. This means that the ideas presented in these scenarios need to be adapted to best fit the local situation of the school where you will be teaching.

You will be given a range of different scenarios.

Agriculture and food project scenarios



Learning activity 1

Your school has set your class some farming and food projects to work on as assignments. The projects are designed to raise funds for the school. There are awards for the groups that manage the most successful projects. Your school will provide the resources and money to set up these projects. But your group has to plan the project thoroughly and present a proposal and budget to be approved before you can start.

Group A

Group A has been offered 15 laying hens to start an egg production project.

Chickens require about $\frac{1}{3}$ square metre per chicken inside the chicken coop and 1 square metre per chicken in an outside run. Limiting space for a flock of chickens can cause stress, cannibalism, pecking, and sometimes even death.

Source: From Dummies.com. Converted to metric

You have to build nests, a chicken house (coop) and enclose the area for the chicken run.

You should design the chicken coop and run and develop a scaled drawing of this design. You should decide what materials you need to build the run and, based on the dimensions, work out how much wood, roofing and fencing and other equipment you would need.

Each chicken will require around 100gm of feed a day. The prices for chicken feed are fluctuating and currently a 25kg bags sells for Ks12,000.

You should calculate how much food you need to buy every month and what your monthly food costs will be for the 15 chickens.

If healthy and well-fed each chicken will produce 320 eggs a year.

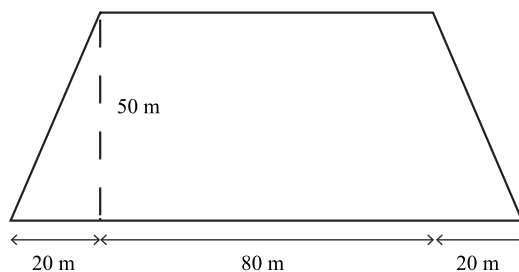
At current prices eggs sell on average for Ks2,600 for 12 in the urban areas.

How many eggs will the chickens produce in a week and in a month on average? What will the weekly income be for eggs if you sell them all? What will the monthly income be?

Group B

Group B has been offered a piece of land and seeds for growing rice.

Below is a diagram of the land available and its dimensions.



What is the area of the land available to produce rice? What length of fencing are you going to have to construct to keep animals out of the field? Present all your calculations clearly on a flipchart.

Farmers in Myanmar produced on average 3.89 tonnes of rice per hectare in 2014.

Using this average, calculate the yield group B is likely to produce with one crop of rice. Present each step of your calculations here with an explanation to justify or explain.

Rice is sold in 49 kg bags. The price for excellent quality rice that you expect is Ks52,000 per bag

Approximately how many bags of rice will you produce from your field? What is your expected income?

What are the costs you will face in producing this rice? What about labour costs? How much will Group B be able to contribute to school funds? Present each step of your calculations here with an explanation to justify or explain.

Adapting the scenarios

In your groups, consider the scenario you have been given. Imagine the primary school you attended and the situation and resources there. How would you adapt this scenario to make it as realistic as possible for this context? Are there any further questions you would ask? Could you ask questions that involves further mathematical thinking or operation.

Annex 19: 3.4.2. - Lecture note

Planning a class trip scenario

Scenario

Your class has a 'music group'. The group made a short video of the group performing and sent it off to enter the local rounds of a national competition. They were selected!

Now your class has been invited to a town 300km away to participate in the National Schools Music Competition to take place during the school holidays. Your class music group will be one of the teams competing to go through to the next round.

The trip will take two days during the school holidays. It will take around five hours of traveling during the first day. The competition performances will take place on the evening of the first day and your class will sleep over for the night. On the second day, your class will return back home on the buses. A local business is able to hire two small 16-people buses for Ks120,000 per day. This cost includes the cost of the driver. An additional fee of Ks 150/km for petrol is also charged.

Your class will sleep in accommodation arranged by the organisers of the competition. You will have to bring and prepare food for the class. There will be facilities to cook food made available but you will have to prepare the food yourselves.

The organisers of the competition will be sponsoring the costs for food, transport and costumes for the music group. Your class has to present them with a budget and some other details to secure the money for this sponsorship.

Assignment 1: Information about the trip

This assignment requires the group to develop a schedule for the class trip so that all the students (and their parents) know what the programme entails. This schedule should be presented in a table of times and activities.

A detailed letter of information containing all the relevant facts (including distances, times and arrangements) must be sent to all the parents. You have to draft this letter.

You need to develop a budget of the costs of transport.

Assignment 2: The catering

The catering group has to plan and budget for three meals and some snacks for 30 students and two teachers over the two days. The sponsors have requested you send them a menu for the meals and a budget with detailed costs. Remember to plan for all the ingredients you will need.

Assignment 3: The outfits

There are nine members of the music group.

You have to design the outfits and develop a rough pattern. You have to calculate how much fabric you need. The fabric ranges between Ks 800/metre and Ks 5,000/metre. You have to purchase the materials for the outfits and commission a tailor/seamstress to make these. The local clothing makers have quoted you Ks 18,000 to make each outfit. Develop a detailed budget for the sponsors.

Annex 20: 3.4.3. – Lecture note

Scenario

This material in this lesson is pitched at teaching in the higher grades of primary. Read through this scenario and answer the questions in your groups.

The seasons, climate, temperature and rainfall in Myanmar

Use the internet to collect the data you require for this assignment. (Ask good questions on a search engine.) Also look at the way this data is presented in the sites that you are referred to during your internet search.

To represent the data to your students use bar graphs, pie charts or line graphs. Use maps with different shadings and colours.

Rainfall

What are the rainfall patterns across the year and in different zones in Myanmar? How are these patterns determined and measured?

What is a good way to present this data in a graph and on a map (or both)? Develop a graphic poster representing rainfall patterns in Myanmar through the year and in different zones.

Temperature

What are the temperature patterns across the year and in different zones in Myanmar? How are these patterns determined and measured?

What is a good way to present this data in a graph and on a map? Develop a graphic poster representing temperature patterns through the year and in different zones in Myanmar.

The seasons

Describe the seasons in Myanmar? Are there different seasons in different areas? What is the best way to represent these? Use the data you have collected.

Climate zones

Does the data you collected suggest that there are different climate zones in Myanmar? What are these zones? What are their characteristics? How would you present these graphically to primary school students?

What are the characteristics of good quality classroom posters?

Discuss this question before you proceed to develop your classroom presentation of graphs and maps of the climate of Myanmar.

Notes

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Notes

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Notes

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The Government of the Republic of the Union of Myanmar
Ministry of Education